What are Gravity Waves?

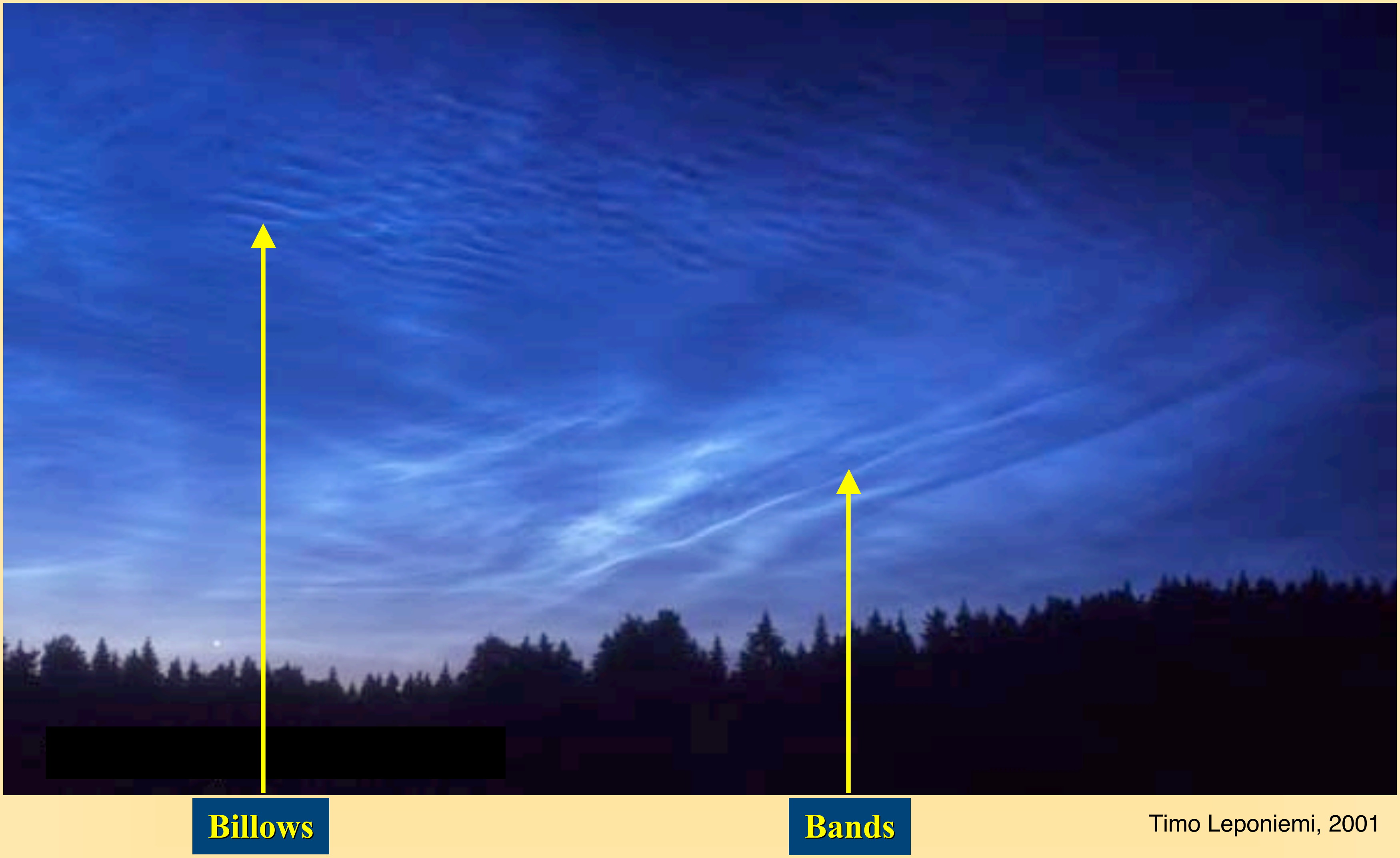
- Gravity waves are buoyancy waves – the restoring force comes from Archimedes's principle.
- They involve vertical displacement of air parcels, along slanted paths
- The waves are transverse with temperature and wind pertubations, δT and δw being the two free parameters that oscillate for a freely propagating wave
- They are found everywhere in the atmosphere
- They can propagate vertically and horizontally, transporting momentum from their source to their sink
- Global circulation models use GW parameterization schemes to represent GW transfer of momentum - major source of controversy

Atmospheric Gravity Waves

- Ubiquitous
- Small scale
- Wavelengths :

  tens to thousands km
- Periods: mins to hrs

PMCs display complicated structure most likely caused by GW activity

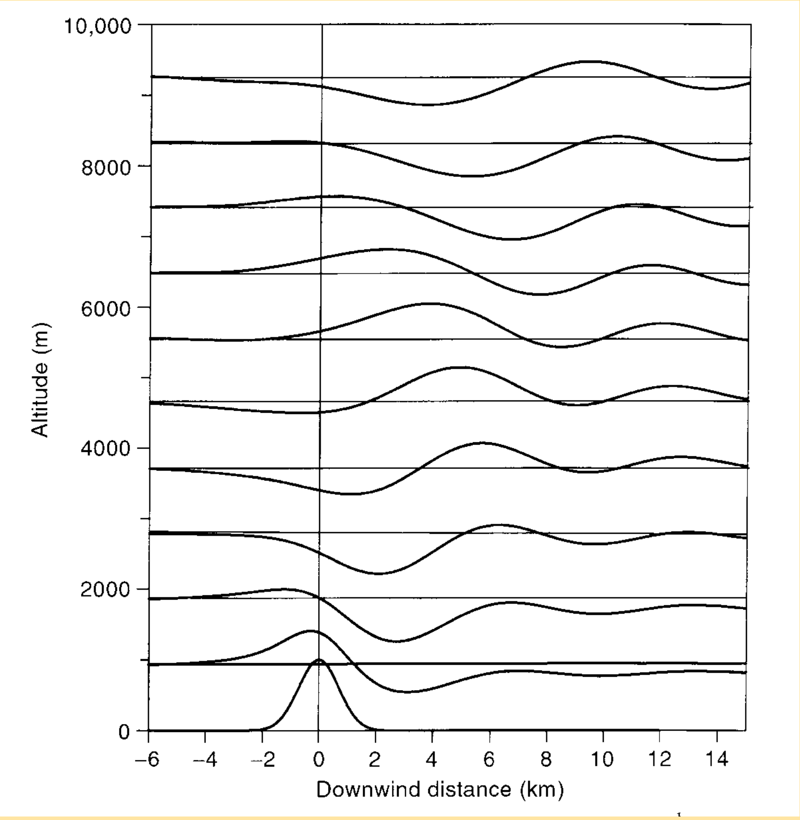

[redacted]

Timo Leponiemi, 2001

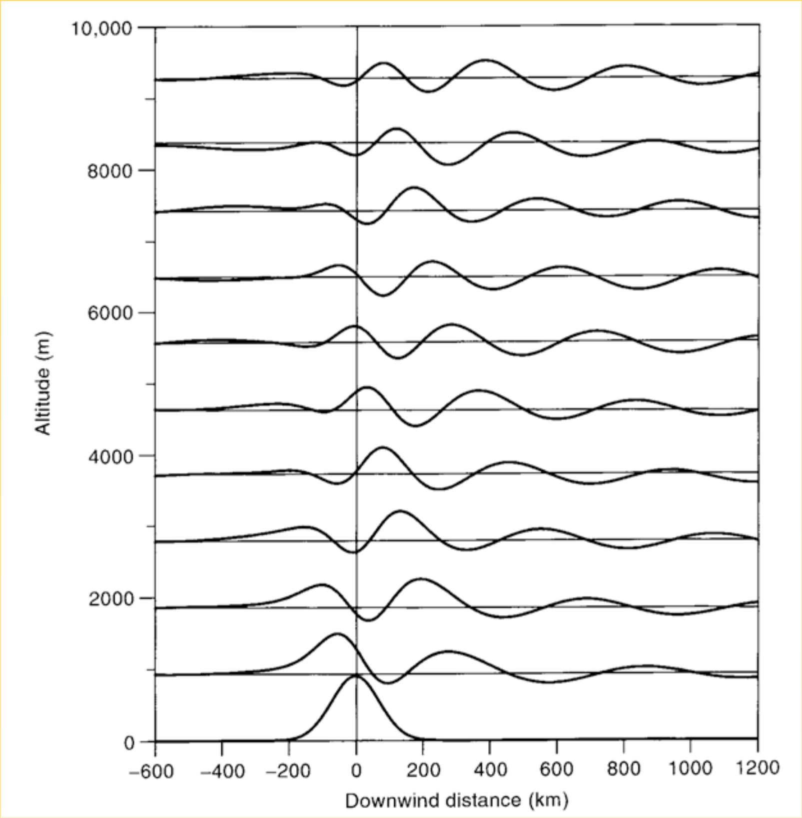
Multiple GW Sources

- Flow over a mountain range
- Flow over convective cloud (moving mountain)
- Kelvin-Helmholtz instability around the jet stream
- Geostrophic adjustment

Calculated wave patterns over a two-dimensional ridge



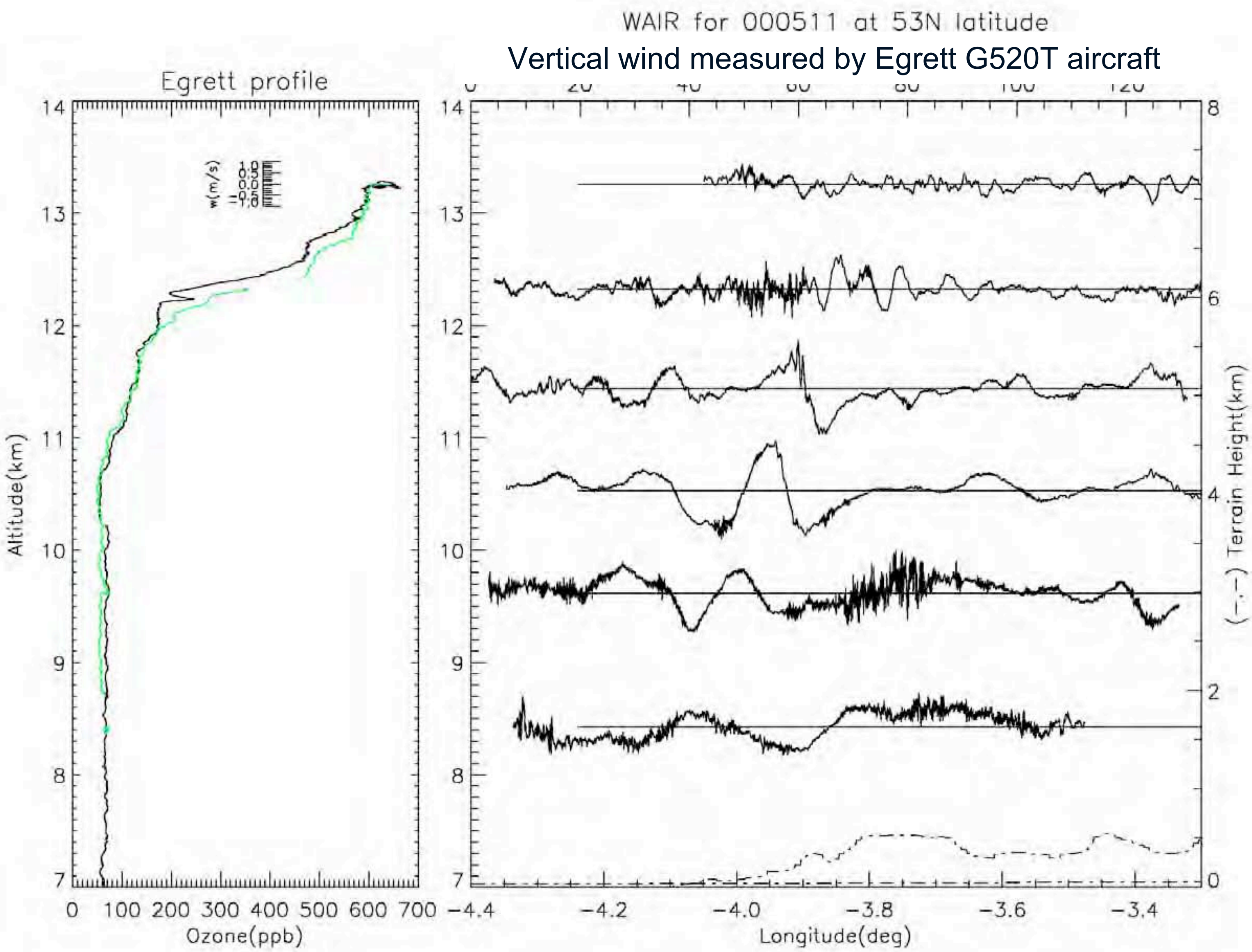
Gaussian-shaped ridge, width 1 km



Gaussian-shaped ridge, width 100 km

From Carmen J. Nappo, Atmospheric Gravity Waves, Academic Press

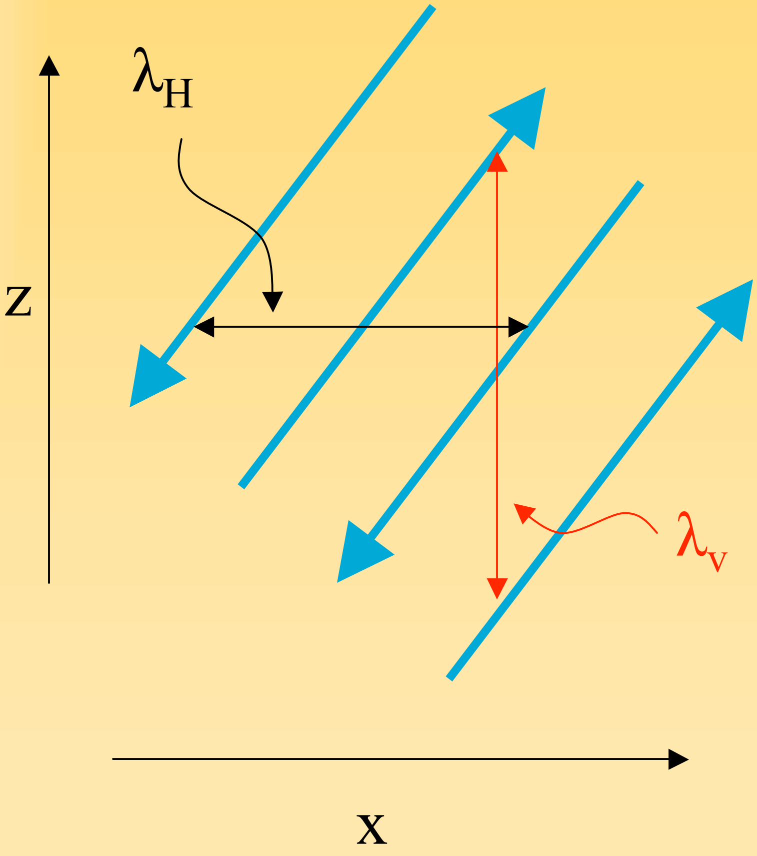
Breaking mountain waves – 11 May 2000

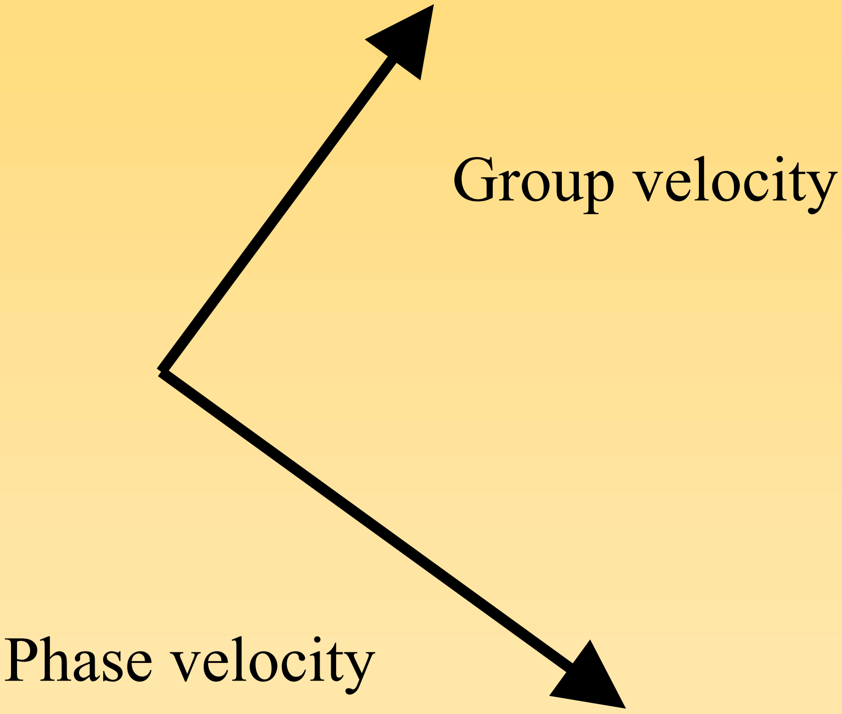


Vertical wind measured by Egrett G520T aircraft

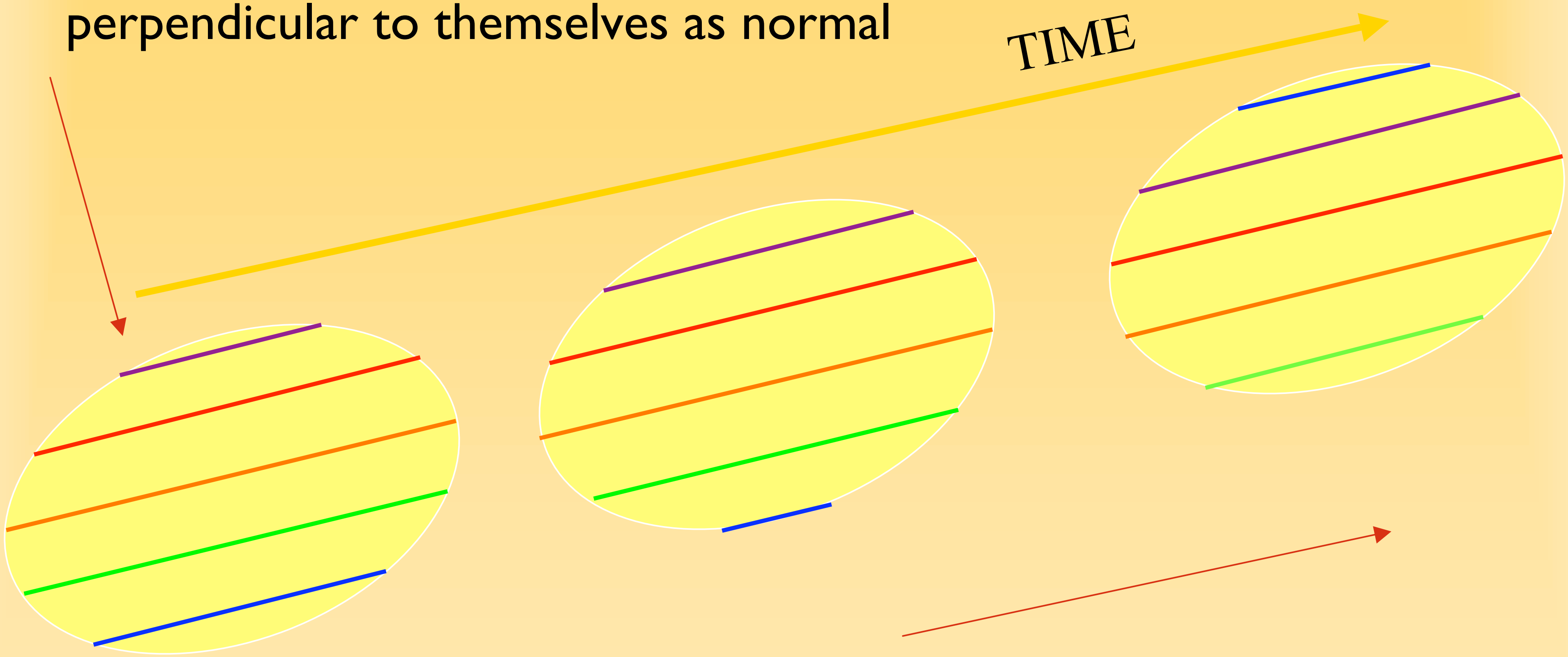
Propagating gravity waves

Buoyancy waves where air parcels oscillate along slant paths





Group and phase velocity



Individual phase fronts propagate perpendicular to themselves as normal

GROUP of waves propagates ALONG phase lines

Typical GW properties

- Frequencies ***greater*** than N (Brunt-Vaisala frequency) and ***less*** than f (Coriolis parameter: periods ~ 5 min – ~ 1 day
- Typical vertical wavelength in mesosphere: 2-3 km to 30 km
- Mountain waves have $C_{gH} = 0$ – fixed w.r.t ground
- Waves propagate vertically into the stratosphere and mesosphere
- Wave amplitudes vary as $\rho^{-1/2}$: density decreases so waves grow in amplitude with height
- Waves can be filtered and dissipated by stratospheric wind system as a result of critical layer interactions when phase speed matches background wind speed

Inertia-gravity waves

Long-period gravity waves, affected by Earth's rotation.

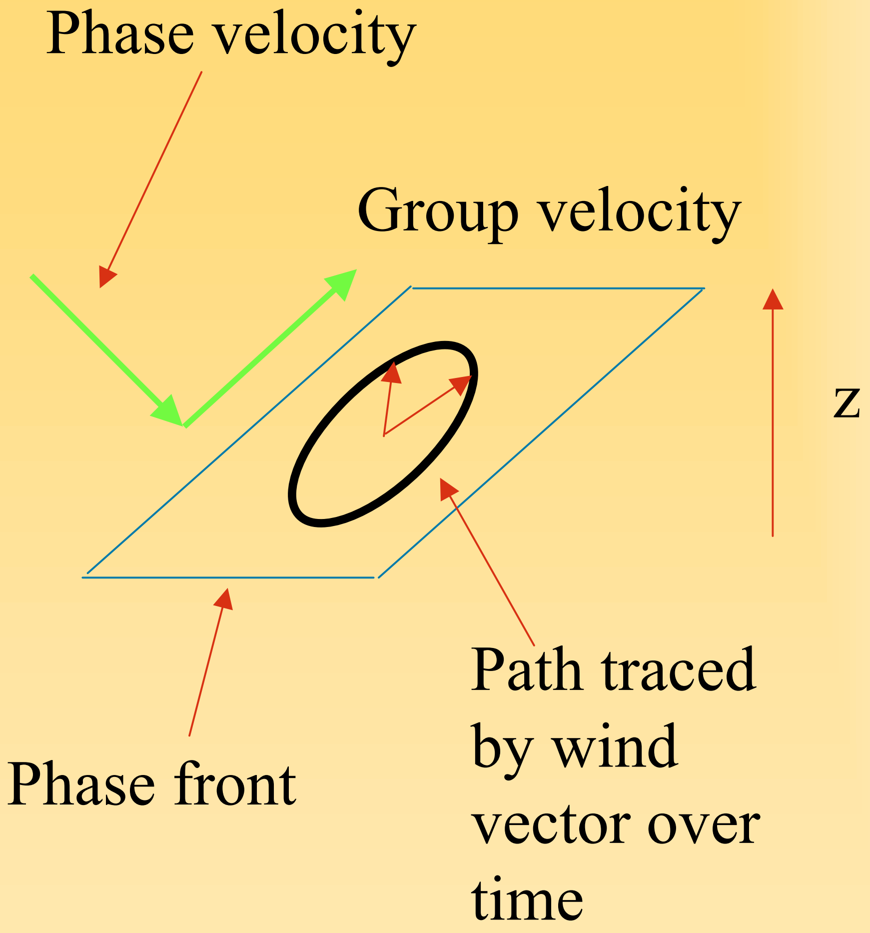
Frequency ~ f   (2Ωsinλ – corr to T ~ 16 hours at 50°N)

Horizontal Wavelength > 100 km

Vertical wavelength ~2 km

Wind vector rotates elliptically with time or ht.

Wave packet = ? km



Why do we care about GWs?

- They transport momentum vertically. This momentum transfer is crucial to the large-scale momentum balance of the stratosphere and mesosphere
- They break, causing mixing of air from different origins.
- Quantifying the influence of GWs is important for simulations of climate change scenarios.

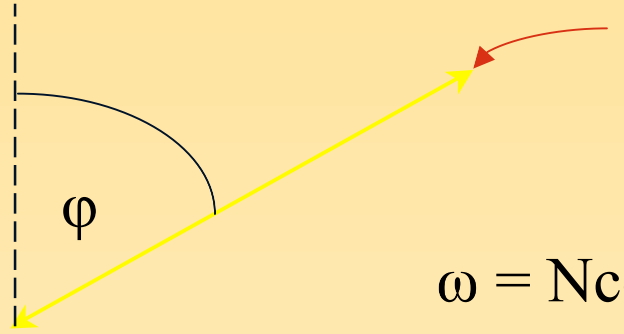
Mathematical theory of gravity waves

- The basic equations of atmospheric dynamics are the three momentum equations, the continuity equation, the thermodynamic energy equation and the equation of state for air. They are non-linear.
- Gravity wave theories start by postulating some background state of the atmosphere, and introducing small departures from the background state. This is a standard technique in mathematical physics for linearising the equations.
- The linear equations have harmonic solutions: $\exp i(kx-\omega t)$
- Actual gravity waves can be represented as superpositions of these harmonic solutions

Properties of harmonic solutions I

- Dispersion equation for short-period waves $$\omega^2 = \frac{N^2(k^2+\ell^2)}{(k^2+\ell^2+m^2)}$$
- Dispersion equation for inertia-gravity waves $$\omega^2 = \frac{f^2m^2+N^2(k^2+\ell^2)}{(k^2+\ell^2+m^2)}$$

Where k, ℓ and m are the wavenumbers in the x, y and z directions, N is the Brunt-Vaisala frequency and f the Coriolis parameter



Plane of oscillation of air parcels

$$\omega = N\cos\varphi$$

Properties of harmonic solutions 2

In an atmosphere with a background wind U, the wave frequency ω is replaced by the intrinsic frequency Ω in the dispersion equation:

$$\Omega = \omega - kU$$

As the wave propagates up in the atmosphere ω remains constant (by definition) so if U changes the intrinsic frequency Ω must change. Thus the horizontal and vertical wavelengths, which are related to Ω, also change.

In the extreme case, Ω can become zero. No gravity wave solutions can exist in this case. A level where $\Omega=0$ is called a critical level – in practice waves tend to break just below it.

Gravity wave spectra

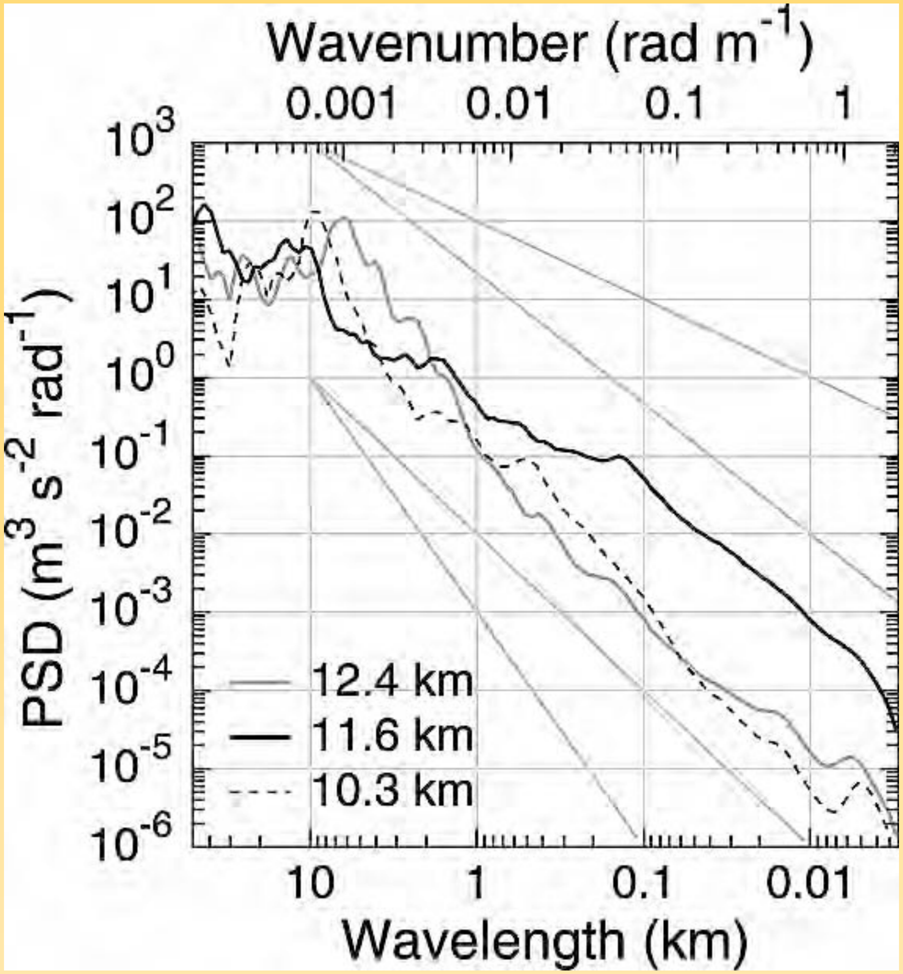
The standard mathematical solutions to the perturbation equations are not gravity waves – the functions are defined for all values of x, y, z and t.

Real waves are always localised in space and time. They must therefore be composed of groups of monochromatic waves (Fourier theory).

Fourier analysis can be used to decompose observed gravity waves to a spectrum of monochromatic components.

These spectra are the subject of considerable attention in the literature.

Observed spectra from aircraft measurements shown earlier

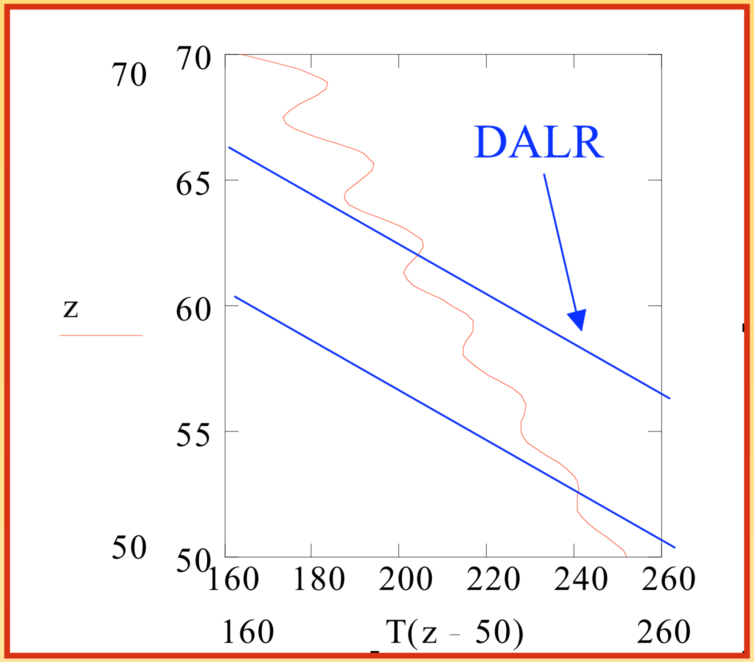


Log-averaged vertical wind kinetic energy spectral densities for each level measured in a frame of reference relative to air. Slanted grey lines show -1, $-5/3$, -2 and -3 power law dependencies.

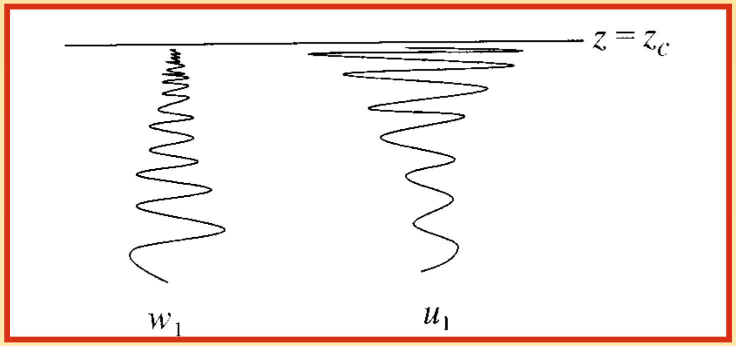
T. Duck and J. A. Whiteway, The spectrum of waves and turbulence at the tropopause, Geophysical Research Letters, 32, L07801, 2005

Breaking gravity waves

- Gravity waves break through setting up either convective or shear instability.
- This can happen either through growth of the wave amplitude with height or through reduction of the vertical wavelength by Doppler-shifting.



- The instabilities generate turbulence and mixing.



Approach to a critical level:
$\lambda_v \to 0$ and $u' \to \infty$

Mesospheric Circulation

- “Anomalous” mesospheric structures suggests need for dynamical forcing (Rayleigh friction) (Murgatroyd and Singleton, 1961; Leovy, 1964).

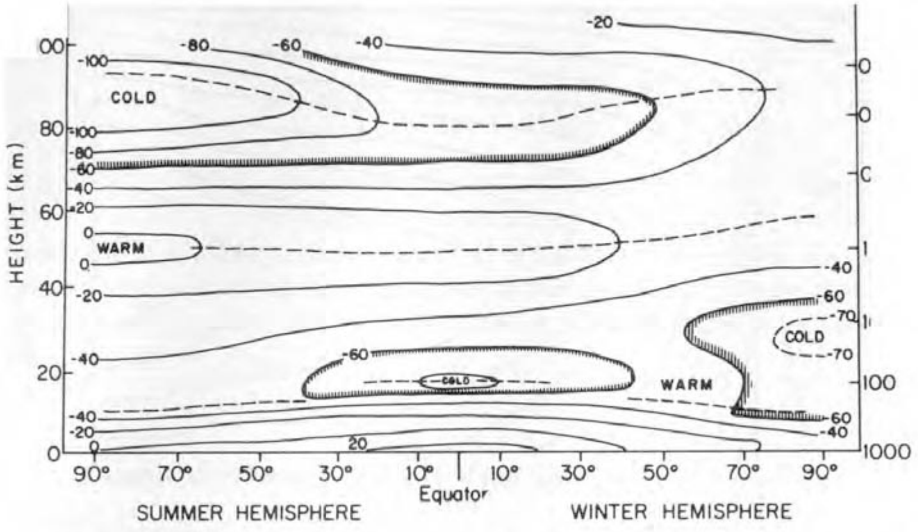


Fig. 1.3. Schematic latitude-height section of zonal mean temperatures (°C) for solstice conditions. Dashed lines indicate tropopause, stratopause, and mesopause levels. (Courtesy of R. J. Reed.)

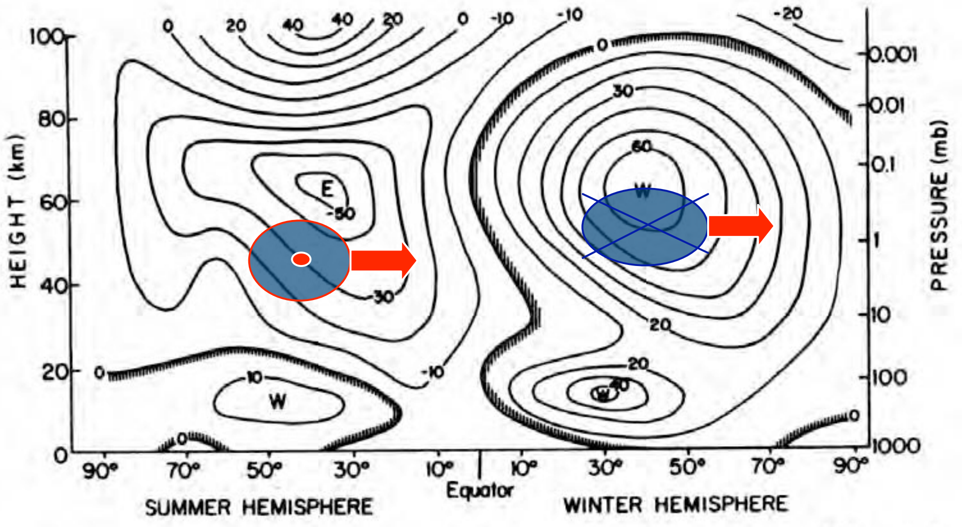


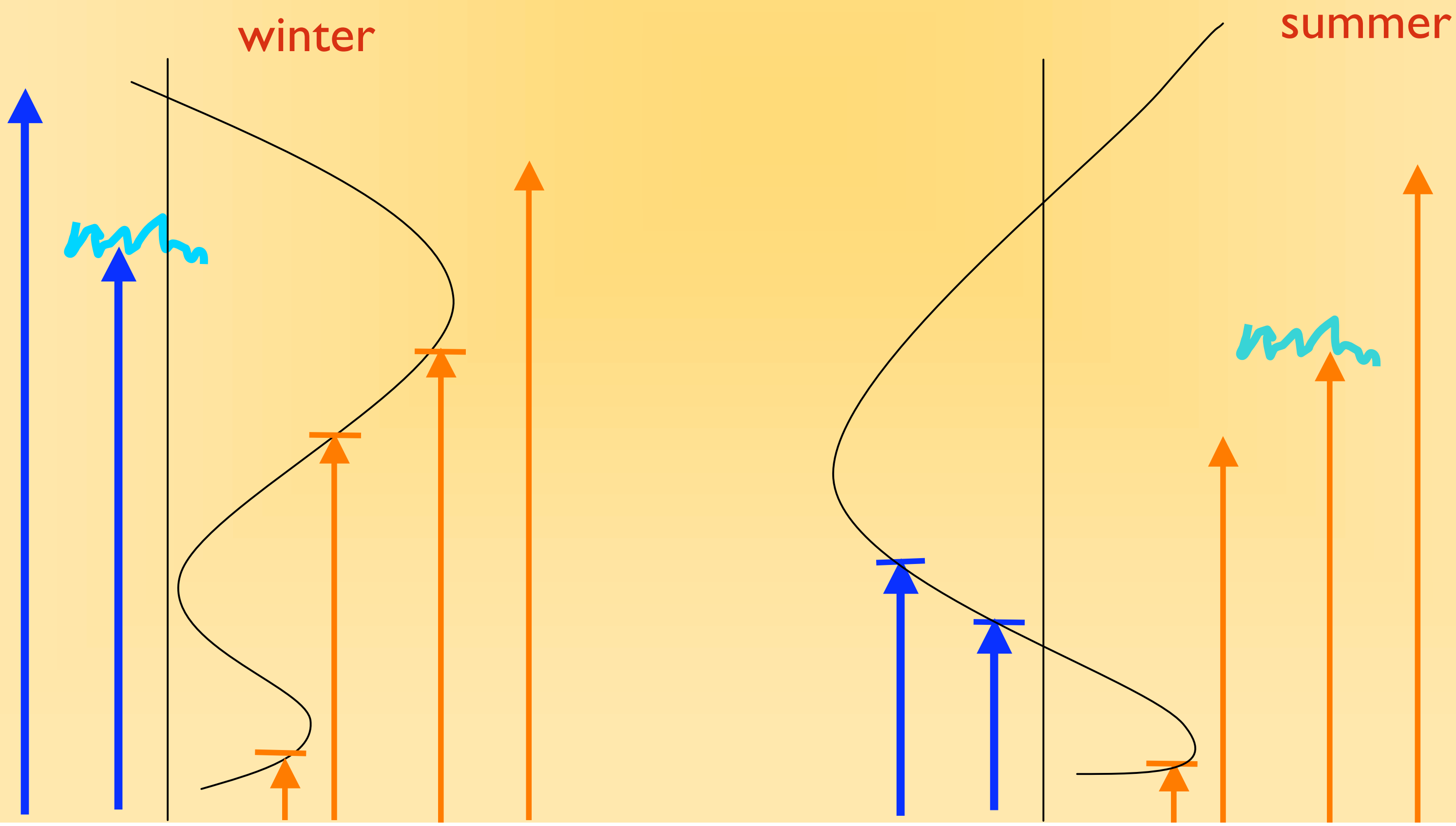
Fig. 1.4. Schematic latitude-height section of zonal mean zonal wind ($m\ s^{-1}$) for solstice conditions; W and E designate centers of westerly (from the west) and easterly (from the east) winds, respectively. (Courtesy o

- Gravity wave impact on the mesospheric circulation (Holton, 1982, 1983).

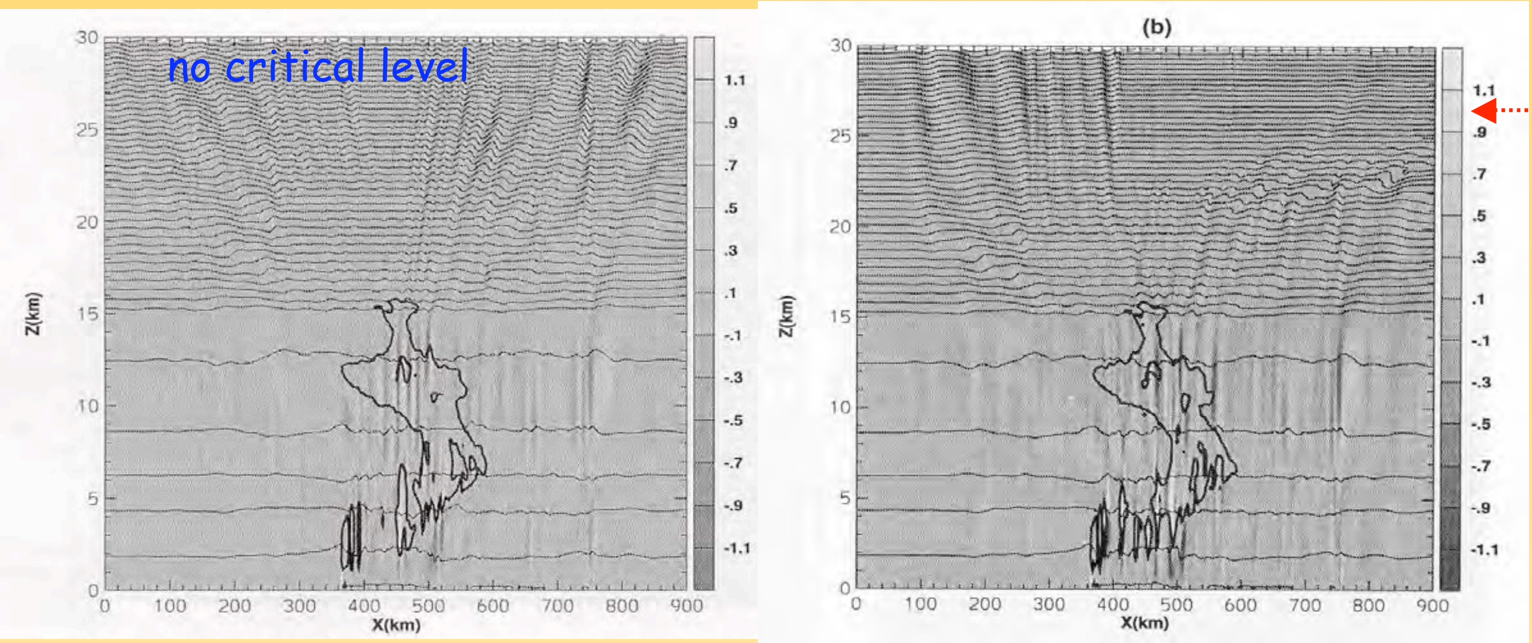
Jim in front of his Mac in 1985 or 86 (taken by Shigeo Yoden)

Change of Gravity Wave Forcing between summer and winter

- Filtering of gravity waves by stratospheric wind system: gravity wave will be reflected or absorbed at critical layer.
  - Eastward stratospheric jet under normal winter conditions: dominant westward propagating gravity waves in the mesosphere.
  - Stratospheric wind reversal during equinox: dominant direction of gravity wave in mesosphere also reverses due to filtering.

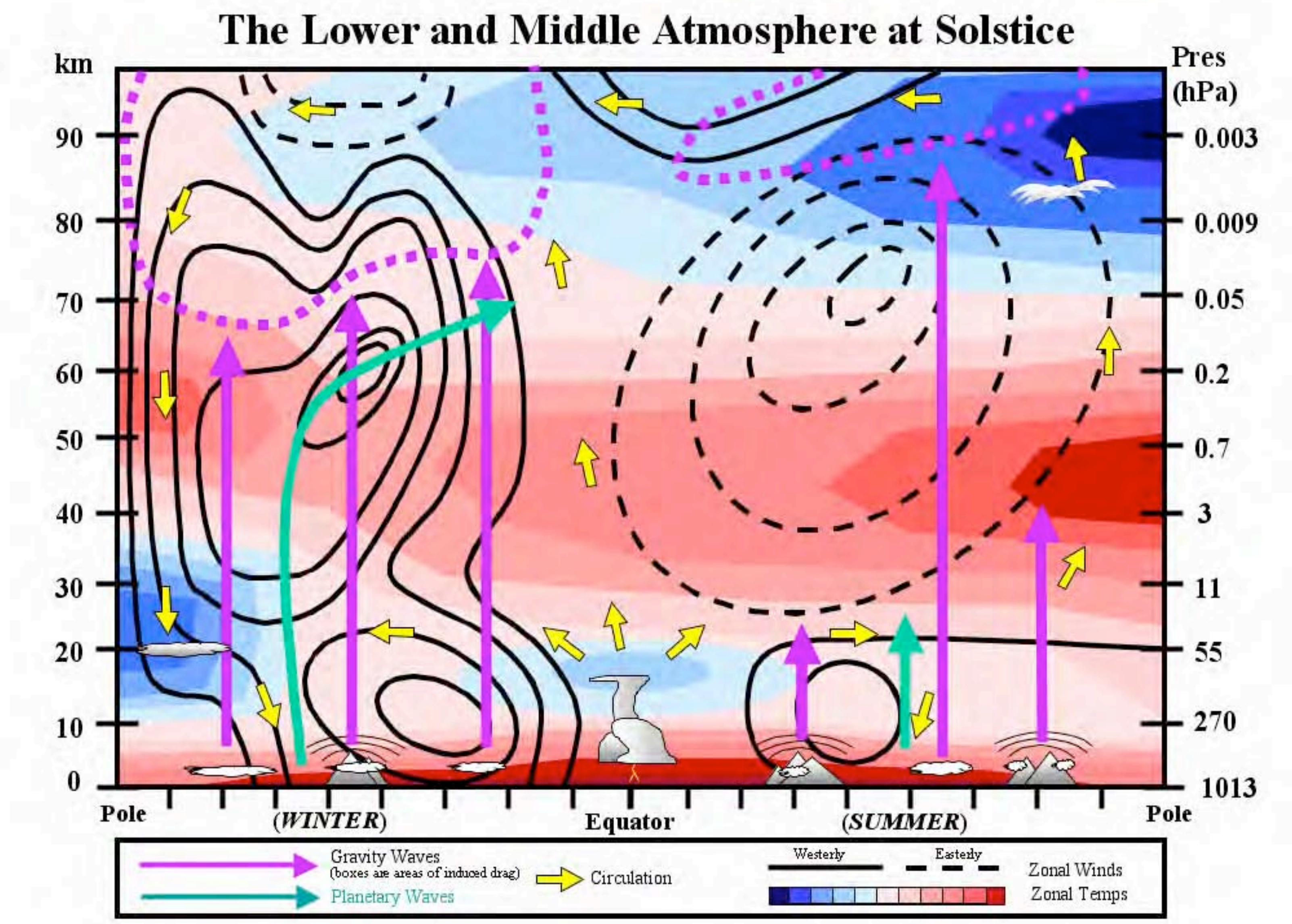


Model simulation of gravity waves forced by deep convection



critical
level
(U>0)

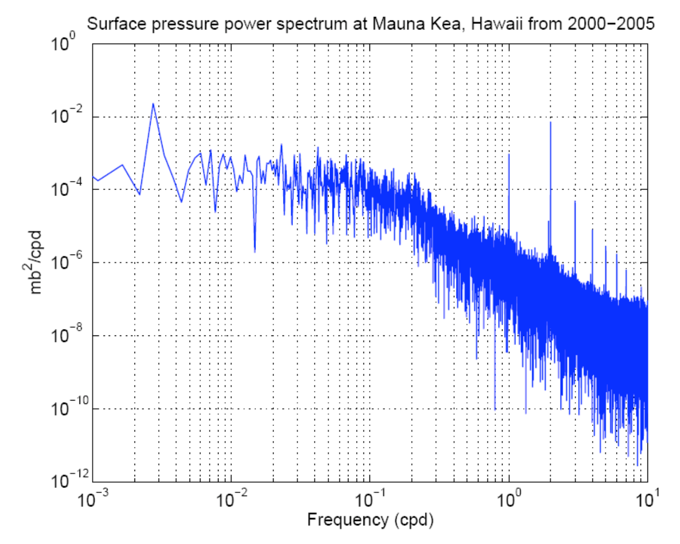
Alexander and Holton, 2000

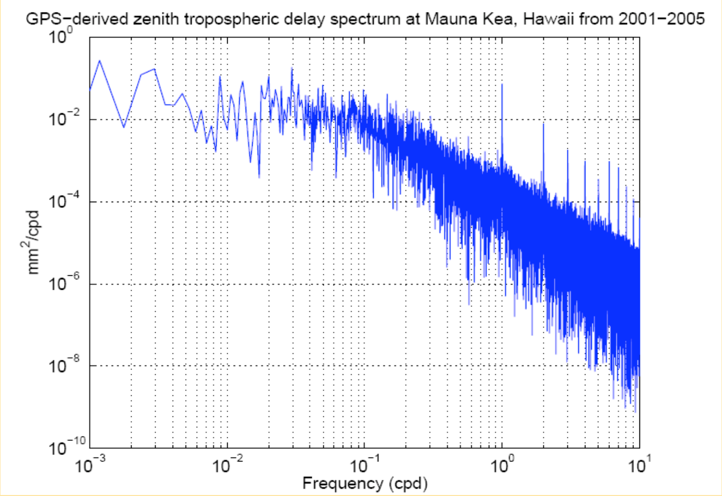


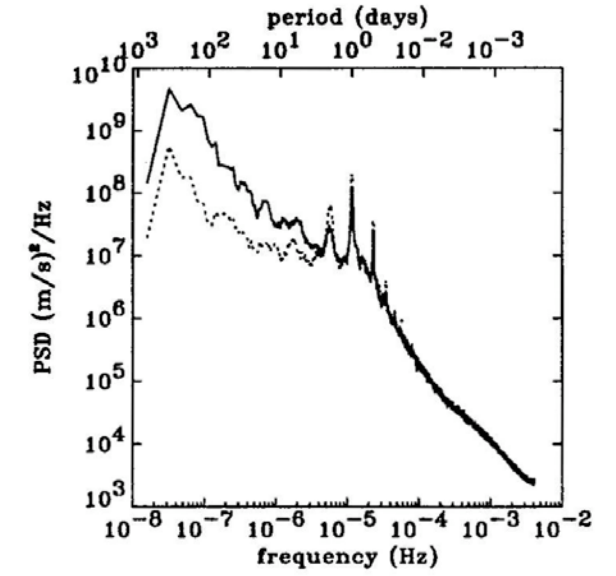
Mesosphere exhibits complex circulation that is far from radiative equilibrium: cold in the summer, warm in the winter

Tides are wave variations with periods of 24,12, and… hrs.

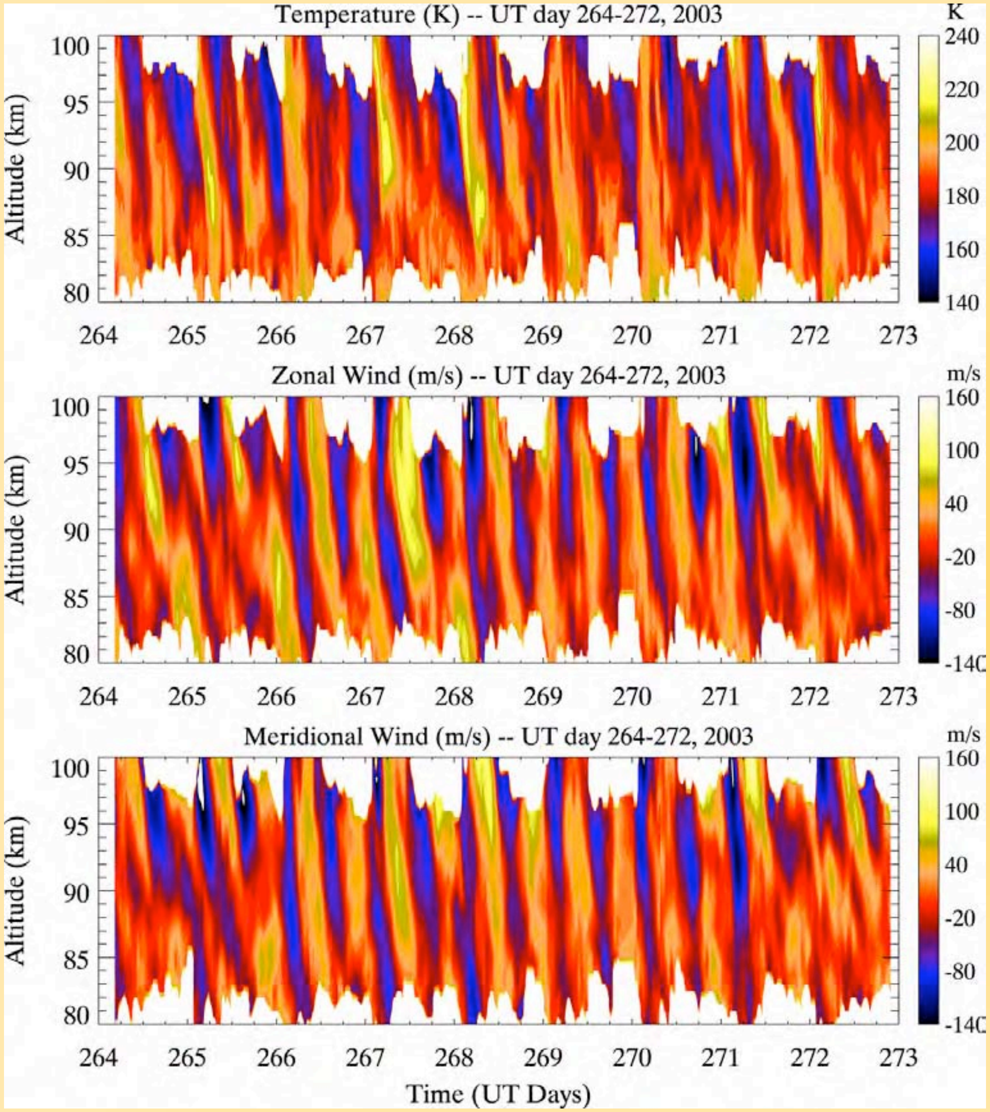
- Annual, daily, and subdaily atmospheric tides are seen in
- surface pressure, GPS-derived tropospheric delay, and mesospheric winds
- Atmospheric tides dominate the dynamics of the mesosphere-lower thermosphere.

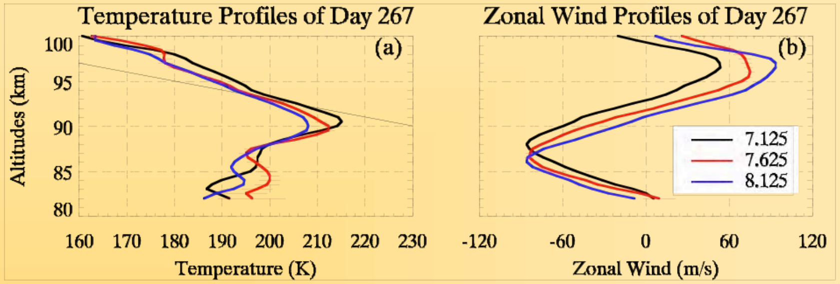

Surface pressure power spectrum at Mauna Kea, Hawaii from 2000−2005

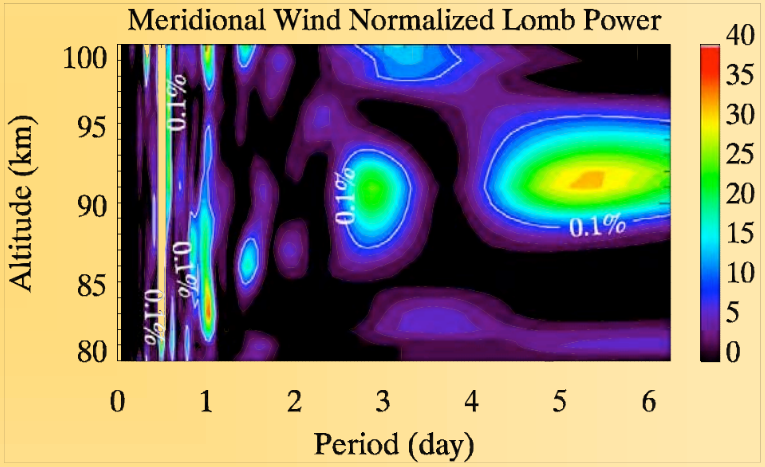

GPS−derived zenith tropospheric delay spectrum at Mauna Kea, Hawaii from 2001−2005



She et al., 2003





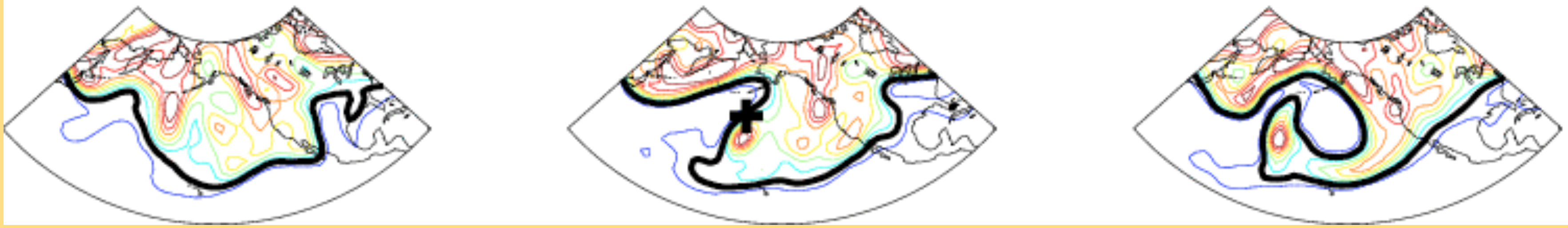


Tidal Variability

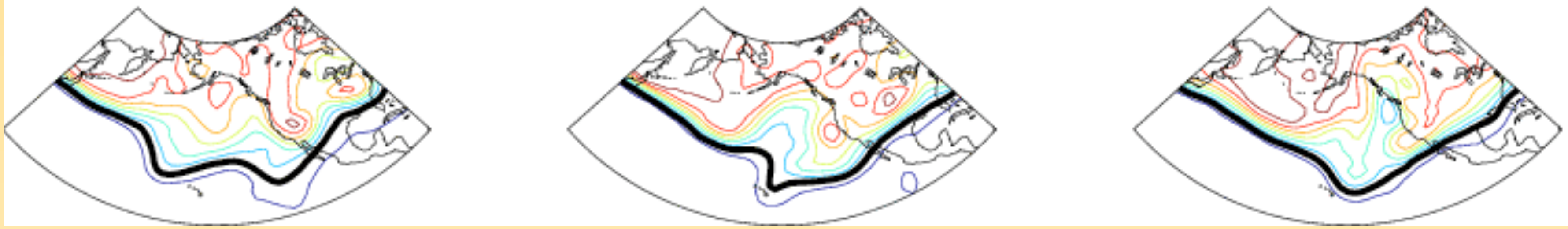
- *Gravity Wave Interactions :*
- *Planetary Wave Interactions*

Results obtained for a 9 day run by the CSU UVT lidar illustrate the variability of the tidal structure in response to GW and tidal fluctuations.

PV on 350K surface on 4, 5 and 6 July 1979



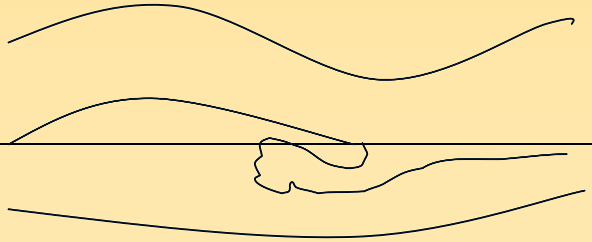
PV on 350K surface on 16, 17 and 18 Dec 1993



Nonlinear theory

- Linear propagation from midlatitudes to lower latitudes

- Waves break as they approach their critical latitude (u=0 stationary waves)
- Rearrangement of PV field in the critical layer (advection around closed streamlines)



HUANG AND REBER: TIDES AND MEAN FLOWS AT 95 KM FROM UARS ACL 5 - 5

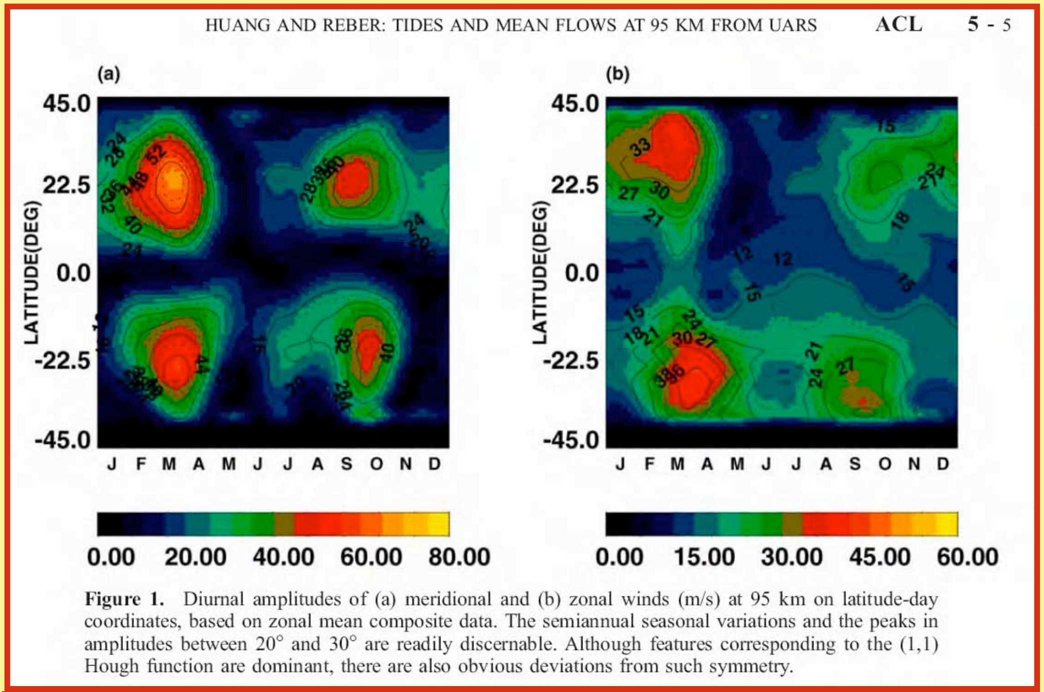


Figure 1. Diurnal amplitudes of (a) meridional and (b) zonal winds (m/s) at 95 km on latitude-day coordinates, based on zonal mean composite data. The semiannual seasonal variations and the peaks in amplitudes between 20° and 30° are readily discernable. Although features corresponding to the (1,1) Hough function are dominant, there are also obvious deviations from such symmetry.

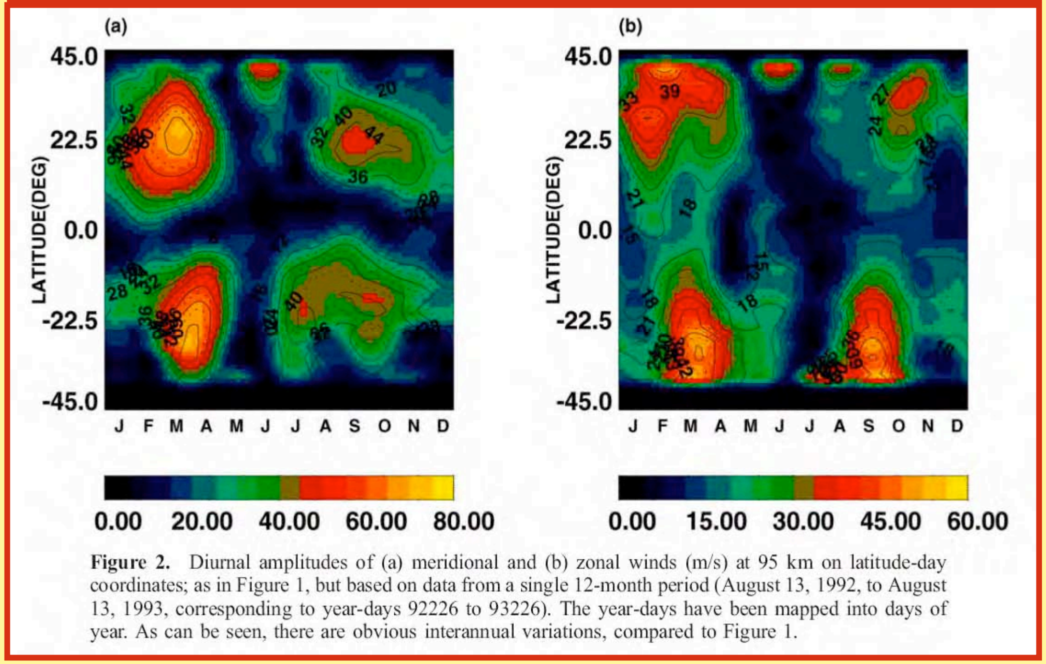


Figure 2. Diurnal amplitudes of (a) meridional and (b) zonal winds (m/s) at 95 km on latitude-day coordinates; as in Figure 1, but based on data from a single 12-month period (August 13, 1992, to August 13, 1993, corresponding to year-days 92226 to 93226). The year-days have been mapped into days of year. As can be seen, there are obvious interannual variations, compared to Figure 1.

Huang and Reber, 2002

Diurnal tides observed by UARS/HRDI at 95 km.

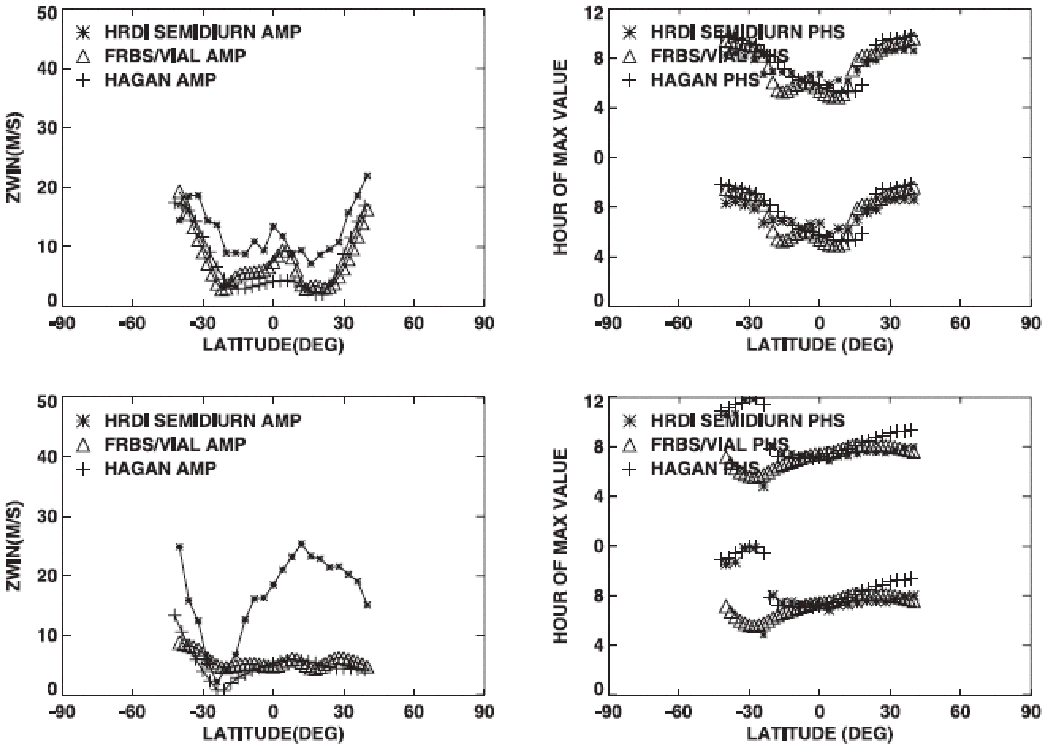


Figure 7. As in Figure 6, but for zonal winds. Semidiurnal amplitudes (m/s) and phases (hour of maximum value) versus latitude at 95 km, based on zonal mean composite data. The days of year correspond to 75 (top row) and 165 respectively. Also plotted are corresponding results (asterisks) from *Forbes and Vial* [1989] and the GSWM-00 by Hagan (plusses). Note the agreement in the phases is especially good compared to that of Hagan.

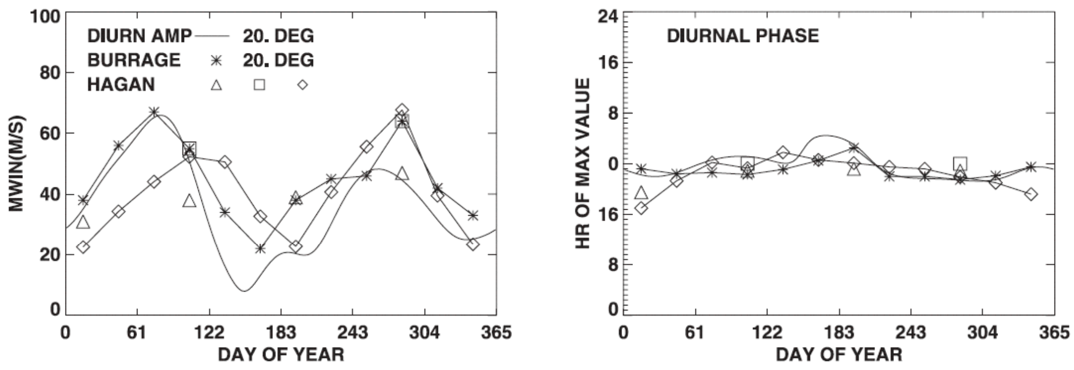


Figure 3. Meridional diurnal amplitudes (m/s) and phases (hour of maximum value) at 20°N latitude versus day, based on zonal mean composite (from several years) data, at 95 km. Also shown are results from Figure 2 of *Hagan et al*. [1999]. Hagan et al. included results from the GSWM-1998 (squares), 1995 (triangles), and results from *Burrage et al.* [1995b] (asterisks), which are based on monthly averages of HRDI data. Diamonds represent updated results by Hagan et al. (GSWM-00, NCAR CEDAR database), at 21°N latitude.

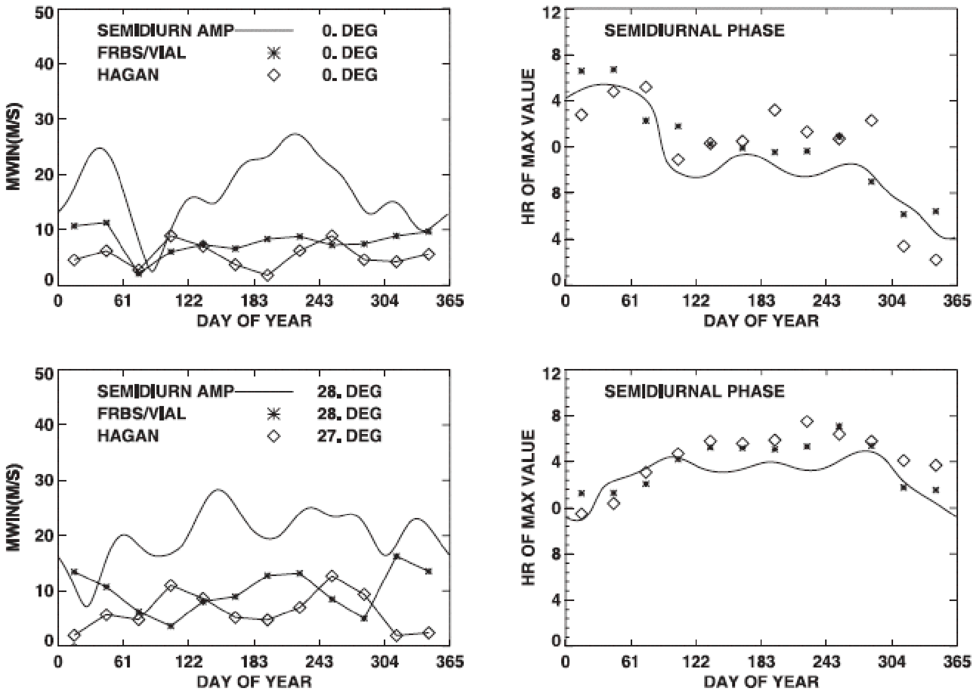


Figure 8. Semidiurnal amplitudes and phases versus day of year at 0° (top row) and 28° latitudes for zonal mean meridional winds at 95 km based on composite HRDI data. Also plotted are corresponding results (asterisks) of *Forbes and Vial* [1989] and Hagan's GSWM-00 (diamonds).

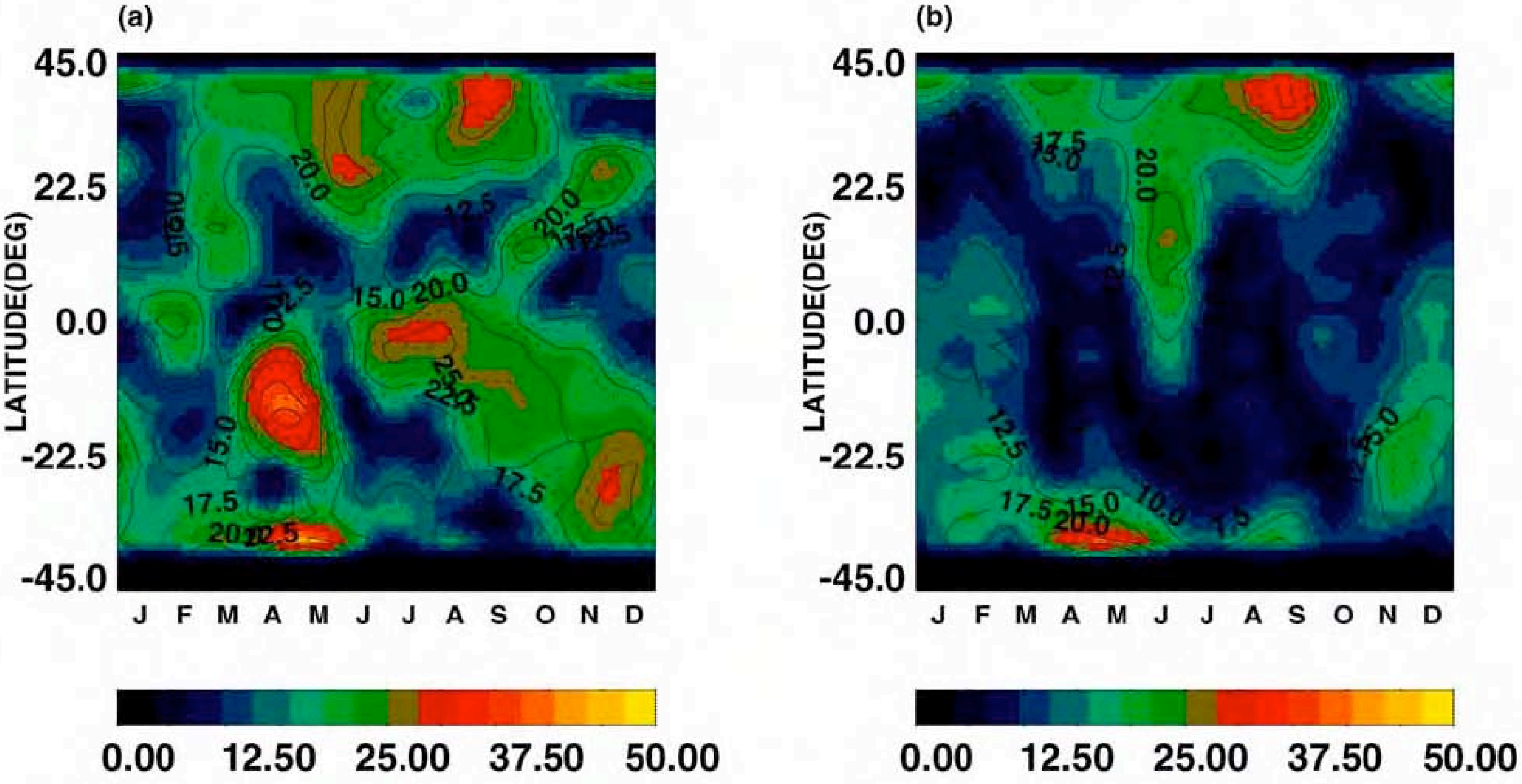


Figure 5. As in Figure 1, but for semidiurnal components. Amplitudes of (a) meridional and (b) zonal winds (m/s) at 95 km on latitude-day coordinates, based on zonal mean composite data. The amplitudes

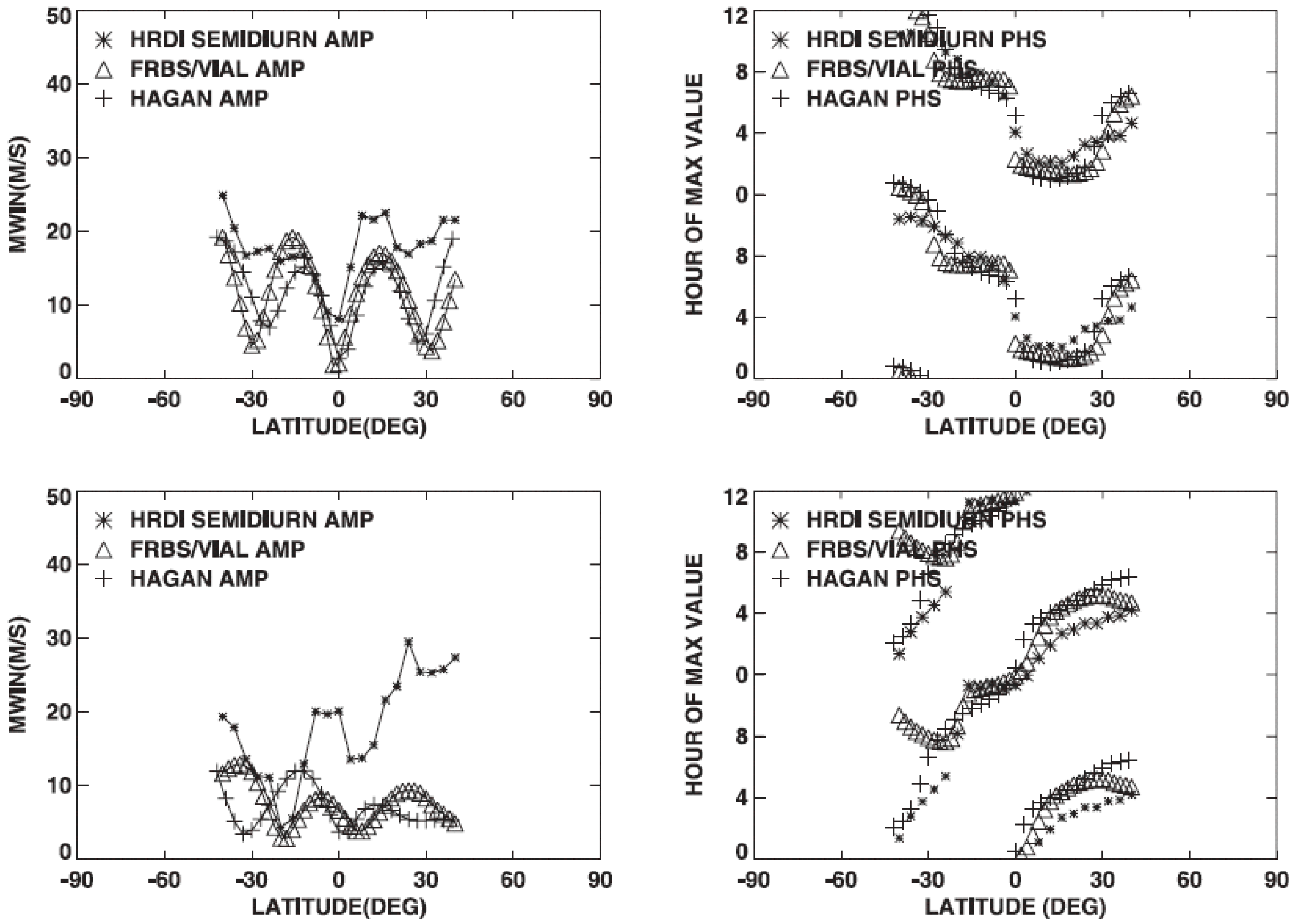


Figure 6. Meridional wind semidiurnal amplitudes (m/s) and phases (hour of maximum value) versus latitude at 95 km, based on zonal mean composite data. The days of year correspond to 75 (March 16, top row), and 165 (June 14) respectively. Also plotted are corresponding results (triangles) from *Forbes and Vial* [1989] at 96 km, and the GSWM-00 of Hagan (plusses) at 94.58 km.