Virtual Observatories from *Swarm*
and their use in data assimilation studies of core dynamics

Chris Finlay[1], Magnus Hammer[1], Olivier Barrois[2] and Nicolas Gillet[2]

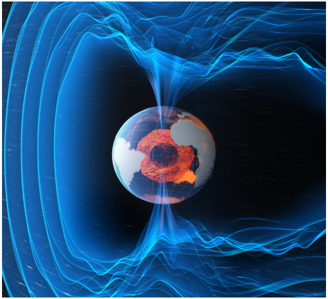
1: DTU Space, Technical University of Denmark

2: ISTerre, Université Grenoble 1, CNRS 1381, France

Motivation: investigating the dynamics of the deep Earth

- $> 98\%$ Earth's $\mathbf{B}$ field originates in the core
- Generated by **dynamo action** in the liquid outer core
- Not steady; continuously changing
  -> Secular Variation (SV)

$$\underbrace{\frac{\partial \mathbf{B}}{\partial t}}_{\text{Secular variation}} = \underbrace{\nabla \times (\mathbf{u} \times \mathbf{B})}_{\text{Advection \& stretching by core flow}} + \underbrace{\eta \nabla^2 \mathbf{B}}_{\text{Ohmic diffusion}}$$



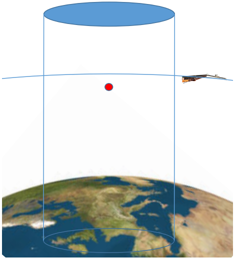
[Image credit: ESA]

Challenges

- Need to combine observations with physics-based models
- Require realistic error estimates for data assimilation and hypothesis testing
- Ideally should have uniform global coverage, spanning decades

Virtual observatories or Monthly Point Estimates (MPEs)

- **Time series of monthly point estimates at satellite altitude**
  [Mandea and Olsen, 2006; Olsen and Mandea, 2007;
  Beggan et al., 2009; Whaler and Beggan, 2015]
- Take all data within cylinder of chosen radius (e.g. 2000km)
- Choose selection criteria e.g. only dark, quiet time data
  ($K_p < 3$, $|dRC/dt| < 3$nT/yr, IMF $B_z > 0$, $E_m < 0.8$ mV/m)
- Remove estimates of crustal, magnetospheric and S_q fields
- Work with sums and differences of data, along and across track
- Threshold for minimum number of data (e.g. 70 per month)



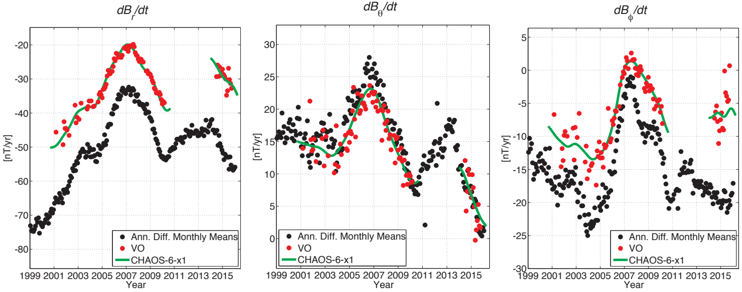
  - Robust (Huber weighted) fit of **local cubic potential** to all data in cylinder

$$
\begin{aligned}
V(x,y,z) = {} & v_x x + v_y y + v_z z + v_{xx} x^2 + v_{yy} y^2 - (v_{xx} + v_{yy}) z^2 \\
& + 2v_{xy} xy + 2v_{xz} xz + 2v_{yz} yz - (v_{xyy} + v_{xzz}) x^3 \\
& + 3v_{xxy} x^2 y + 3v_{xxz} x^2 z + 3v_{xyy} xy^2 + 3v_{xzz} xz^2 + 6v_{xyz} xyz \\
& - (v_{xxy} - v_{yzz}) y^3 + 3v_{yzz} y^2 z + 3v_{yzz} yz^2 - (v_{xxz} + v_{yyz}) z^3
\end{aligned} \quad (1)
$$

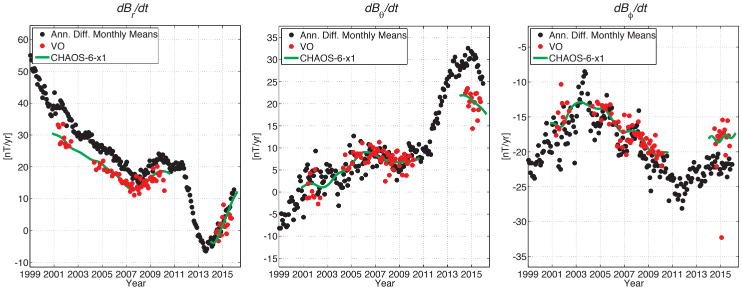
  - Then calculate prediction at chosen reference point using $\mathbf{B} = -\nabla V$

Comparisons with selected ground observatories I

HER

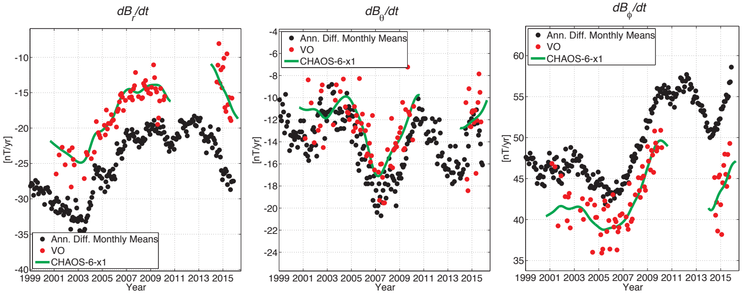


HON

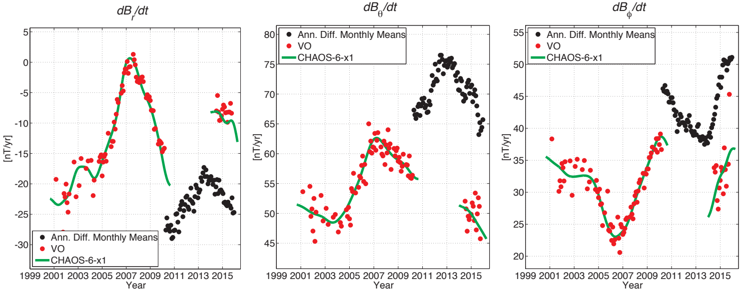


Comparisons with selected ground observatories II

CLF

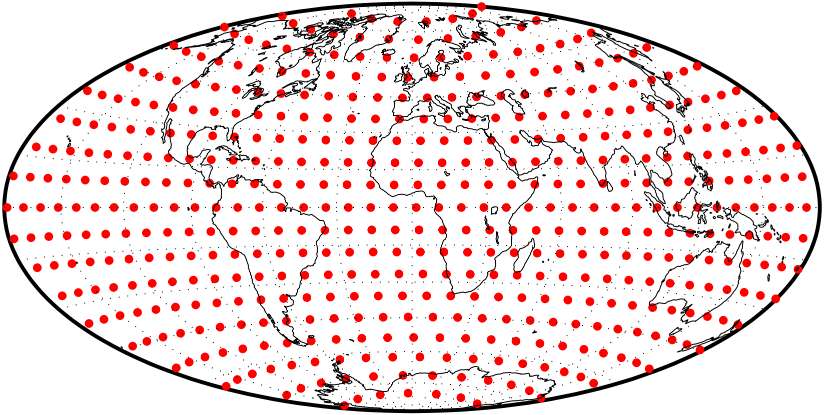


TDC



Global grid of Monthly Point Estimates

- Approx equal area grid, Recursive Zonal Equal Area Sphere Partitioning (Leopardi, 2006)

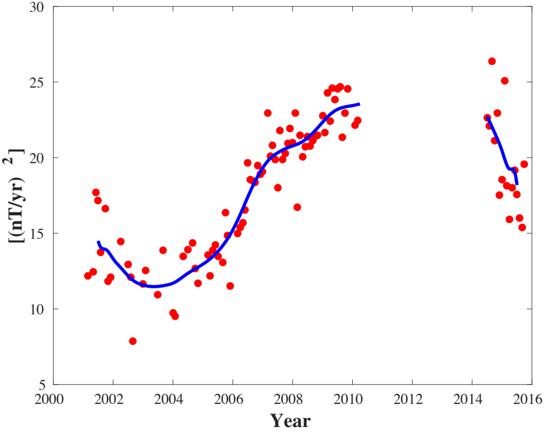


A full data covariance matrix $\mathbf{C_e}$

- Time series of 3 components (e.g. dB_r/dt,dB_θ/dt,dB_ϕ/dt) at P locations, -> $3P$ series in all, each of length N_T

- Detrend each times series using cubic smoothing spline and Generalized Cross Validation

$$\mathbf{x}_i = \mathbf{d}B_i/dt - \widetilde{\mathbf{d}B_i/dt}$$



A full data covariance matrix $\mathbf{C}_e$

- Place these $3P$ time series into columns of N_T x $3P$ matrix

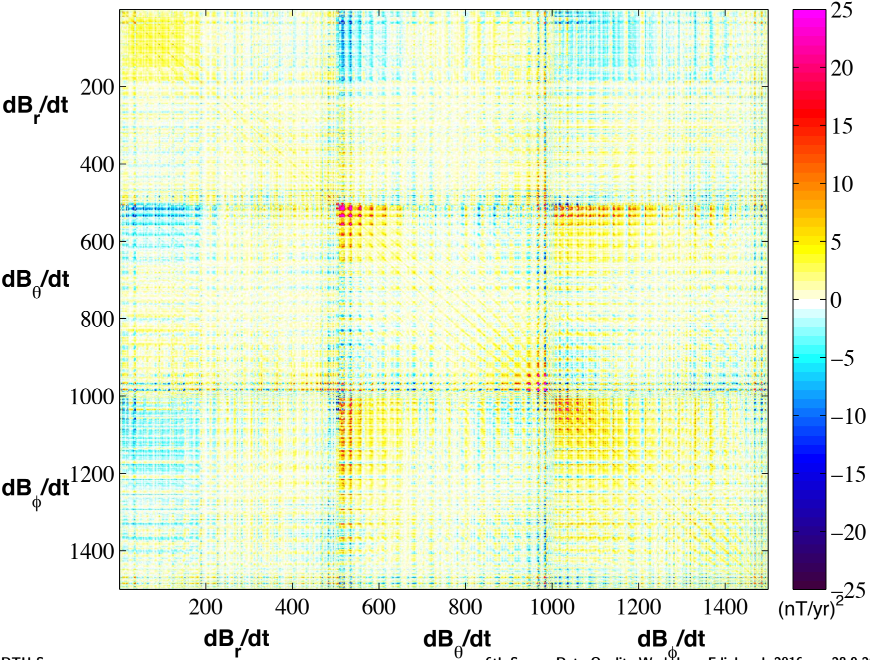
$$\left(\begin{array}{c|c|c|c} \cdot & \cdot & \cdots & \cdot \\ \mathbf{x}_1 & \mathbf{x}_2 & \cdots & \mathbf{x}_{3P} \\ \cdot & \cdot & \cdots & \cdot \end{array}\right)$$

- Compute covariances between columns of this matrix

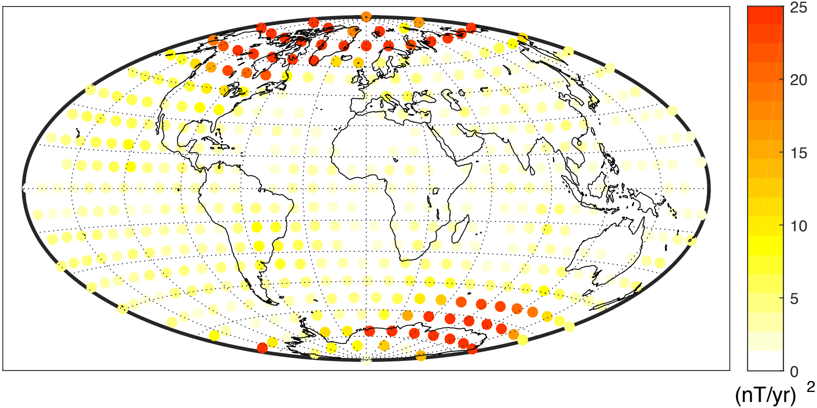
$$\mathbf{C}_e = \mathrm{Cov}(\mathbf{x}_i, \mathbf{x}_j) = \frac{1}{N_T} \sum_{k=1}^{N_T} x_{i,k}\, x_{j,k}$$

- $\mathbf{C}_e$ has size $3P$ x $3P$=1500 x 1500 (manageable in inversions)

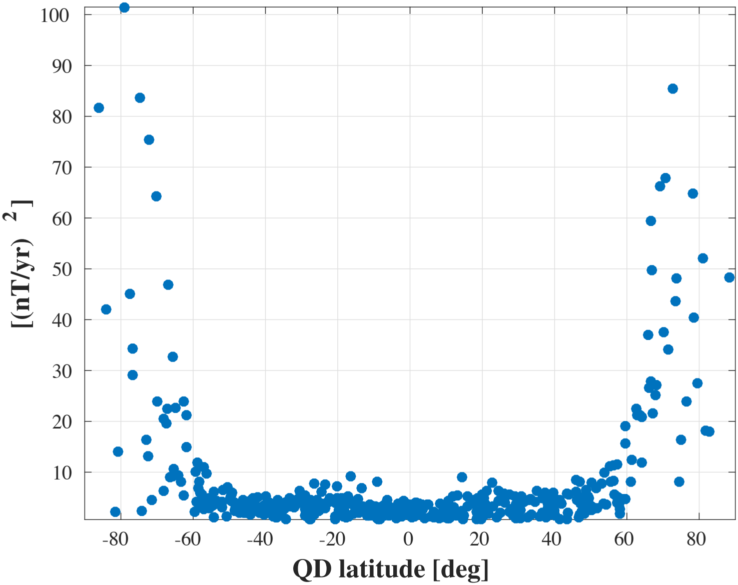
A full covariance matrix C_e



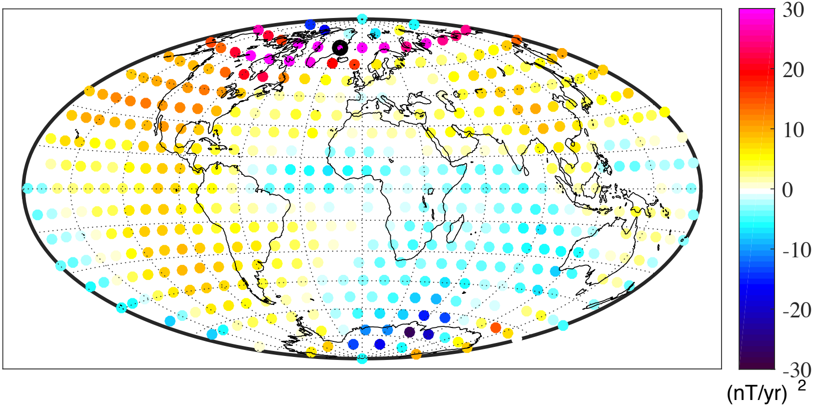
Data error variances for each location: dB_θ/dt



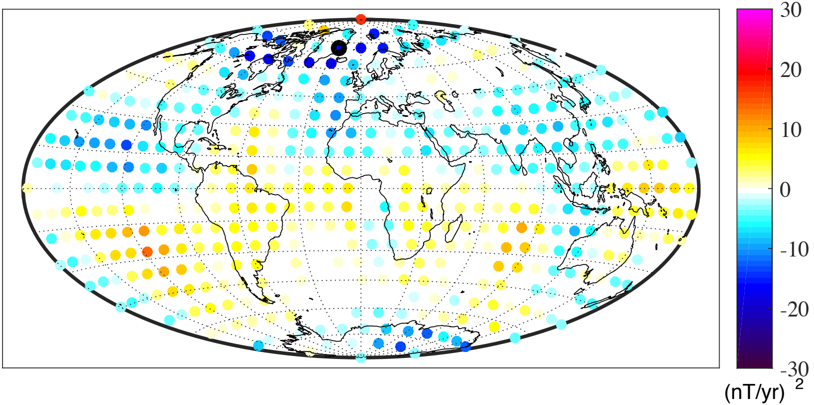
Data error variances for dB_θ/dt, dependence on QD latitude



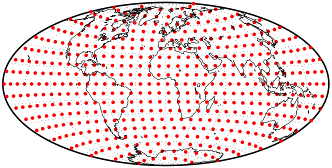
Spatial covariances: btw dB_θ/dt and dB_θ/dt



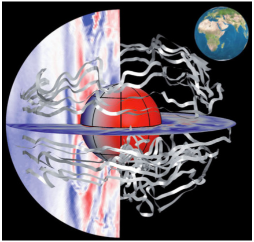
Spatial covariances: btw dB_θ/dt and dB_r/dt



Example application: Kalman Filter of VO data & dynamo simulation statistics



&



[Image courtesy of J. Aubert]

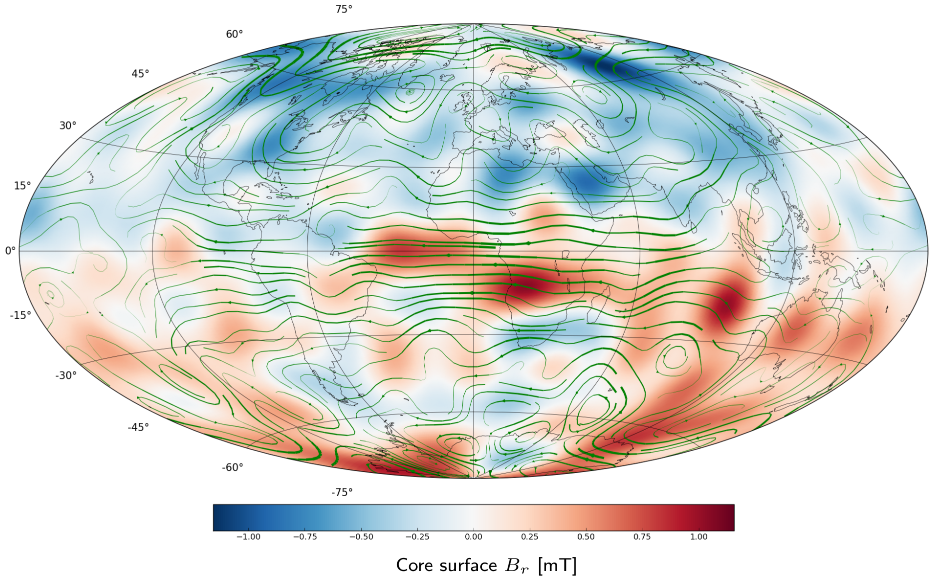
- Data at time t_k, $\mathbf{d}_k = \{B_r; B_\theta; B_\phi; dB_r/dt; dB_\theta/dt; dB_\phi/dt\}$
- Model at time t_k: $\mathbf{m}_k = \{\mathbf{g}_k, \mathbf{u}_k\}$
- Prior model covariances $\mathbf{C_m}$ from CE dynamo model [Aubert et al., 2013]

- Model estimation using a Kalman Filter algorithm [Fournier et al., 2013, Gillet et al., 2015]
- Forecast carried out using a stochastic equation using an ensemble approach $=>$ posterior model pdf available

$$\mathbf{m}_{k+1} = \mathbf{A}\mathbf{m}_k + \mathbf{C}_m\mathbf{G}^T\left(\mathbf{G}\mathbf{C}_m\mathbf{G}^T + \mathbf{C}_e\right)^{-1}(\mathbf{d}_{k+1} - \mathbf{G}\mathbf{A}\mathbf{m}_k) \quad (2)$$

- Prelim. expts.: analyses of data every 6 months, diagonal $\mathbf{C}_e$, frozen model covariances

Example application: Field and flow at core surface in 2008



Summary

Summary

(1) **Monthly point estimates** of satellite magnetic data on a grid of locations i.e. 'Virtual Observatories' or MPEs are well suited for global monitoring of SV

Summary

(1) **Monthly point estimates** of satellite magnetic data on a grid of locations i.e. 'Virtual Observatories' or MPEs are well suited for global monitoring of SV

(2) Advantages include:

- Provide time series with high temporal resolution and uniform global coverage
- Combining data from CHAMP and *Swarm*, series already span 1.5 decades
- Relatively mature, and compare well with ground observatory records
- Full data error covariance information available (not previously exploited.....)

Summary

(1) **Monthly point estimates** of satellite magnetic data on a grid of locations i.e. 'Virtual Observatories' or MPEs are well suited for global monitoring of SV

(2) Advantages include:

- Provide time series with high temporal resolution and uniform global coverage
- Combining data from CHAMP and *Swarm*, series already span 1.5 decades
- Relatively mature, and compare well with ground observatory records
- Full data error covariance information available (not previously exploited.....)

(3) Don't be confused:
Are not proper 'observatories' - only useful for studies of long term field evolution!

Summary

(1) **Monthly point estimates** of satellite magnetic data on a grid of locations
i.e. 'Virtual Observatories' or MPEs are well suited for global monitoring of SV

(2) Advantages include:

- Provide time series with high temporal resolution and uniform global coverage
- Combining data from CHAMP and *Swarm*, series already span 1.5 decades
- Relatively mature, and compare well with ground observatory records
- Full data error covariance information available (not previously exploited.....)

(3) Don't be confused:
Are not proper 'observatories' - only useful for studies of long term field evolution!

(4) Already implemented in prototype data assimilation schemes, more to come