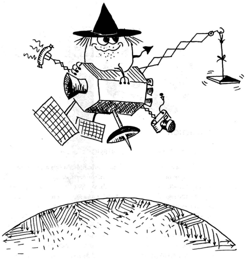
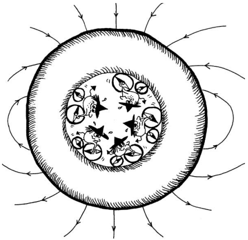
Conductivity Distribution of the Earth Derived from Long Period Magnetic Data

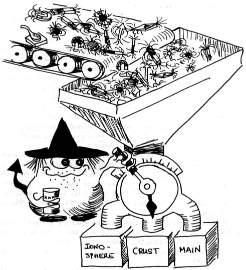
Nils Olsen

Neustadt/Weinstrasse, 27^{th} June 2009

DTU Space
National Space Institute







Long Period Transfer Functions Derived from Magnetic Data

Nils Olsen

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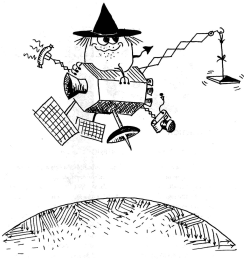

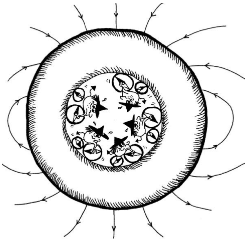

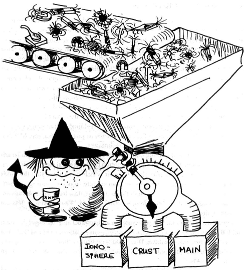

Outline of Talk

1. Transfer functions
2. Source Fields
3. Transfer functions determinations before 1985
4. Transfer functions determinations 1985 - 1999
5. Latest attempts and new concepts

Outline of Talk

Response functions for induction studies

Spherical Harmonic domain

$$Q_n(\omega) = \frac{\iota_n^m(\omega)}{\epsilon_n^m(\omega)}$$

Spatial domain

$$C(\omega) = \frac{B_z(\omega)}{\nabla_H \cdot \mathbf{B}_H(\omega)}$$

ϵ_n^m and ι_n^m are spherical harmonic expansion coefficients of external (inducing), resp. internal (induced), sources

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Response functions for induction studies

Spherical Harmonic domain

$$Q_n(\omega) = \frac{\iota_n^m(\omega)}{\epsilon_n^m(\omega)}$$

$$Q_n = \frac{n}{n+1} \frac{1 - \frac{n+1}{a} C_n}{1 + \frac{n}{a} C_n}$$

Spatial domain

$$C(\omega) = \frac{B_z(\omega)}{\nabla_H \cdot \mathbf{B}_H(\omega)}$$

$$C_n = \frac{a}{n+1} \frac{1 - \frac{n+1}{n} Q_n}{1 + Q_n}$$

Earth's radius $r = a$

ϵ_n^m and ι_n^m are spherical harmonic expansion coefficients of external (inducing), resp. internal (induced), sources

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These responses are only valid for 1D Earth, $|\nabla_H \sigma| \ll \sigma/|C|$

Global vs. regional sounding

- Spherical Harmonic Domain (global) sounding:
  Spherical Harmonic analysis of B_x, B_y, B_z yields $\epsilon_n^m(\omega), \iota_n^m(\omega)$

$$Q_n(\omega) = \frac{\iota_n^m(\omega)}{\epsilon_n^m(\omega)}$$

- Spatial domain (regional) sounding:

$$C(\omega) = \frac{B_z(\omega)}{\nabla_H \cdot \mathbf{B}_H(\omega)}$$

Global vs. regional sounding

- Spherical Harmonic Domain (global) sounding:
  Spherical Harmonic analysis of B_x, B_y, B_z yields $\epsilon_n^m(\omega), \iota_n^m(\omega)$

$$Q_n(\omega) = \frac{\iota_n^m(\omega)}{\epsilon_n^m(\omega)}$$

  Both horizontal and vertical components are used *globally*

- Spatial domain (regional) sounding:

$$C(\omega) = \frac{B_z(\omega)}{\nabla_H \cdot \mathbf{B}_H(\omega)}$$

  Vertical component is used *locally*, horizontal components are used *regionally*

Two methods for determining $\nabla_H \cdot \mathbf{B}_H(\omega)$

- *Z/H*:
  Assumption about source structure,
  e.g. P_1^0 for RC or P_{m+1}^m for mth daily harmonic (Sq)

- Gradient sounding:
  Estimation of source structure from observed $\mathbf{B}_H = (B_x, B_y)$
  - (regional) polynomial fit yields

$$\mathcal{G} = \nabla_H \cdot \mathbf{B}_H$$

  - $Z : \mathcal{Y}$
    (global) spherical harmonic fit yields $v_n^m = \epsilon_n^m + \iota_n^m$

$$\mathcal{Y} = \nabla_H \cdot \mathbf{B}_H = \sum_{n,m} n(n+1) v_n^m P_n^m e^{im\phi}$$

Two methods for determining $\nabla_H \cdot \mathbf{B}_H(\omega)$

- *Z/H*:
  Assumption about source structure,
  e.g. P_1^0 for RC or P_{m+1}^m for mth daily harmonic (Sq)
  allows determination of response from data of a *single* site
- Gradient sounding:
  Estimation of source structure from observed $\mathbf{B}_H = (B_x, B_y)$
  - (regional) polynomial fit yields

$$\mathcal{G} = \nabla_H \cdot \mathbf{B}_H$$

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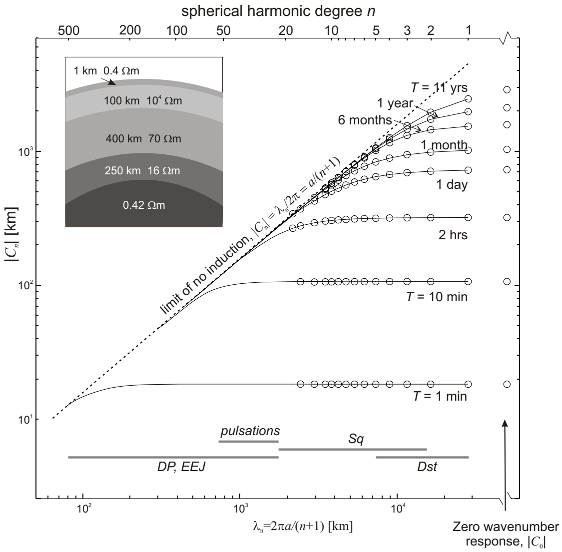
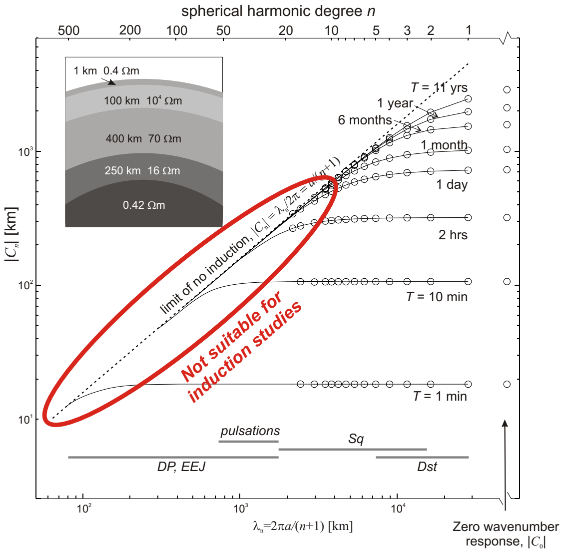
Vertical component is used *locally*, horizontal components are used *regionally*

The C-Response



The *C*-Response



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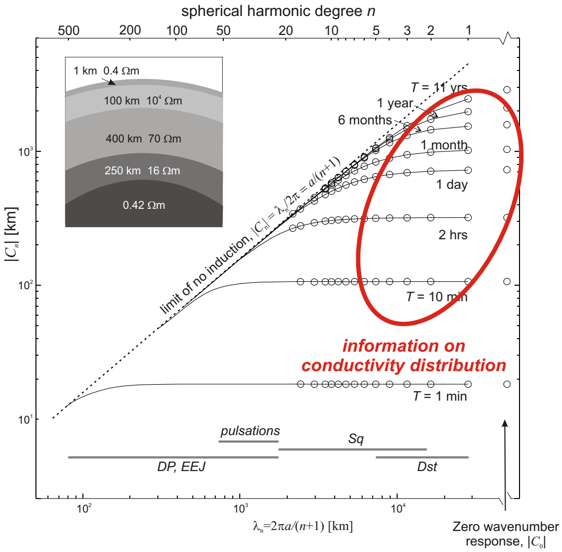
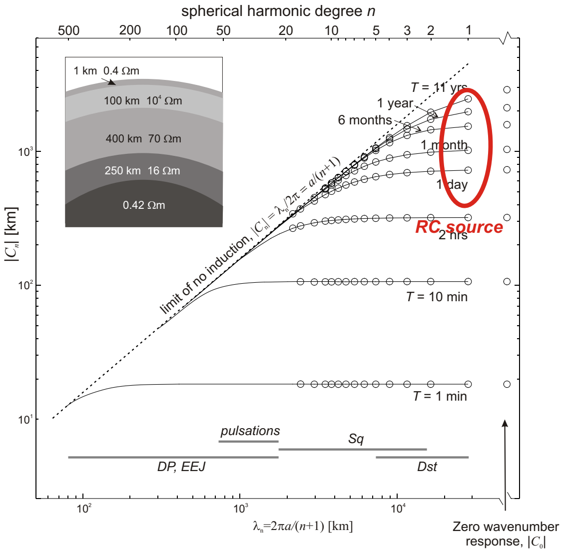
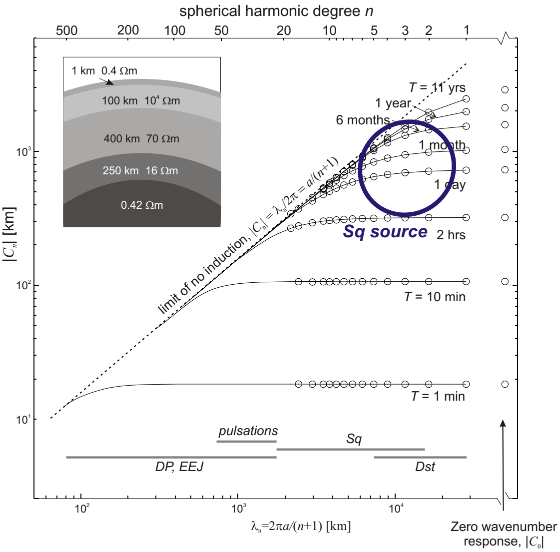


The *C*-Response

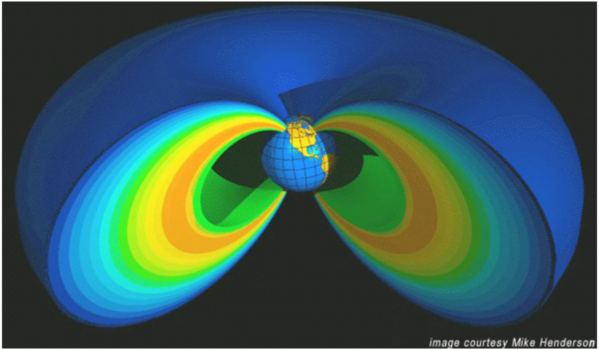


The C-Response



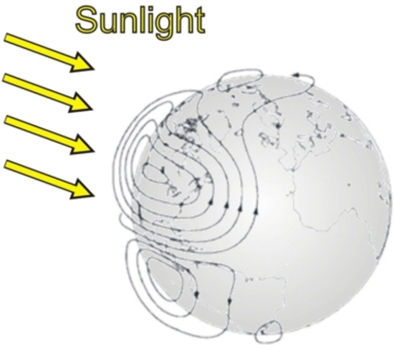
Outline of Talk

RC, the magnetospheric ring-current



- Source structure is mainly given by $P_1^0 = \cos\theta_d$ (θ_d is dipole co-latitude)
- Period range between a few hours and 100 days
- Depth range 300 km to 1500 km
- Can be used for induction studies with satellite data (source is external to observation point!)

Sq, the ionospheric regular daily variation



- Source structure of mth daily harmonic ($m = 1, 2 \ldots 6$) dominated by $P_{m+1}^{m} e^{imT}$ with local time T
- Period range between 4 hrs (for $m = 6$) and 24 hours ($m = 1$)
- Depth range 300 to 600 km

Outline of Talk

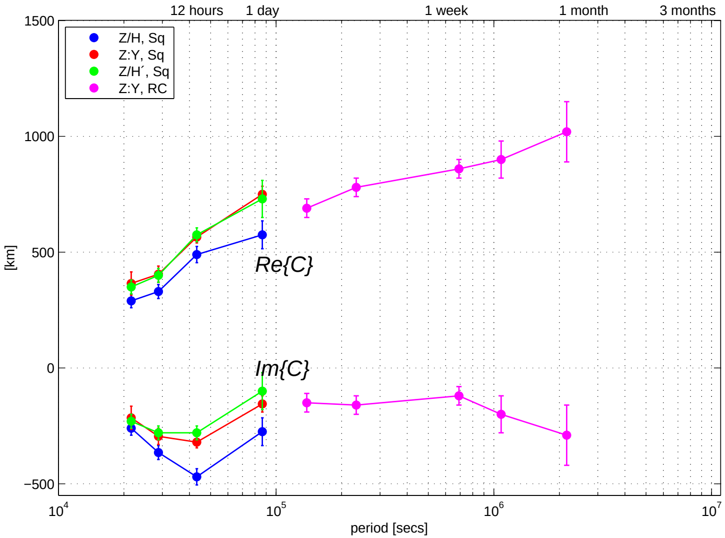
Transfer functions determinations before 1985

Important publications by Ulrich Schmucker:

- *Erdmagnetische Variationen und die elektrische Leitfähigkeit in tieferen Schichten der Erde*, Sitzungsbericht und Mitteilungen Braunschweigische Wiss. Gesellschaft, Sonderheft, Band **4**, p. 45-102, 1979
- *Magnetic and electric fields due to electromagnetic induction by external sources*, Landolt-Börnstein, Springer Verlag, 1985

Development of $Z : \mathcal{Y}$ method (spherical version of gradient method)
Application of the various methods to observatory hourly mean values

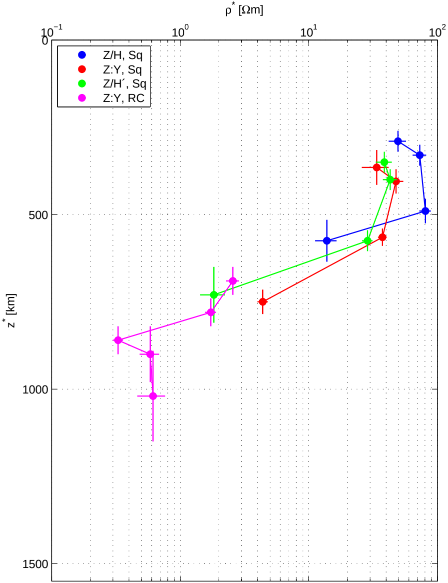
Ulrich Schmucker's Responses as of 1985



Ulrich Schmucker's Responses as of 1985

$$\rho^\star(\omega) = 2\mu_0\omega \, \mathrm{Im}\{C(\omega)\}^2$$
$$z^\star(\omega) = \mathrm{Re}\{C(\omega)\}$$

after U. Schmucker, *Landolt-Börnstein*, 1985



Outline of Talk

Transfer functions determinations 1985 - 1999

- Data mining: recovery of “old” data sets
  - Hourly mean values of IGY/C (“Chapman-Gupta collection”)
  - Throughout quality check of data from 100-130 observatories from 1957.5-1960.0 (IGY/C), 1964-65 (IQSY), and 1979-80 (IMS)
  - These hourly mean values became basis for digital data collection at WDC-C (Copenhagen, now Edinburgh)

Transfer functions determinations 1985 - 1999

- Data mining: recovery of "old" data sets
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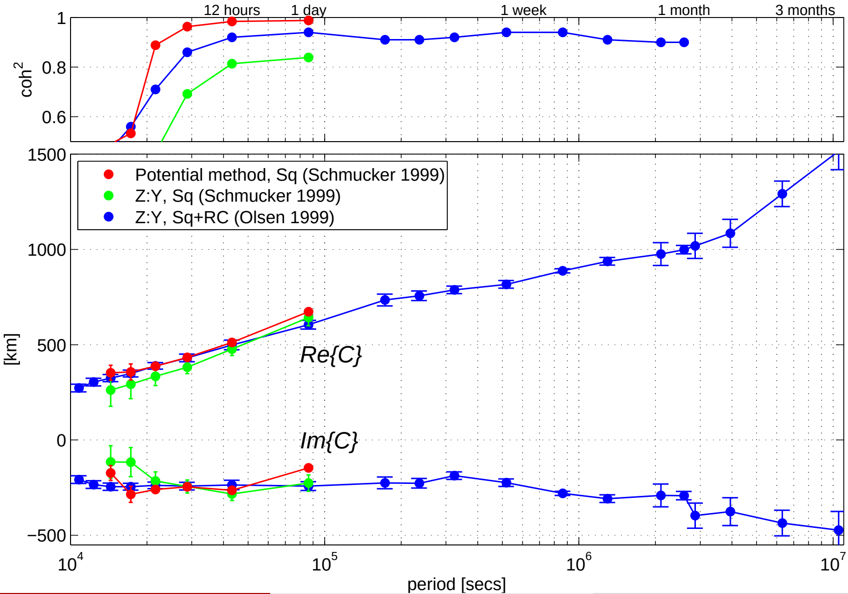
  - These hourly mean values became basis for digital data collection at WDC-C (Copenhagen, now Edinburgh)
- Accounting for day-to-day variability when estimating transfer functions
  - *Sq* analysis using potential and $Z : \mathcal{Y}$ method (Schmucker 1999)
  - *Sq* and *RC* analysis using $Z : \mathcal{Y}$-method (Olsen 1992, 1998, 1999)

Responses as of 1999



Responses as of 1999

$$\rho^\star(\omega) = 2\mu_0\omega \, \mathrm{Im}\{C(\omega)\}^2$$
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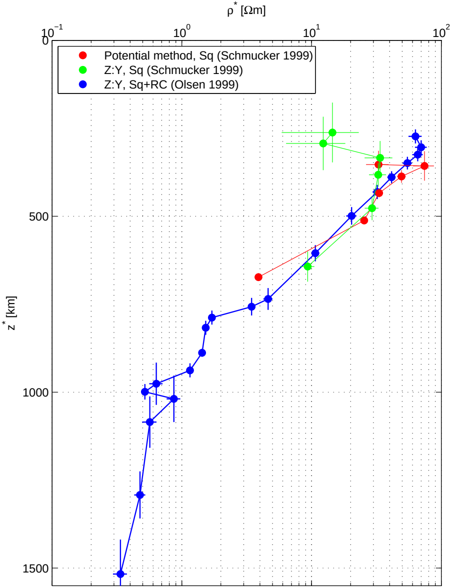

Outline of Talk

Two incompatible approaches

- *Z/H* method, assumption about source structure (here: $P_1^0 = \cos\theta$)

$$\begin{aligned} B_z(\omega) &= +C(\omega) \cdot \mathcal{G}(\omega) \\ &= -C(\omega) \frac{a \tan\theta}{2} B_x(\omega) \end{aligned}$$

  with $\mathcal{G} = \nabla_H \cdot \mathbf{B}_H = -\frac{a \tan\theta}{2} B_x$
  Assumption: 1D Earth (no lateral variation of conductivity)

- Induction arrows, to detect lateral conductivity variations

$$B_z(\omega) = T_x(\omega) \cdot B_x(\omega) + T_y(\omega) \cdot B_y(\omega)$$

  Assumption: homogeneous source ($|\mathbf{k}|, n \to 0$)

Two incompatible approaches

- Z/H method, assumption about source structure (here: $P_1^0 = \cos\theta$)

$$
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B_z(\omega) &= +C(\omega)\cdot\mathcal{G}(\omega) \\
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\end{aligned}
$$

  with $\mathcal{G} = \nabla_H \cdot \mathbf{B}_H = -\frac{a\tan\theta}{2}B_x$
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$$B_z(\omega) = T_x(\omega)\cdot B_x(\omega) + T_y(\omega)\cdot B_y(\omega)$$

  Assumption: homogeneous source ($|\mathbf{k}|, n \to 0$)

In both approaches $B_z \propto B_x$
Interpretation depends on chosen assumptions

How to make them compatible ...

Observed vertical magnetic field $B_z = B_{nz} + B_{az}$ consists of normal part B_{nz} and anomalous part B_{az}

$$\begin{aligned} B_{nz} &= C \cdot \mathcal{G}, \quad \mathcal{G} = \nabla_H \cdot \mathbf{B}_{nH} \\ B_{az} &= T_x \cdot B_{nx} + T_y \cdot B_{ny} \\ B_z = B_{nz} + B_{az} &= C \cdot \mathcal{G} + T_x \cdot B_{nx} + T_y \cdot B_{ny} \end{aligned}$$

U. Schmucker, *Ein Kontinent erwacht*, EMTF 2005

Generalized Gradient Sounding 1/3

U. Schmucker: Electromagnetic induction studies with long-periodic geomagnetic variations in Europe – 1. Methods, unpublished manuscript, 2004

$$\begin{aligned} \mathbf{E}_H &= \underline{\underline{Z}}\mathbf{B}_{nH} \\ [\nabla \times \mathbf{E}]_z &= -i\omega B_z \end{aligned}$$

Lateral variation of conductivity:

$$B_z = C_{xx}\frac{\partial B_{nx}}{\partial y} + \frac{\partial C_{xx}}{\partial y} B_{nx} + \dots$$

with C-response tensor $\underline{\underline{C}} = i\omega\underline{\underline{Z}}$
Eight terms on right side

Generalized Gradient Sounding 2/3

Using $\nabla \times \mathbf{B} = 0$ and rearranging reduces this to five terms:

$$B_z = C_1 \left(\frac{\partial B_{nx}}{\partial x} + \frac{\partial B_{ny}}{\partial y} \right) + C_2 \left(\frac{\partial B_{ny}}{\partial y} - \frac{\partial B_{nx}}{\partial x} \right) + C_3 \frac{\partial B_{nx}}{\partial y} + T_x B_{nx} + T_y B_{ny}$$

with

$$\begin{aligned} C_1 &= \frac{C_{xy} - C_{yx}}{2} \\ C_2 &= \frac{C_{xy} + C_{yx}}{2} \\ C_3 &= \frac{C_{xx} - C_{yy}}{2} \\ T_x &= \frac{\partial C_{xx}}{\partial y} - \frac{\partial C_{yx}}{\partial x} \\ T_y &= \frac{\partial C_{xy}}{\partial y} - \frac{\partial C_{yy}}{\partial x} \end{aligned}$$

Generalized Gradient Sounding 2/3

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with

$$\begin{aligned}
C_1 &= \frac{C_{xy} - C_{yx}}{2} \\
C_2 &= \frac{C_{xy} + C_{yx}}{2} \\
C_3 &= \frac{C_{xx} - C_{yy}}{2} \\
T_x &= \frac{\partial C_{xx}}{\partial y} - \frac{\partial C_{yx}}{\partial x} \\
T_y &= \frac{\partial C_{xy}}{\partial y} - \frac{\partial C_{yy}}{\partial x}
\end{aligned}$$

5 transfer functions, collected in $\underline{C}$ and $\underline{T}$

Generalized Gradient Sounding 3/3

- $B_z = C_0 \cdot \mathcal{G}$
  - univariate
  - corresponds to 1D isotropic conductivity

$$\mathcal{G} = \mathcal{G}_1 \quad = \quad \left(\frac{\partial B_{nx}}{\partial x} + \frac{\partial B_{ny}}{\partial y} \right)$$

Generalized Gradient Sounding 3/3

- $B_z = C_0 \cdot \mathcal{G}$
  - univariate
  - corresponds to 1D isotropic conductivity

- $B_z = C_1 \cdot \mathcal{G}_1 + C_2 \cdot \mathcal{G}_2 + C_3 \cdot \mathcal{G}_3 + T_x \cdot B_{nx} + T_y \cdot B_{ny}$
  - 5-variate
  - general 3D anisotropic conductivity

$$\mathcal{G} = \mathcal{G}_1 = \left(\frac{\partial B_{nx}}{\partial x} + \frac{\partial B_{ny}}{\partial y} \right)$$

$$\mathcal{G}_2 = \left(\frac{\partial B_{nx}}{\partial x} - \frac{\partial B_{ny}}{\partial y} \right)$$

$$\mathcal{G}_3 = \frac{\partial B_{nx}}{\partial y}$$

Generalized Gradient Sounding 3/3

- $B_z = C_0 \cdot \mathcal{G}$
  - univariate
  - corresponds to 1D isotropic conductivity
- $B_z = C_1 \cdot \mathcal{G}_1 + C_2 \cdot \mathcal{G}_2 + C_3 \cdot \mathcal{G}_3$
  - tri-variate
  - corresponds to 1D anisotropic conductivity
- $B_z = C_1 \cdot \mathcal{G}_1 + C_2 \cdot \mathcal{G}_2 + C_3 \cdot \mathcal{G}_3 + T_x \cdot B_{nx} + T_y \cdot B_{ny}$
  - 5-variate
  - general 3D anisotropic conductivity

$$\begin{aligned}
\mathcal{G} = \mathcal{G}_1 &= \left(\frac{\partial B_{nx}}{\partial x} + \frac{\partial B_{ny}}{\partial y} \right) \\
\mathcal{G}_2 &= \left(\frac{\partial B_{nx}}{\partial x} - \frac{\partial B_{ny}}{\partial y} \right) \\
\mathcal{G}_3 &= \frac{\partial B_{nx}}{\partial y}
\end{aligned}$$

Generalized Gradient Sounding 3/3

- $B_z = C_0 \cdot \mathcal{G}$
  - univariate
  - corresponds to 1D isotropic conductivity
- $B_z = C_1 \cdot \mathcal{G}_1 + C_2 \cdot \mathcal{G}_2 + C_3 \cdot \mathcal{G}_3$
  - tri-variate
  - corresponds to 1D anisotropic conductivity
- $B_z = C_0 \cdot \mathcal{G} + T_x \cdot B_{nx} + T_y \cdot B_{ny}$
  - tri-variate
  - corresponds to "moderate deviation" from 1D conductivity
- $B_z = C_1 \cdot \mathcal{G}_1 + C_2 \cdot \mathcal{G}_2 + C_3 \cdot \mathcal{G}_3 + T_x \cdot B_{nx} + T_y \cdot B_{ny}$
  - 5-variate
  - general 3D anisotropic conductivity

$$\begin{aligned} \mathcal{G} = \mathcal{G}_1 &= \left(\frac{\partial B_{nx}}{\partial x} + \frac{\partial B_{ny}}{\partial y} \right) \\ \mathcal{G}_2 &= \left(\frac{\partial B_{nx}}{\partial x} - \frac{\partial B_{ny}}{\partial y} \right) \\ \mathcal{G}_3 &= \frac{\partial B_{nx}}{\partial y} \end{aligned}$$

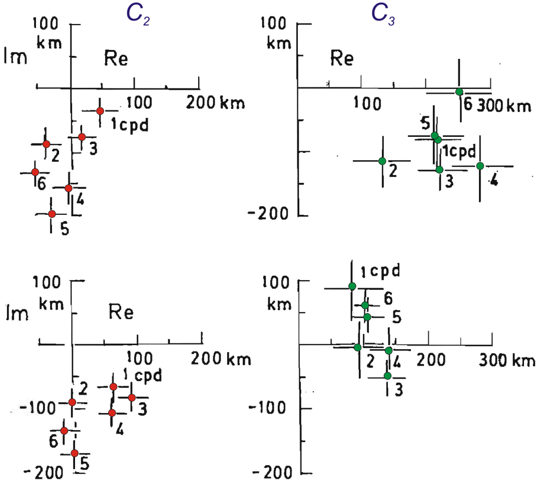
Application to European Hourly Mean Values 1964-65

Observatory Wingst (near Hamburg/Germany)

C_2 show systematic non-zero values, increasing with frequency
indicate anisotropy of the C-response at Wingst

tri-variate

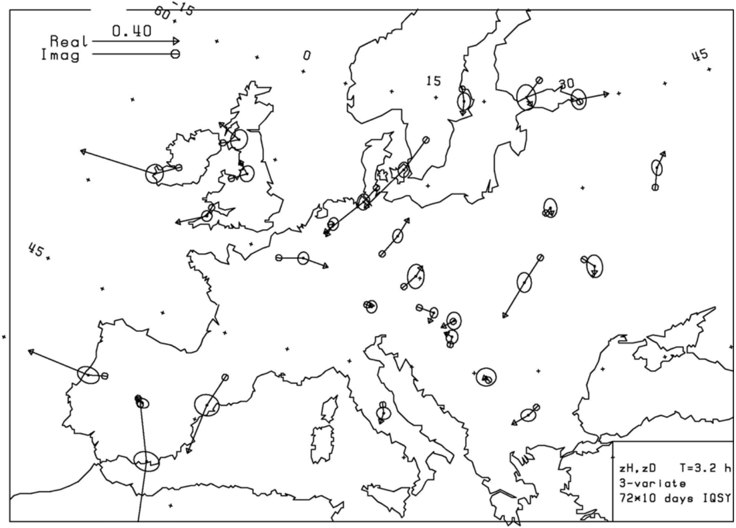
5-variate



U. Schmucker: Electromagnetic induction studies with long-periodic geomagnetic variations in Europe 1.

Long-period induction arrows

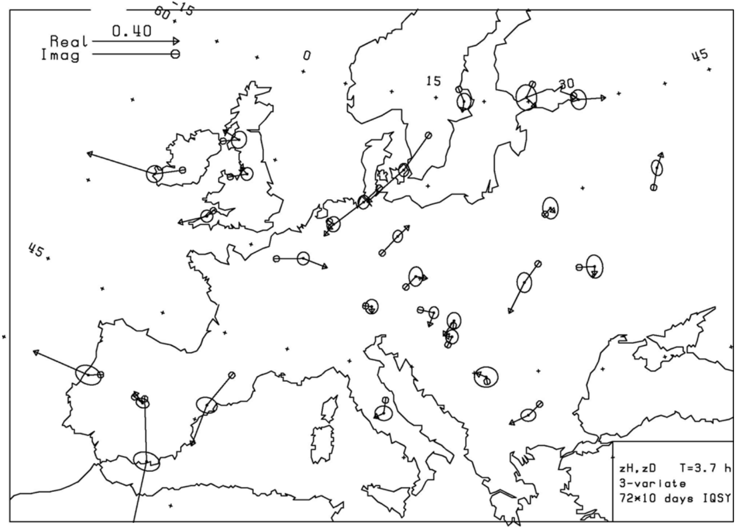
7.5 cpd, $T = 3.3$ hrs, $z^\star = 260$ km



U. Schmucker, *Ein Kontinent erwacht*, EMTF 2005

Long-period induction arrows

6.5 cpd, $T = 3.7$ hrs, $z^\star = 280$ km



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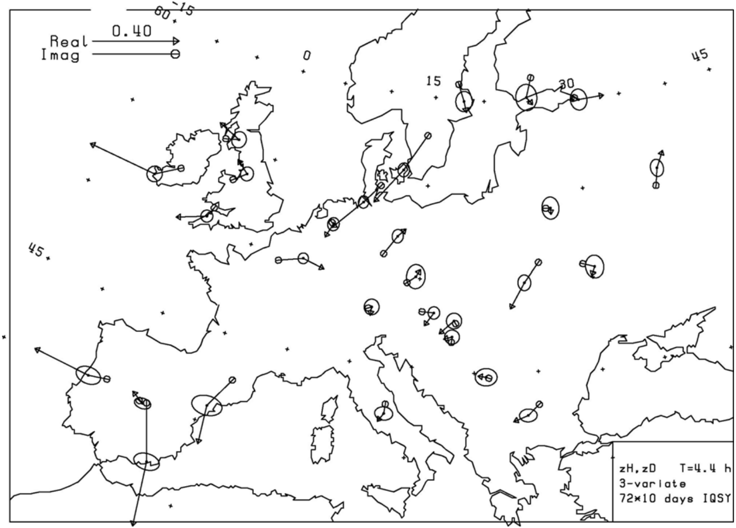
Long-period induction arrows

5.5 cpd, $T = 4.4$ hrs, $z^\star = 315$ km



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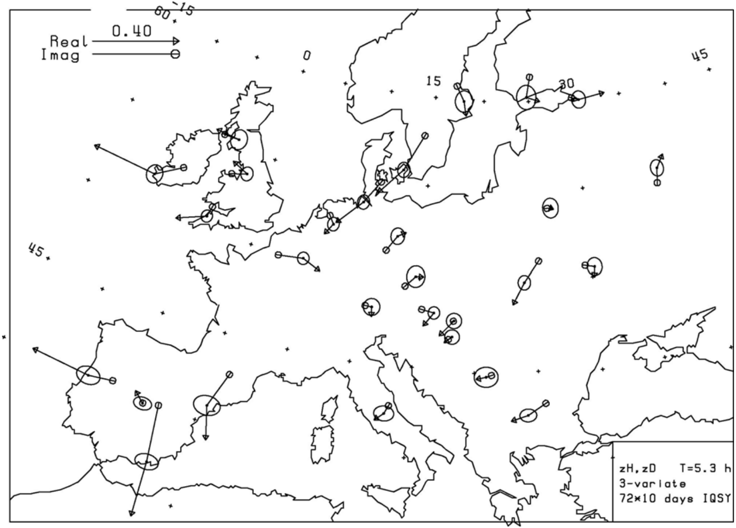
Long-period induction arrows

4.5 cpd, $T = 5.3$ hrs, $z^\star = 360$ km



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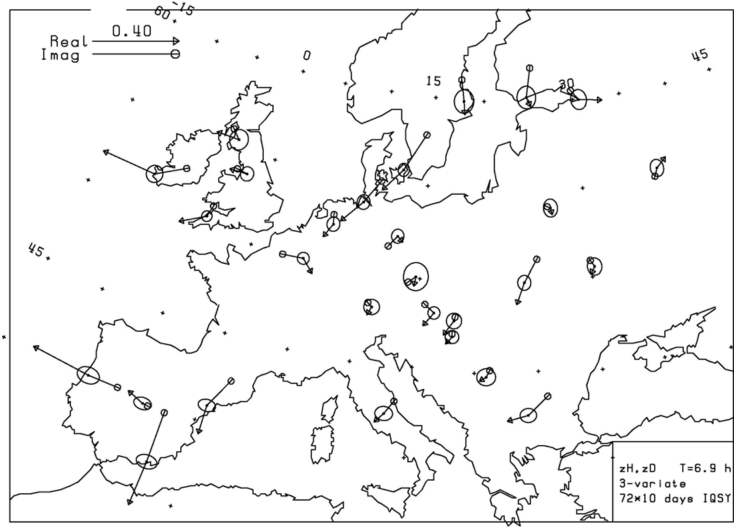
Long-period induction arrows

3.5 cpd, $T = 6.9$ hrs, $z^{\star} = 415$ km



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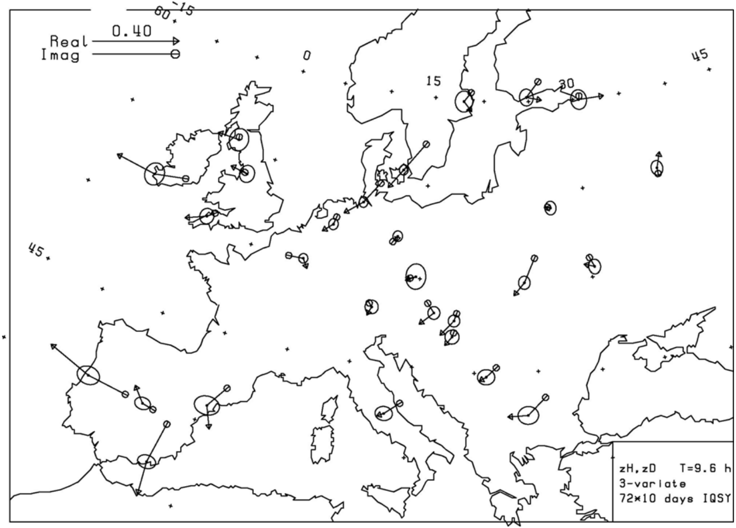
Long-period induction arrows

2.5 cpd, $T = 9.6$ hrs, $z^\star = 500$ km



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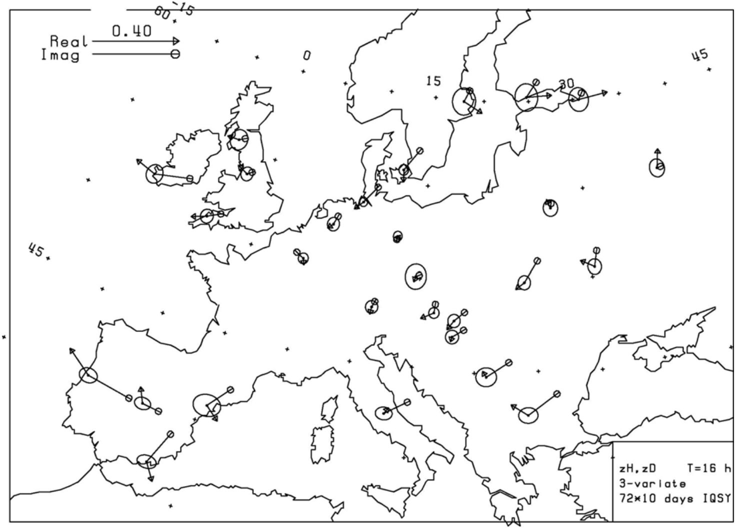
Long-period induction arrows

1.5 cpd, $T = 16$ hrs, $z^\star = 610$ km



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