30552 – Lecture 9.

The Shape of the Earth - MSS and gravity field models and "geophysical applications"

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Before we start:

If you feel ill, go home

Keep your distance to others

Wash or sanitize your hands

Disinfect table and chair

Respect guidelines and restrictions

Content:

Introduction

The Mean sea surface, Geoid and
Mean Dynamic Topography

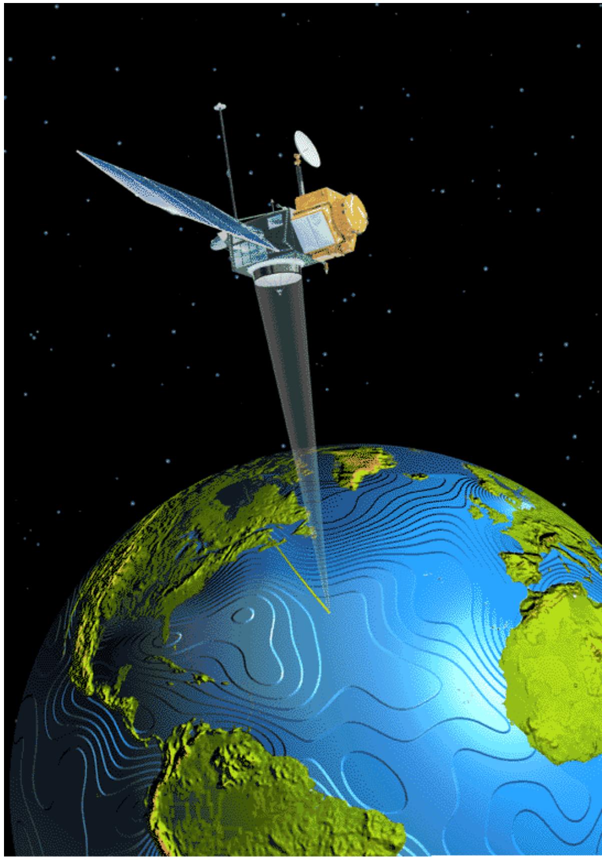
Background

The reference Potential of the Earth
Normal gravity and Earth Rotation.
The geoid.
Disturbing Potential and Bruns formula
Height and Height systems.

Computing the MSS

Repeated Tracks
Geodetic Tracks (crossover adjustment)

Gravity
Bathymetry and Plate Tectonics
Mean Dynamic Topography and Ocean Currents



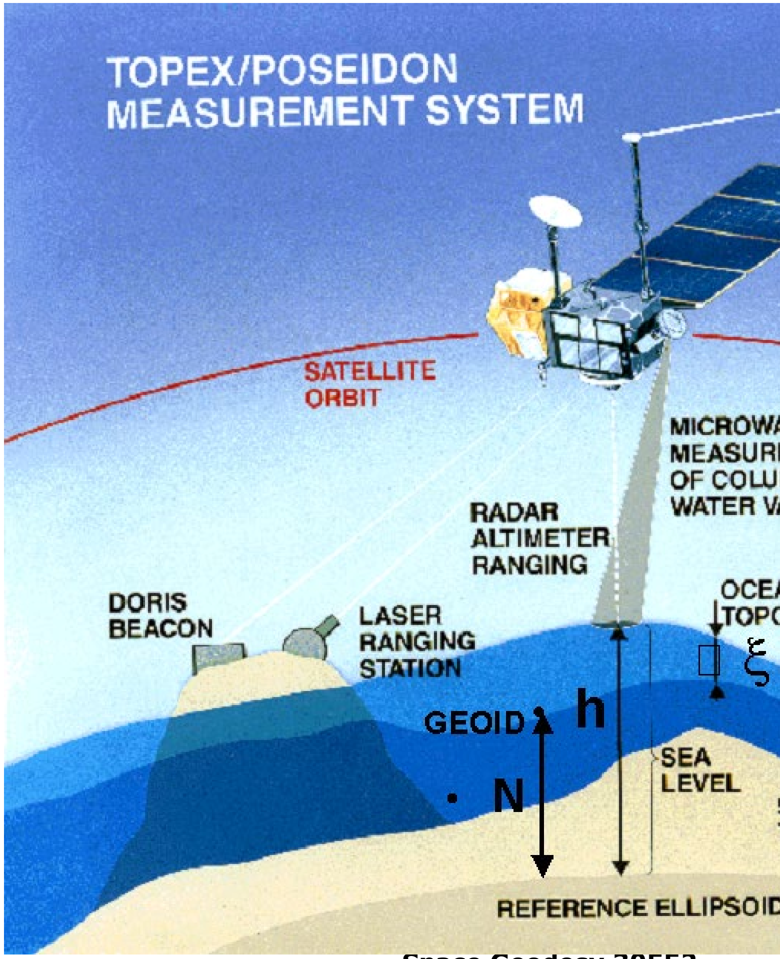
Satellite observes sea level (SWH and wind speed)

Sea Level is key-parameter for geodesy

Sea level h = *MSS* + dynamic sea level *ξ(t)* + error (e).

$$h(t) = MSS + \xi(t) + e$$

$$MSS = \frac{1}{N}\sum_{1}^{N}\xi(t)$$



Mean sea surface (MSS).

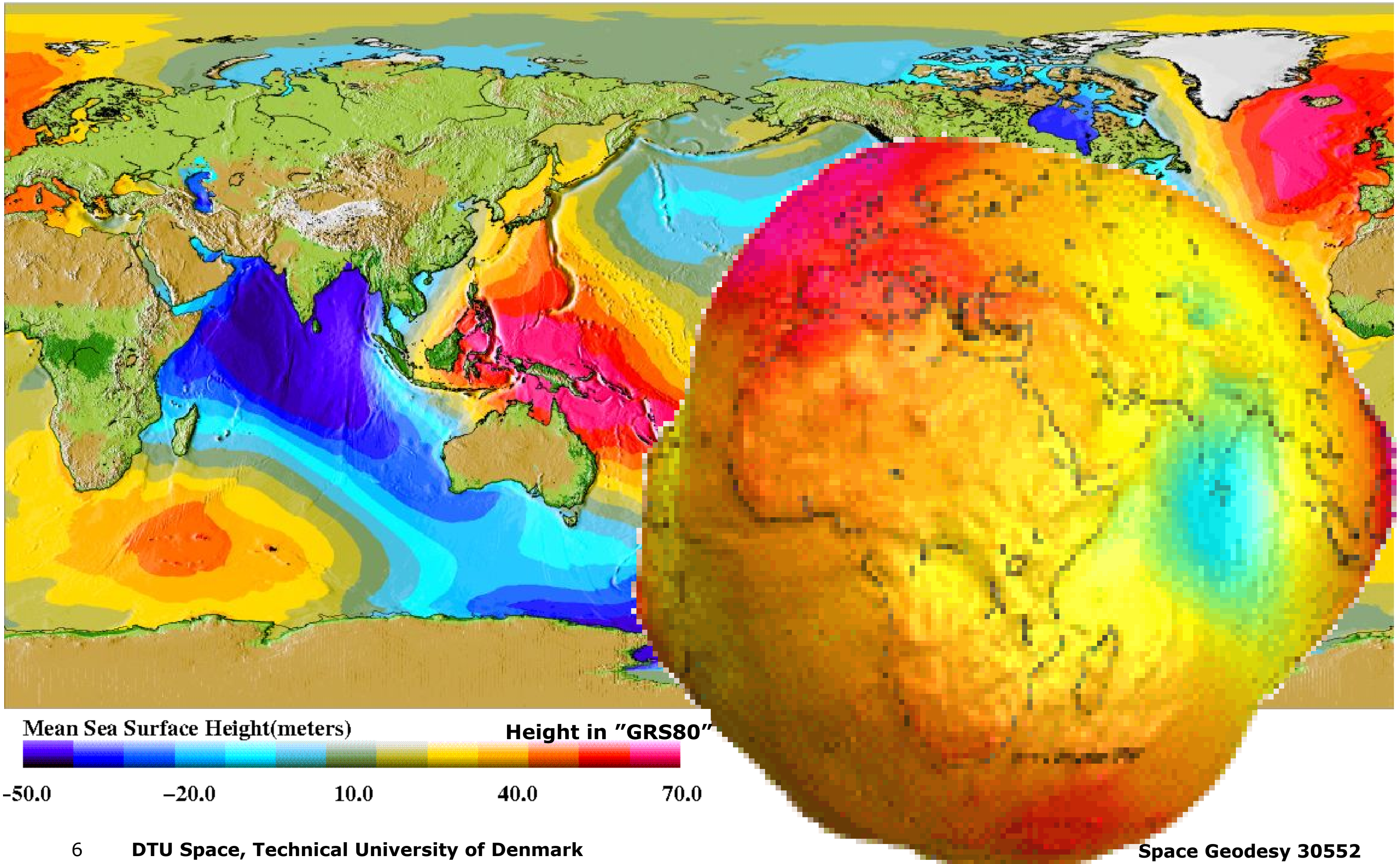
Mean sea surface or mean sea level (GEOMETRICAL).

- **NOT DEFINED EVERYWHERE (ONLY OCEANS)**
- **Conveniently given on grid**
- **Only ocean grid cells**
- **"spherical harmonics are no good"**

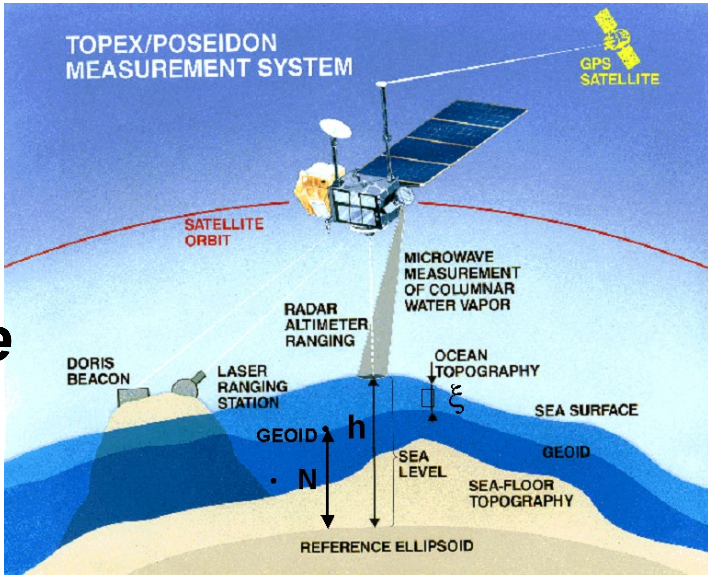
- **ITS AVERAGED OVER A CERTAIN TIME.**
- **"It changes a little with time".**
- **Global Mean Sea Level (GMSL) rise……**

- **Fundamental to height systems (it defines t**
- **Fundamental to determining flooding/storm**

Geoid or Mean Sea Surface



MSS and the Geoid(N)



MSS is the physical mean sea surface

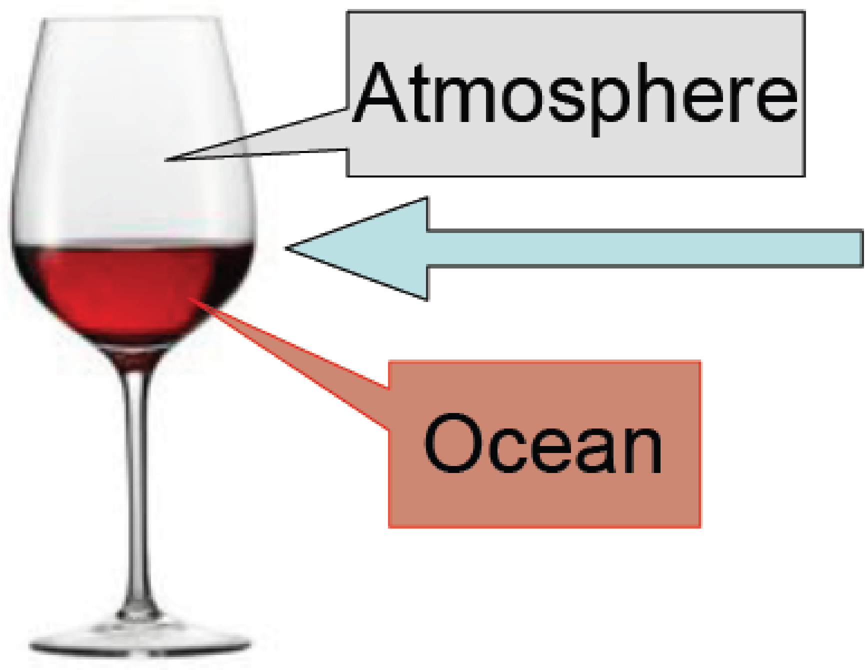
Geoid is the sea surface at rest.

Difference is the Mean Dynamic Topography

$$MSS = N + MDT$$

If there was no currents (ocean at rest) and temperature differences then

MSS = N and MDT = 0.

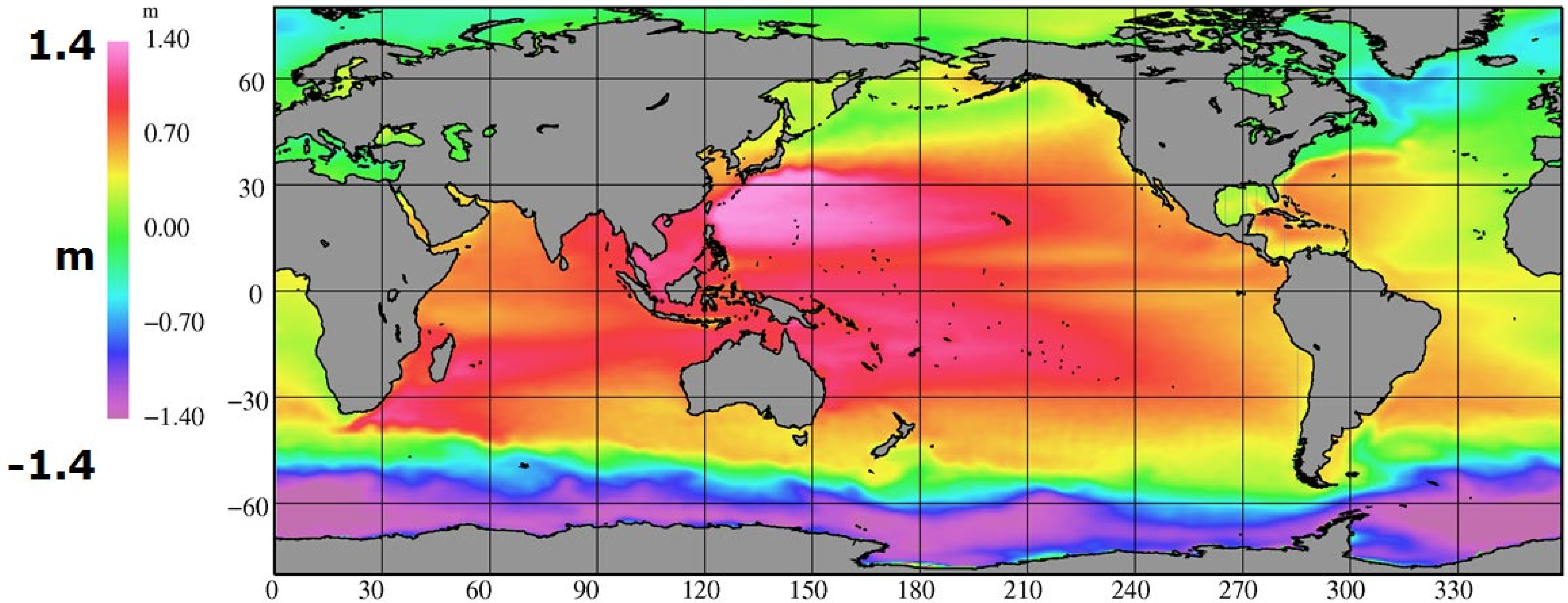


Discussion – Geoid + height

Where is the geoid important?.
What happens if the geoid is wrong?

Which height do you want?
 What your preferred height of the ocean?
 Whats the height of "flooding"

How big is the Mean Dynamic Topography.



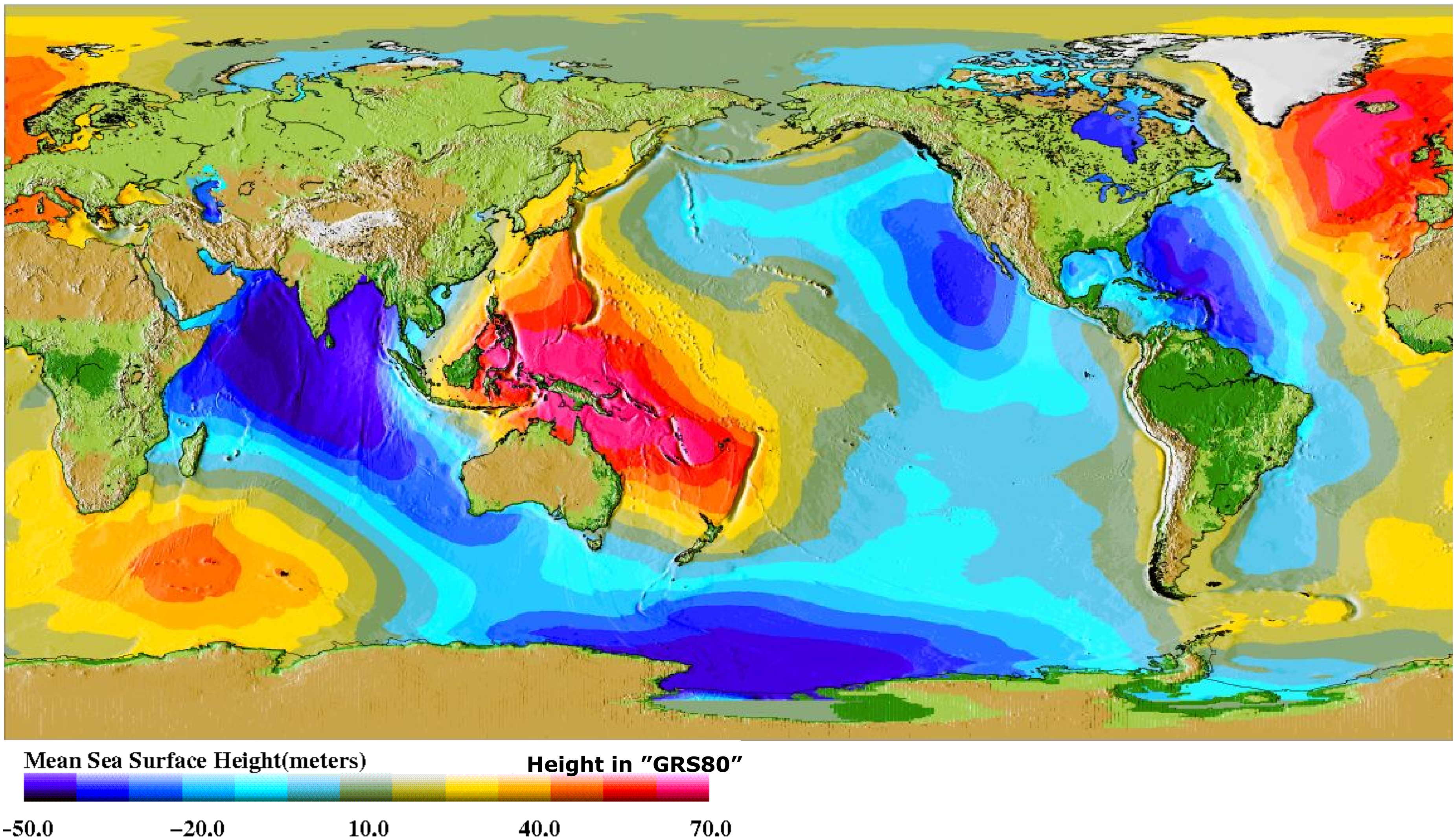
$$MSS = N + MDT$$

The MDT contains info on all major Currents in the world.

MDT also reflect sality and temperature (warmer water stands higher due

To thermal expansion). Change in the MDT will be due to climate change..

Geoid or Mean Sea Surface



Noticing

- The geoid is "height" of a certain constant potential surface.
- Its given relative to the reference Ellipsoid.
- So it can not represent the "full potential" / full gravity field of the Earth.
- Only the "local" or some anomalous or disturbing potential.

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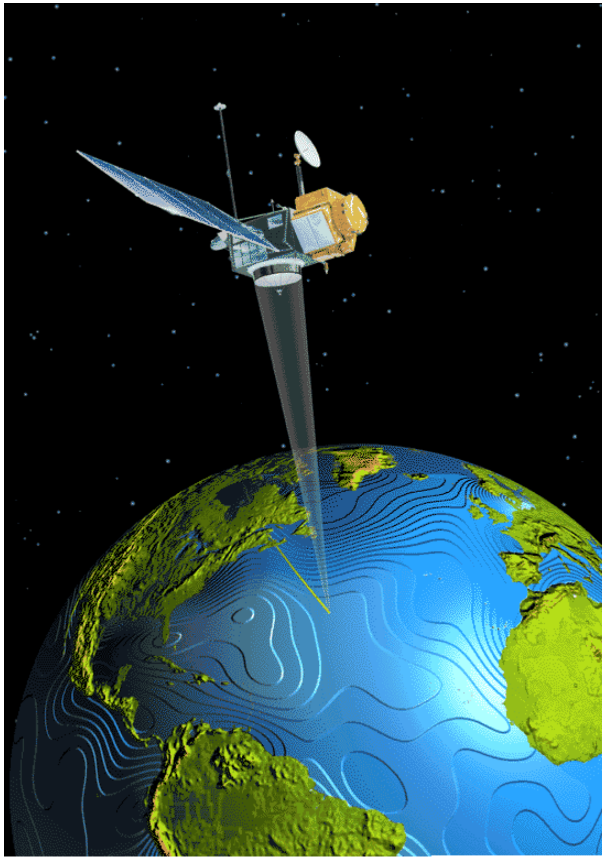


Gravity field of the Earth

- The Gravitational potential W is given as the sum of the gravitational potential V and the Centrifugal acceleration Φ like

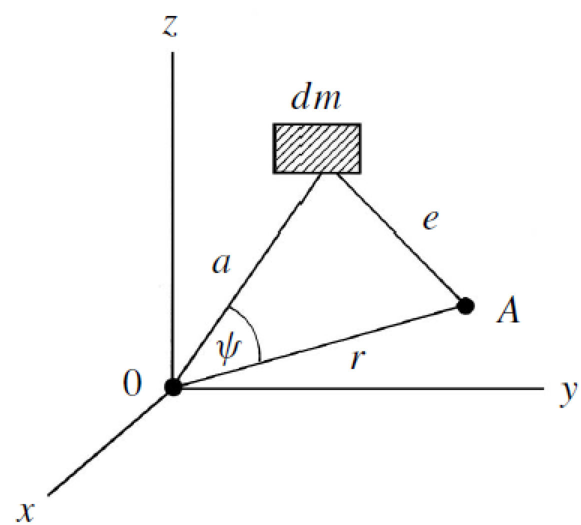
$$W = V + \Phi,$$

- Where

$$V = G \iiint_{\text{Earth}} \frac{dm}{e}, \qquad (12.7)$$

where

A	attraction point,
dm	mass element,
G	gravitational constant,
e	distance of the mass element from the attraction point,
a	distance of the mass element from the geocenter, and
ψ	central angle, corresponding to the spherical distance on a unit sphere.

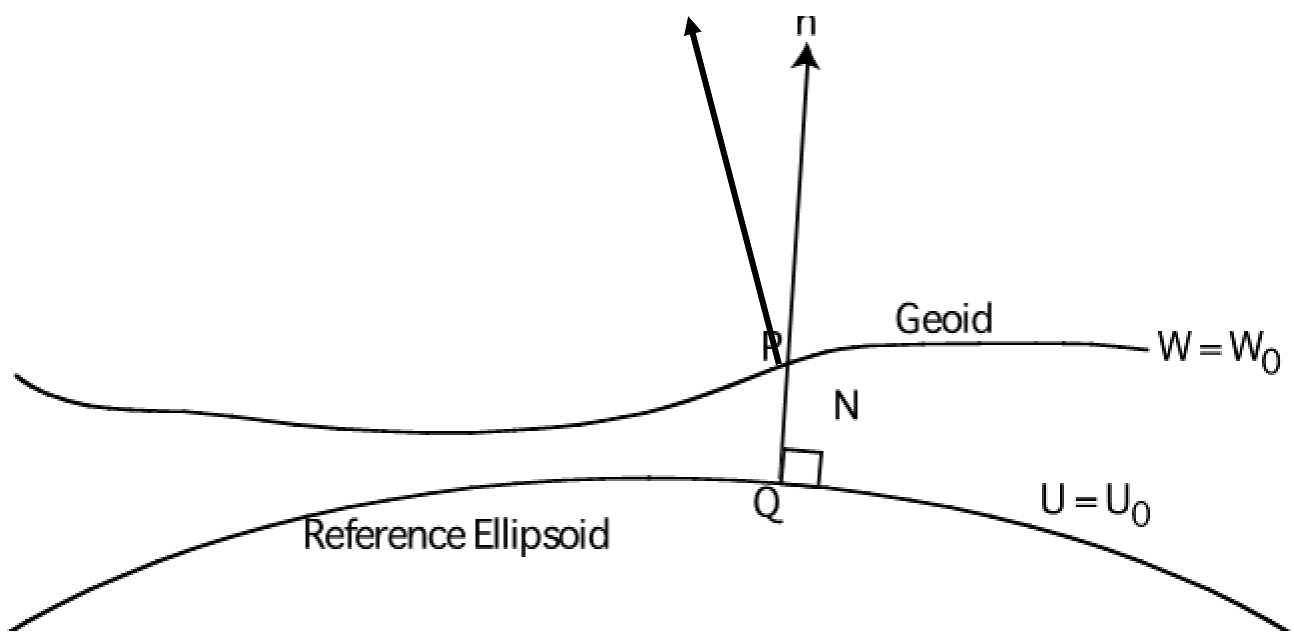


The geoid

If we define the actual potential on the geoid to be W_p and the normal potential as U_P, then the difference will be:

$$T_P = W_P - U_P$$

which we call the *disturbing potential* (the rotational term differences out).



Bruns formula

The value of the *geoid height, N,* can be determined from the disturbing potential in the following way. The reference (normal) gravity at point P will be:

$$U_P = U_Q + \left(\frac{\partial U}{\partial n}\right)_Q N$$

$$\gamma_Q = -\frac{\partial U}{\partial n}$$

$$U_P = U_Q - \gamma_Q N$$

Since

$$W_P \cong U_Q$$

$$N = \frac{T_P}{\gamma_Q}$$

which is *Brun's formula* .

Geoid and Gravity.

Gravity on the geoid and the reference ellipsoid will be given by:

$$\bar{g} = \nabla W_P \quad \text{(on the geoid)}$$

$$\bar{\gamma} = \nabla U_Q \quad \text{(on the reference ellipsoid)}$$

The difference in the direction of the two vectors $\vec{g} - \vec{\gamma}$ is called the *deflection of the vertical*. The difference in magnitude:

$$\Delta g = |\vec{g}| - |\vec{\gamma}|$$

is called the *gravity anomaly*. The gravity anomaly can be directly observed — measure gravity on the surface of the Earth, reduce it to the geoid, and then difference with respect to reference gravity γ.

The reference Ellipsoid

Reference ellipsoid is the (first order) Mathematical shape of the Earth (using a, e etc). **This enable establishing coordinates of our OBSERVATIONS and eg., determining the height of the spacecraft.**

Rather than working in Height of
6000 km +/- 100 meters
We isolate +/- 100 meters

We reference height etc
To the ellipsoid

Normal Gravity.

Normal gravity (aka *reference gravity*) is the gravity field that would result if the Earth was a perfect rotating ellipsoid. Normally, such an ellipsoid is defined (e.g. Geodetic Reference System of 1980, GRS1980) by the following parameters:

a = semi-major axis = 6378137 m = R_E

f = flattening = 1/298.257

GM = 398600.5 x 10^9 m^3/s^2 (or g_e = equatorial gravity = 978.03267 gal)

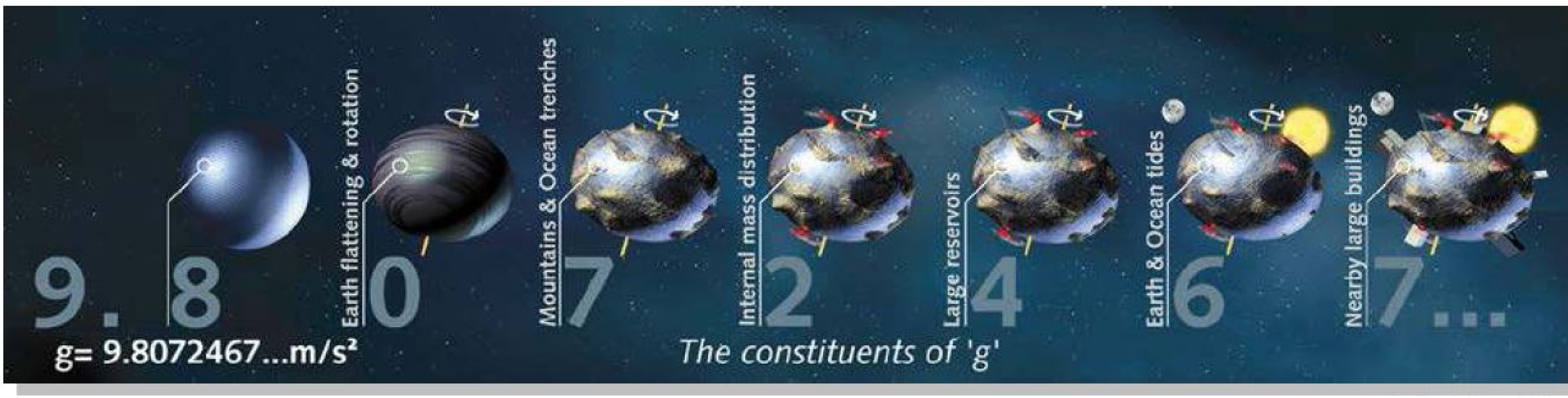
ω = Earth's rotation rate = 0.72921151 x 10^{-5} rad/sec

We can calculate the *gravitational acceleration, b,* on the surface of a spherical Earth:

$$b = \frac{GM}{R_E^2} \cong 9.7982 \text{ ms}^{-2} \approx 980 \text{ gals}$$

We define the unit gal in cgs units as 1 gal = 1 cm s^{-2} = 0.01 m s^{-2}. Surveying and static gravity observations require accuracies of a milligal, 1 mgal = 0.001 gal. To measure changes in gravity over time, the changes are typically measured in microgal, 1μ = 10^{-6} gal.

Gravity: 10m/s²  **or ...?**



- Look at more local things that changes conditions where we are. (Local Mountain, Building, Reservoir, Sub-suface geological structure (oil/mineral)

Gravity with height

Let's look at the how this acceleration changes with radial distance near the Earth:

$$b = \frac{GM}{r^2}$$

$$db = -2\frac{GM}{r^3}dr$$

$$db = \frac{-2g}{r}dr$$

$$db = \frac{-2(980 \text{ gals})}{6378127 \text{ m}} = \frac{-0.3 \text{ mgal}}{\text{meter}}$$

At the top of Himalaya (9 km) gravity will be 9000 meters times -0.3 mgal/m= - 3 gal (less than at the geoid).

If we change the height with 3 mm we change the gravity with -1 micro Gal. **So you can use accurate gravity instruments to measure height.**

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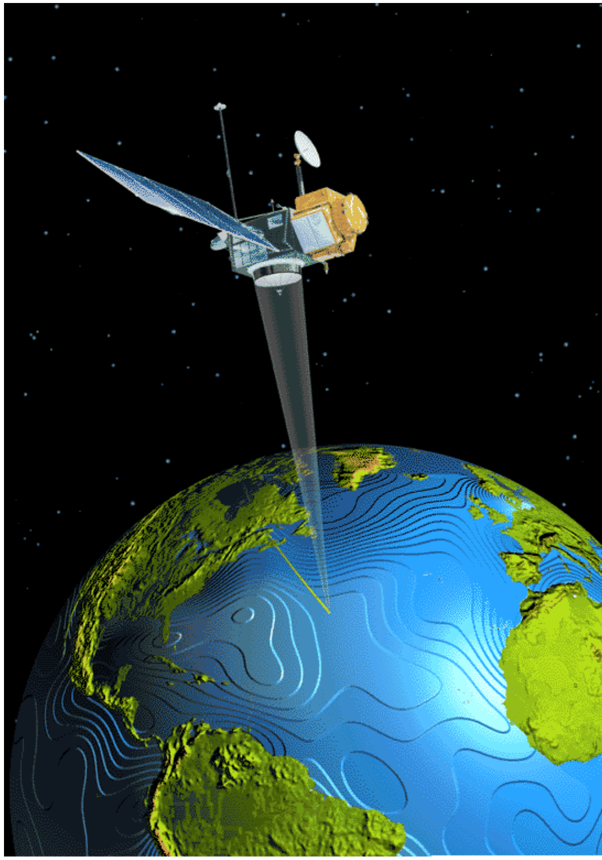
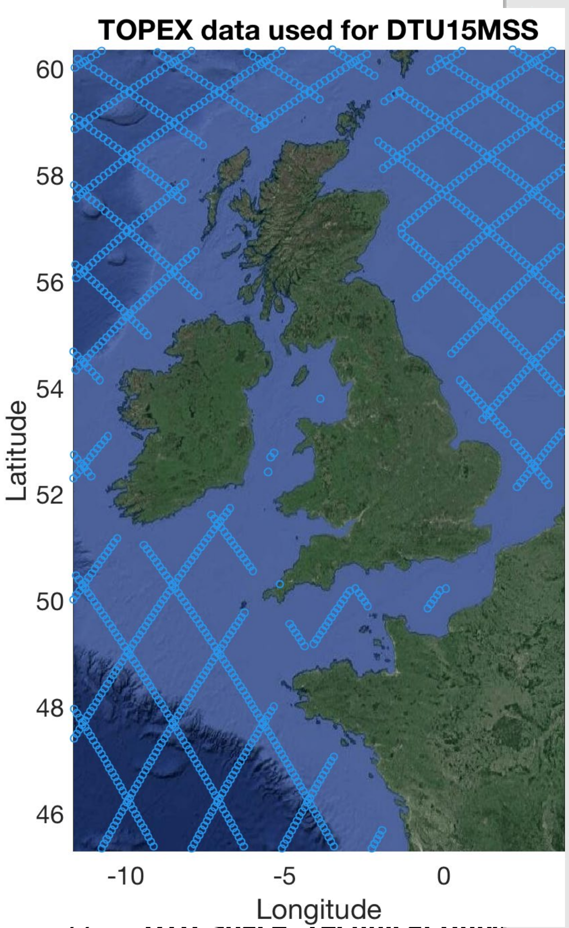
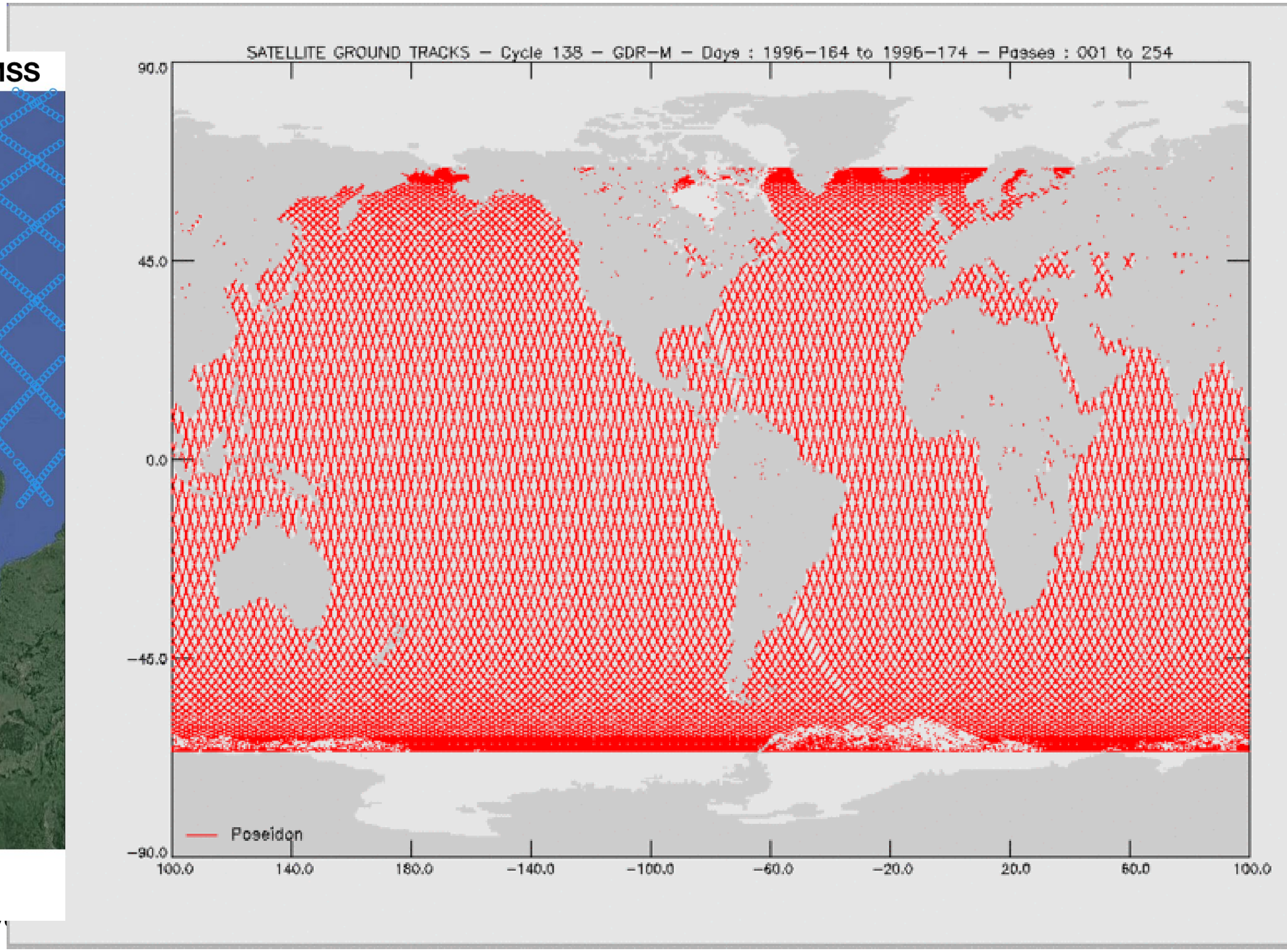


Lets Derive the Mean Sea Surface

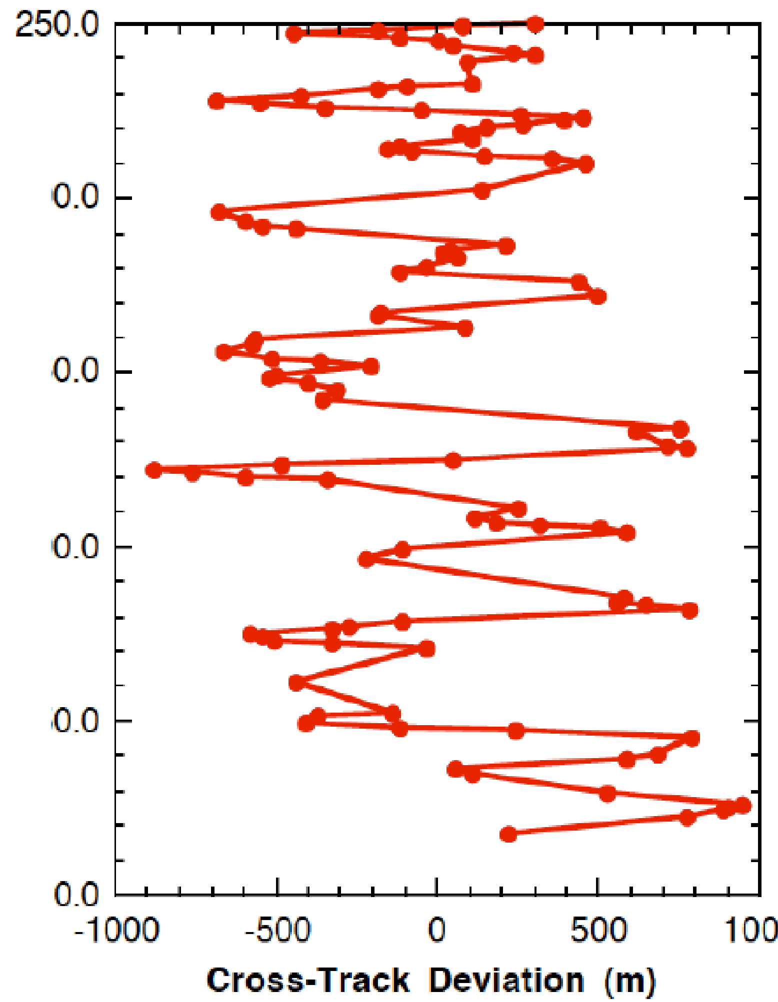
- Use the reference satellite tracks.
- Sampled every 10th day for more than 25 years





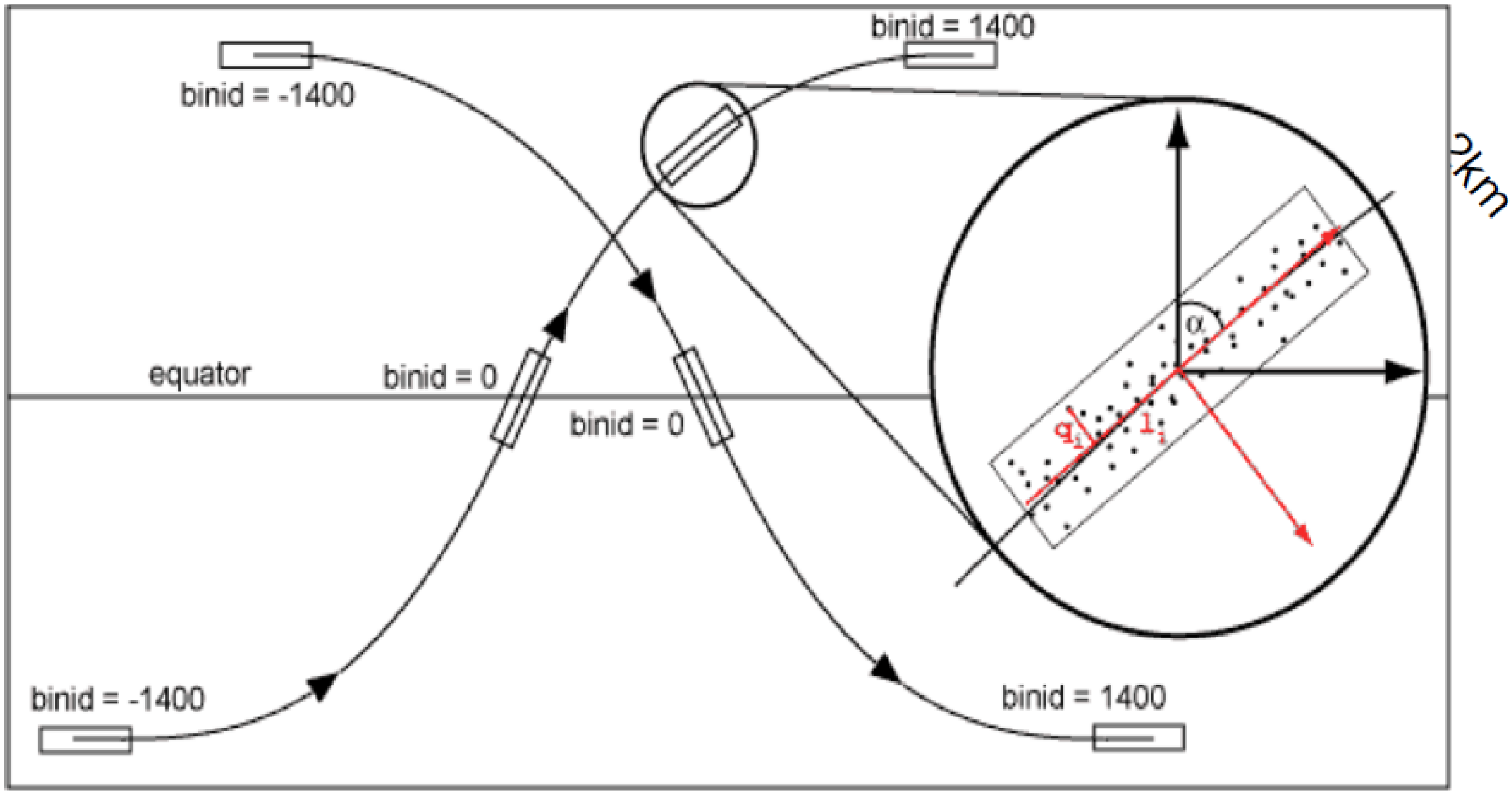
MSS

- Altimeter samples the SSH along the satellite ground-tracks every ~7 km (1 Hz data). Hence, using data of a single satellite, we can't expect to get the MSS with a better resolution than 7 km along-track. Across-track, the resolution is much lower.
- If the altimeter would take repeated measurements above the same sub-satellite point, it would be straightforward to compute the MSS at the sub-satellite point.
- In reality, the ground-track does not repeat exactly:
    - non-conservative forces (e.g., atmospheric drag) and time-dependent gravitational perturbations cause the ground-track to drift eastward over time.
    - Orbit maintenance maneuvers cause the ground-track to drift back westward, until drag again begins to cause an eastward drift.
- Hence, repeated ground-tracks scatter around a nominal ground-track (cross-track deviation).



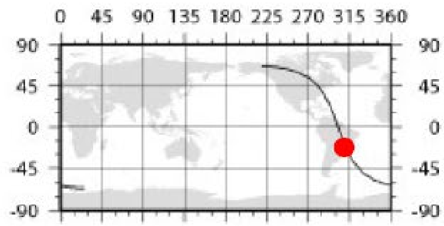
Cross-track deviation of TOPEX/Poseidon (T/P)

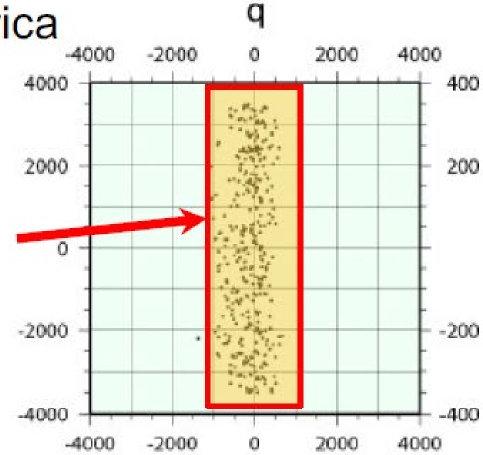
Nominal ground track, maintained within ± 1 km, is used to define geographically fixed bins of 2 km width on ascending and descending tracks.
Length of the bins is adjusted to the distance of 1 Hz measurements, which is ~7 km.
Hence, every bin gets at least one data per cycle. In the course of time, every bin is filled with measured sea surface heights, which refer to different epochs.
Bins are numbered relative to the equator crossing.



There is a clear annual signal.

Example for a bin close to the coast of South America
(Topex satellite, 10-day repeat, 1992-2003)

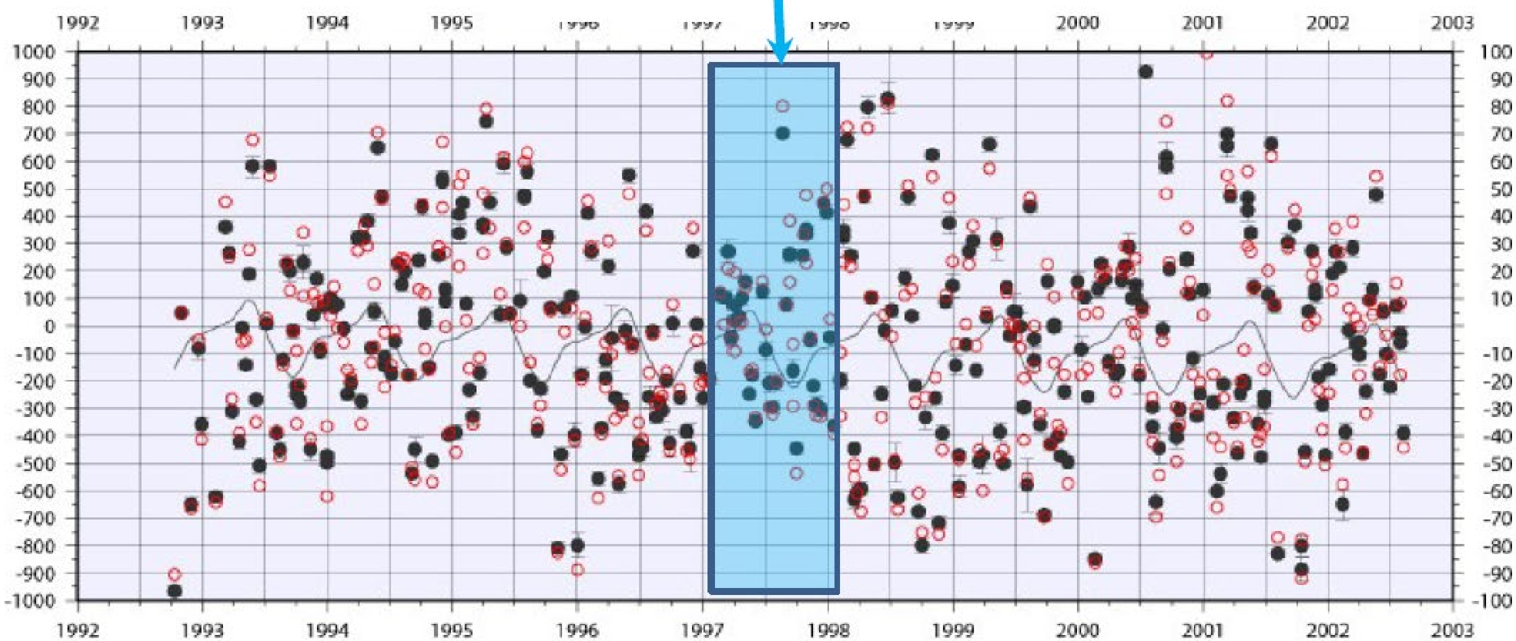




7x2 km bin

10-day repeat -> ~ 36
data points per year

432

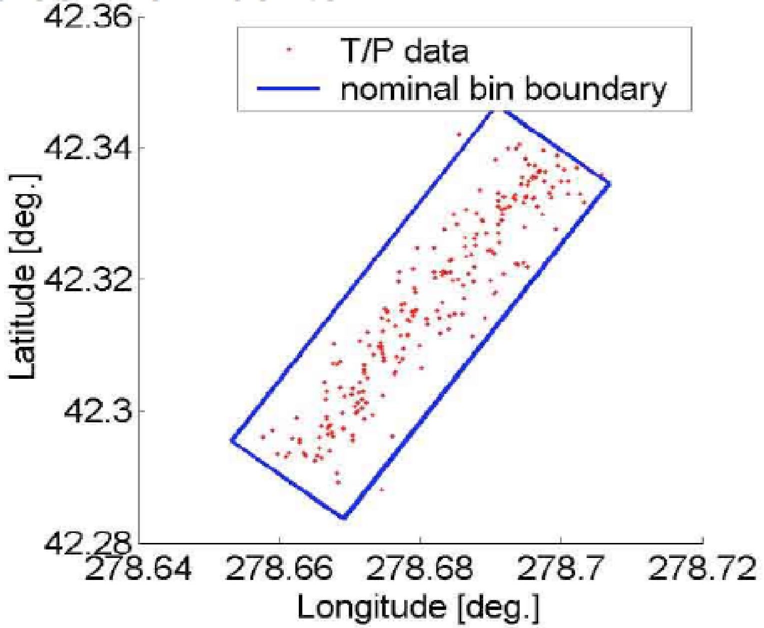


Model of temporal variations for data inside a bin

In reality, the sea surface changes a function of time and the afore-mentioned approach cannot be used directly to estimate the MSS at the bin center (parameter a). This can only be done if we remove the time variability from the observed SSHs. A simple model of the time variations at an arbitrary point within the bin comprises a secular rate, annual term, and semi-annual term:

$$SSH = g + k\,t + c\,\sin(2\pi t) + d\,\cos(2\pi t) + e\,\sin(4\pi t) + f\,\cos(4\pi t)$$

- g = offset
- k = secular rate
- c,d = amplitudes of annual term
- e,f = amplitudes of semi-annual term
- t = time relative to the reference epoch t_0 as a fraction of a year



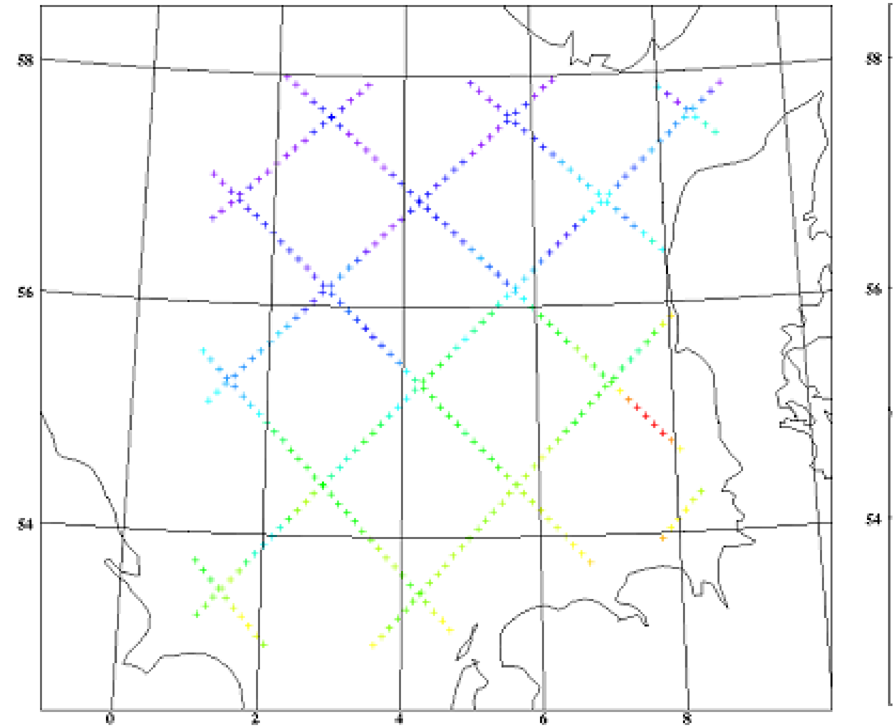
Extremely accurate MSS along the repeated tracks

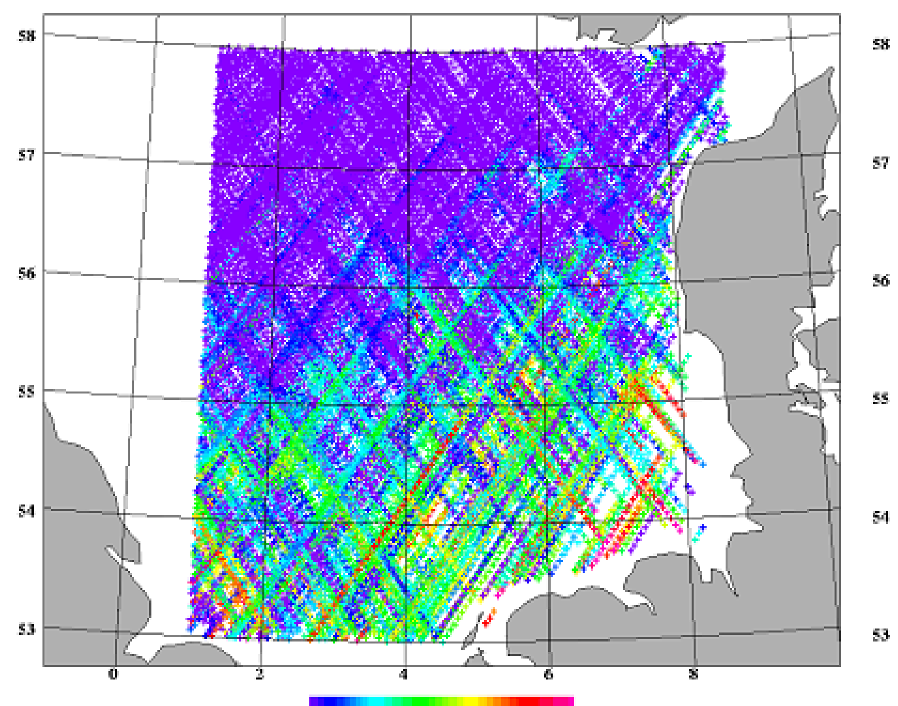
Inside the diamonds we use the data from the Geodetic Missions

These are not repeated. Hence SSH = MSS + ξ(t)

Solution:

Say that MSS must be the same at crossing loations
and that ξ(t) must be responsible for the differences.





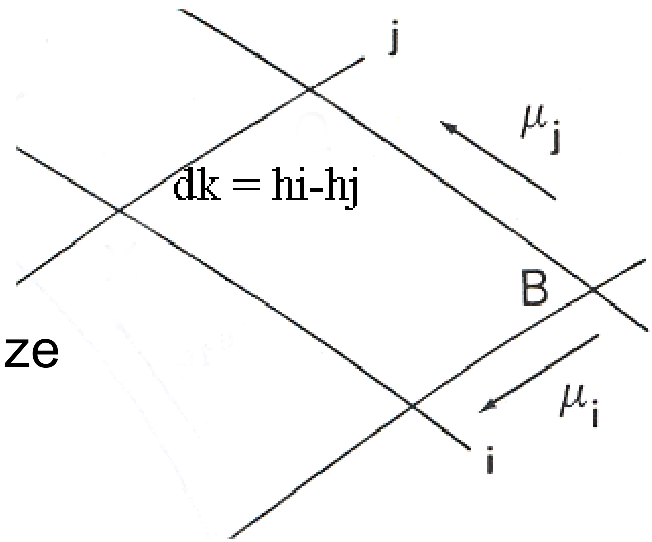
Crossover adjustment

- $\underline{d_k} = \underline{h_i} - \underline{h_j}$
- $\underline{d} = A\underline{x} + v$
- where $\underline{x}$ is vector containing the unknown parameters for the track-related errors.

  v is residuals that we wish to minimize
- Least Squares Solution to this is

$$\underline{x} = (A^T C_d^{-1} A + cc^T)^{-1} A^T C_c^{-1} \underline{d}$$

- Constraint is needed $\mathbf{c}^T\underline{x}=0$
- Case of bias – mean bias is zero



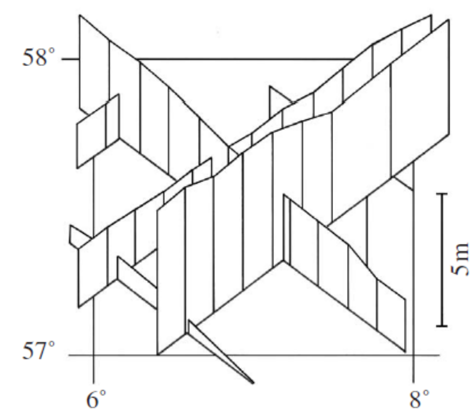
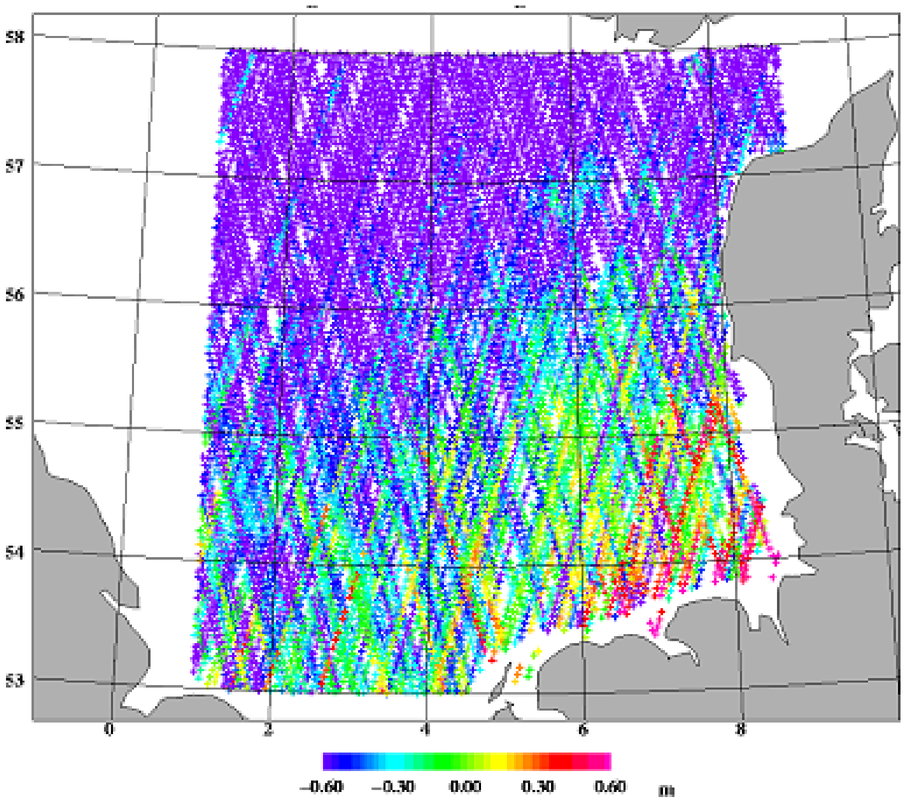
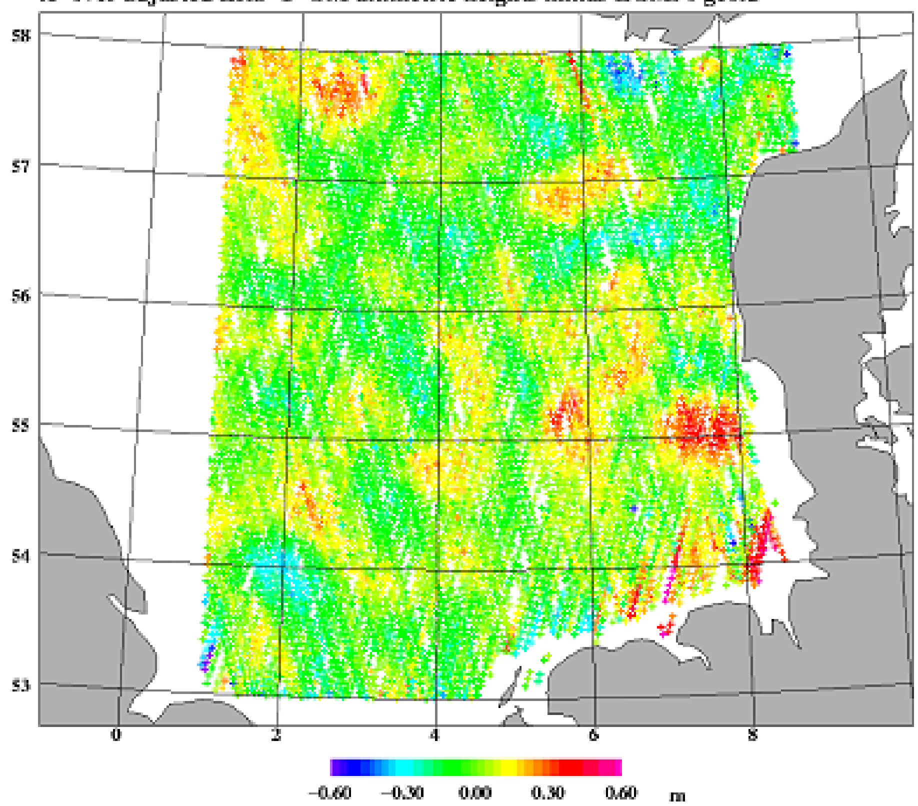


Figure 9.12. Example of altimeter data in the North Sea

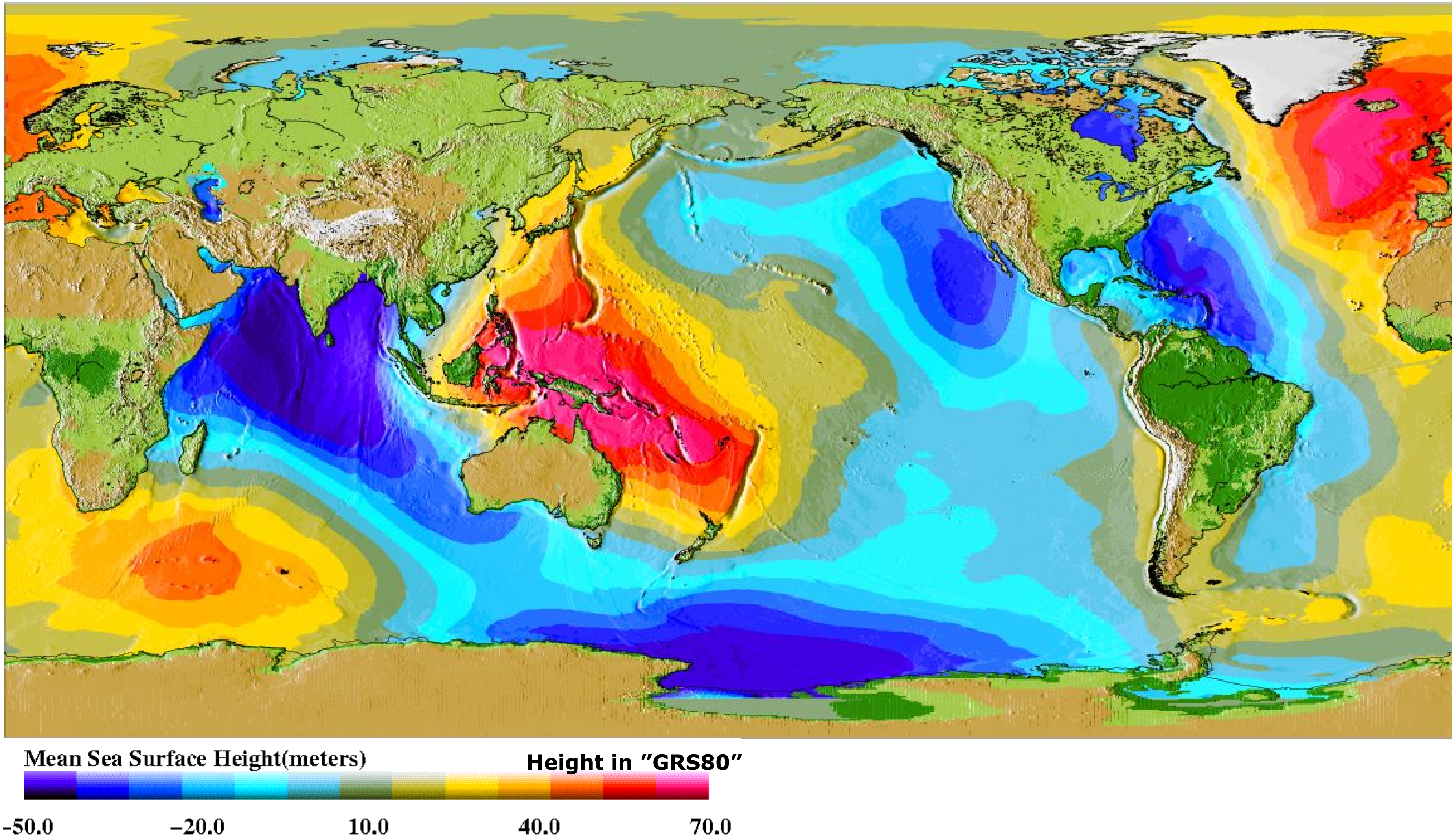
X-over adjustment (ex curriculum)

- This way you can determine the fine structure of the MSS inside the bins and resolve signal down to around 12 km.





DTU15 Mean Sea Surface



Mean sea surface (MSS).

Mean sea surface (GEOMETRICAL).

- **NOT DEFINED EVERYWHERE (ONLY OCEANS)**
- **Given on a 1 minute global grid**
- **Only ocean grid cells**

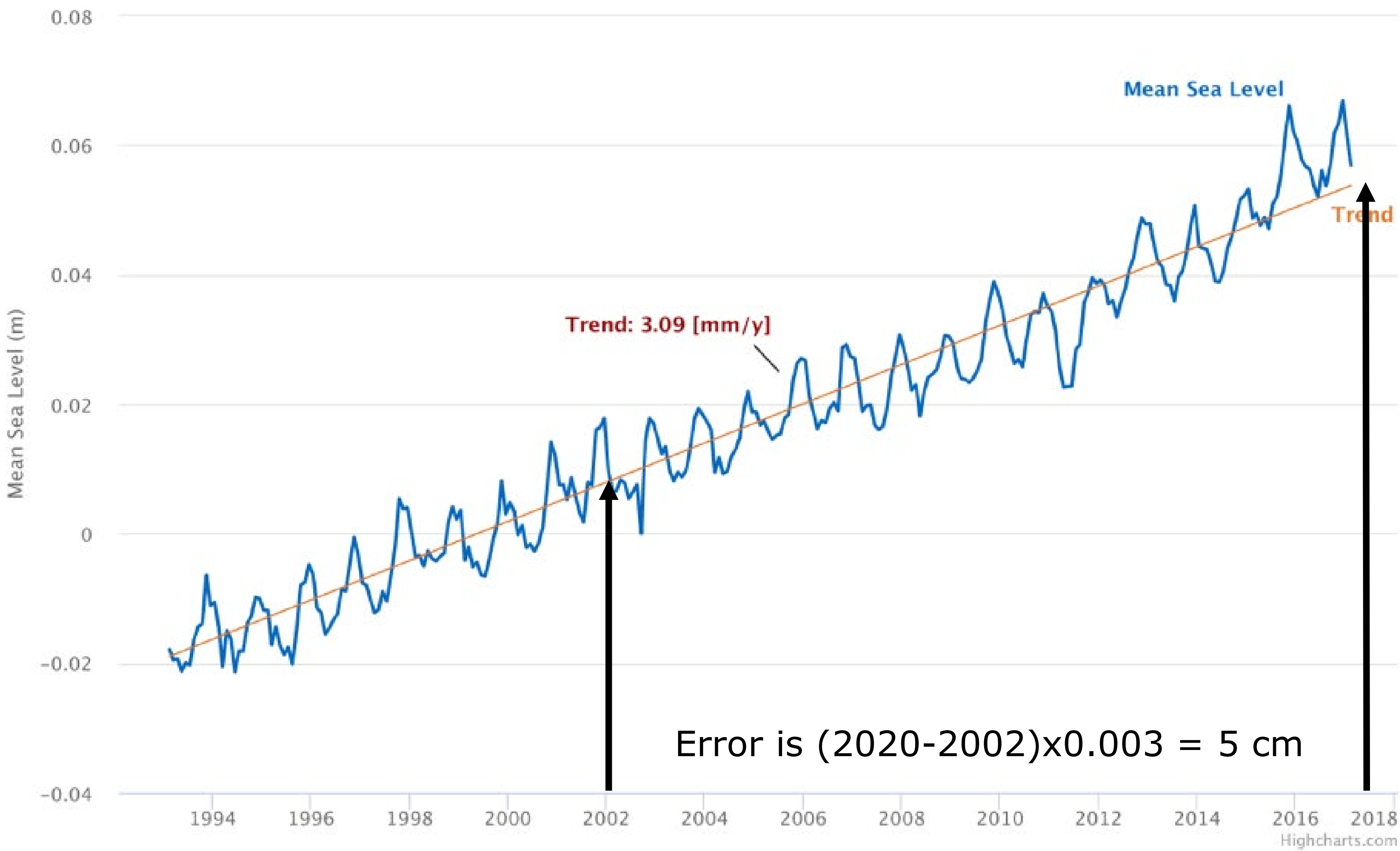
- **SEA LEVEL ANOMALY**

$$h(t) = MSS + \xi(t) + e$$

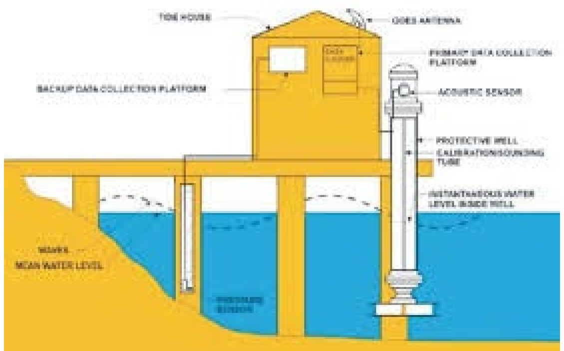
- **YOU HAVE AVERAGED ALL SATELLIE OBS OVER 20 YEARS.**
- **Typically 1993-2012 (Mean time is 2002)**

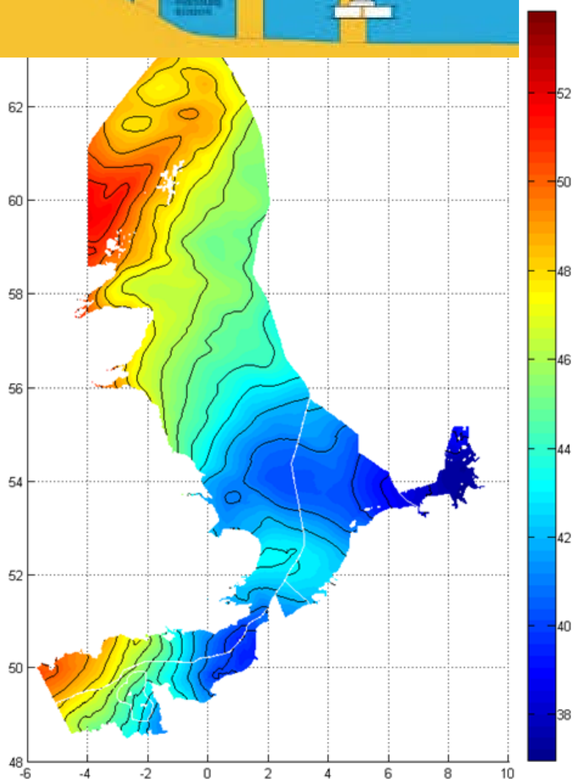
- **“It changes a little with time”.**
- **Global Mean Sea Level (GMSL) rise......**

Global mean sea level (GMSL) time series and estimated trend from multi-mission satellite altimetry (Jan 1993-Feb 2017)



Height systems
Denmark and Europe





Local Heights relative to EVRF2007
EVRF2007 transformed to ITRF 2008 (GRS80)

Country	VD	Offset (cm)
D	DHHN92 (normal heights)	1
F	NGF-IGN69 (normal heights)	-47
DK	DVR90 (orthometric heights)	0
N	NN1954 (orthometric heights)	-1
NL	NL_AMST / UNCOR (orthometric)	+2
UK	Newlyn (ODN) (ortometric heights)	+5
B	DNG (pure leveled heights)	-232

DVR90 is fitted to **mean sea level** in Aarhus Denmark in 1990…….
NOTHING TO DO WITH SATELLITES
"Its what we want locally that
(height zero=Mean sea level/surface)"

DVR90 -> GRS80 40 meters.

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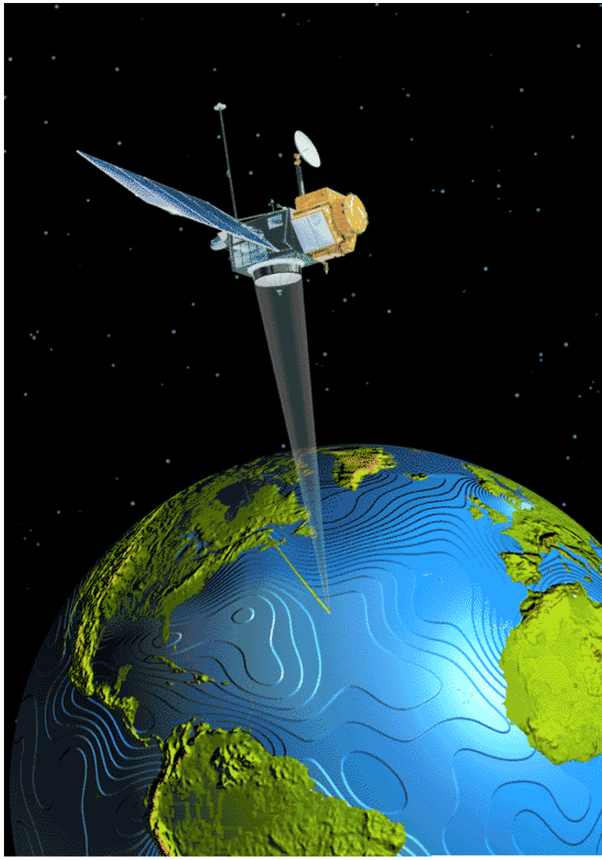
Applications
Predicting Local Gravity
Bathymetry and Plate Tectonics
Mean Dynamic Topography and Ocean Currents



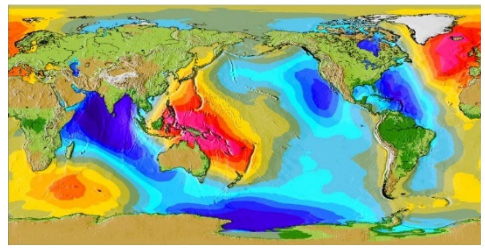
Application of satellite altimetry to Local Gravity and Geophysics

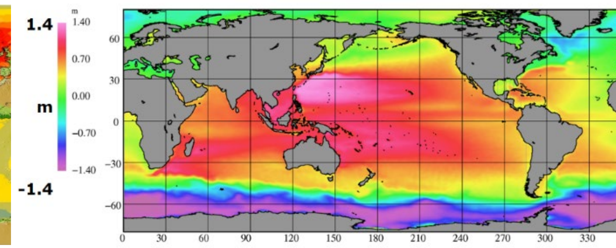
If we consider

$$MSS = N + MDT$$

Then

$$N = MSS - MDT$$





$$N = \frac{T_P}{\gamma_Q}$$

For the gravity anomaly, we can use the definition (given without proof):

$$\Delta g = -\frac{\partial T}{\partial r} + \frac{\partial \gamma}{\partial r} N$$

Again taking a spherical approximation:

$$\frac{\partial \gamma}{\partial r} = -2\frac{GM}{r^3} = -\frac{2\gamma}{r_E}$$

$$\Delta g \; : \qquad) = -\frac{\partial T}{\partial r} - 2\frac{T}{r} \approx -\frac{1}{\gamma}\left(\frac{\partial N}{\partial r} + 2\frac{N}{r}\right)$$

The Anomalous Potential.

- T is a harmonic function outside the masses of the Earth satisfying
- ($\nabla^2 T = 0$) Laplace (outside the masses) $\frac{\partial^2}{\partial r^2} + \frac{\partial^2}{\partial \varphi^2} + \frac{\partial^2}{\partial \lambda^2} = 0$
- ($\nabla^2 T = -4\pi\gamma\rho$) Poisson (inside the masses (ρ is density))

$$\Delta g = -\frac{\partial T}{\partial r} - 2\frac{T}{r} \approx -\frac{1}{\gamma}(\frac{\partial N}{\partial r} + 2\frac{N}{r})$$

- Where we used Bruns' formula

Gravity from Geoid

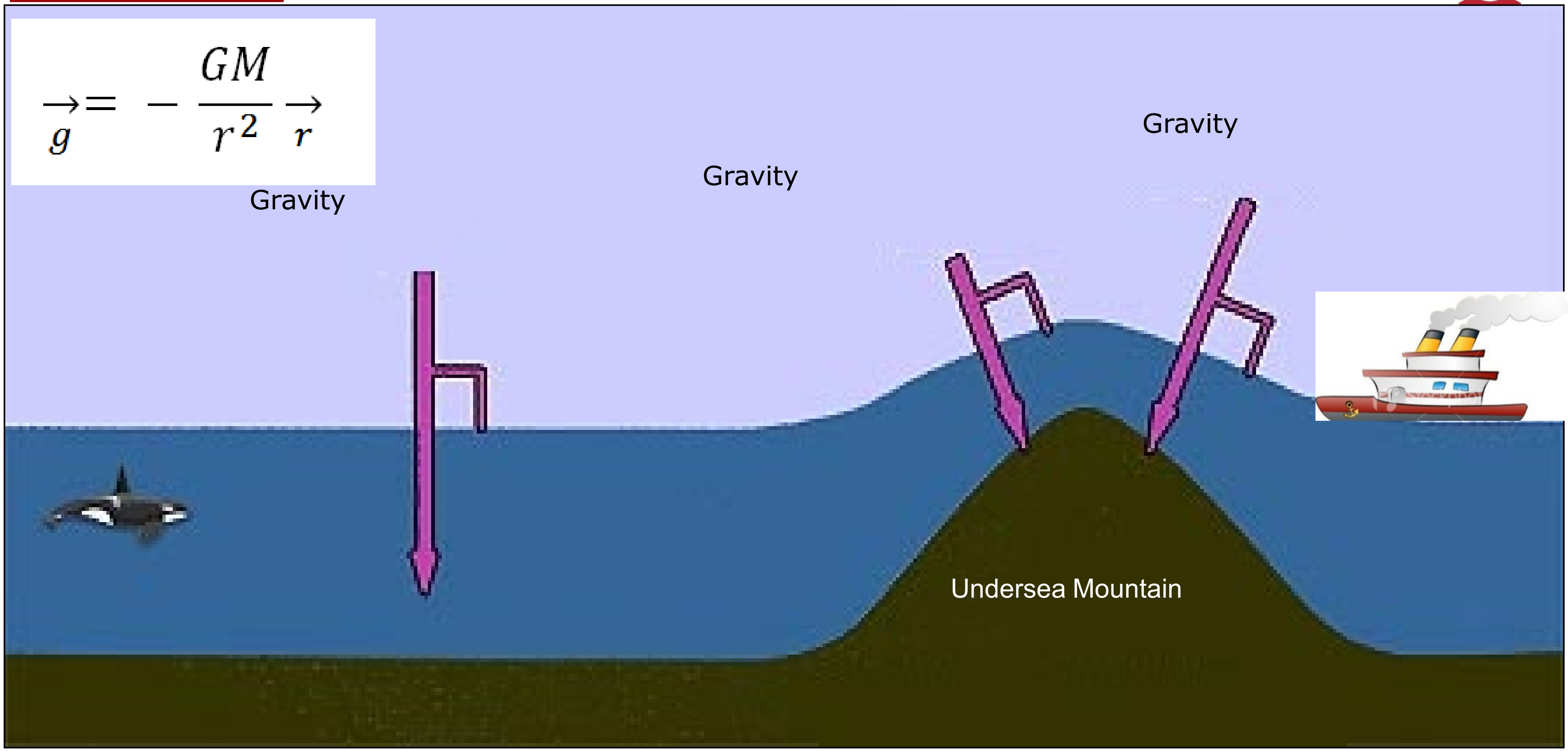
1) Integral formulas (Inverse Stokes + Vening Meinesz)
Requires extensive computations over the whole earth.

$$\Delta g_p = \gamma \frac{N_p}{r} - \frac{\gamma}{16\pi r} \iint_\sigma \frac{N - N_p}{\sin^3(\psi/2)} d\sigma$$

2) Fast Fourier Techniques.
Requires gridded data (using least squares collocation, using err stat)
Very fast computation.
Requires a Flat Earth Approximation.
Presently the most widely used method.

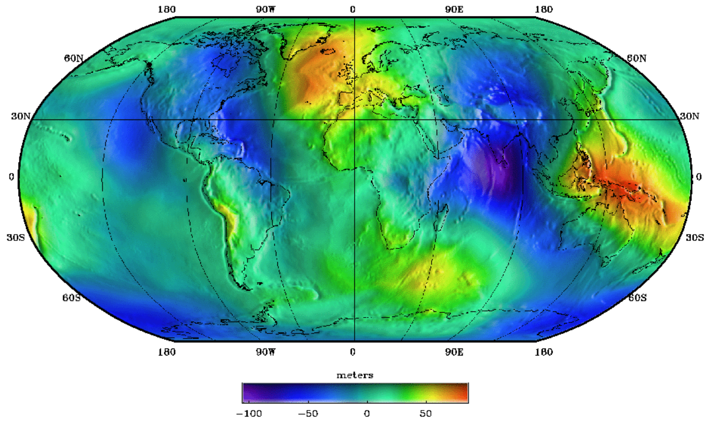
Getting Gravity from geoid

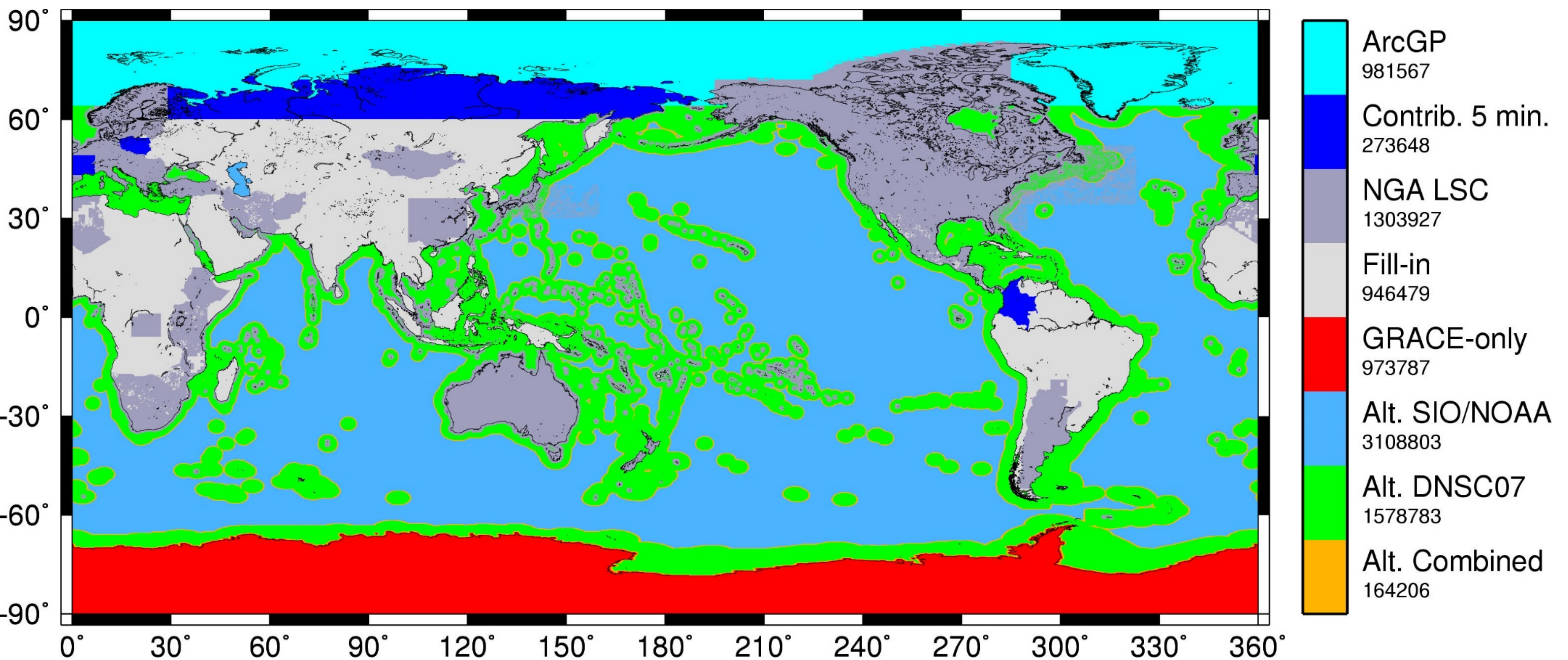
$$\vec{g} = -\frac{GM}{r^2}\vec{r}$$



- *Geoid is an equipotential surface of the Earth gravity potential N=W/γ.*
- ***Locally** Change in gravity/potential is related to local density change*
- *within the outer part of the Earth.*
- *Geoid reflects how the earth looks like beneath the ocean........*

The best global Geoid Model Earth Geopotential Model (EGM2008).





Residual gravity functionals

Model of the gravity potential of the Earth W_{GGM}

Residual gravity $F := g - g_{\mathrm{GGM}}$

Residual disturbing potential $h := T - T_{\mathrm{GGM}}$

- **Assumes GGM are "perfect" down to 100 km and remove these.**
- **Technique is called "REMOVE / RESTORE Technique.**
- **So we really look locally**

(outside pensum)

Flat earth approximation

In the map domain, we introduce Cartesian coordinates (x,y) (should not be confused with the coordinates (x,y) of the 3D terrestrial reference system used in the previous slide). Moreover, we add a third axis (the z-axis), which is orthogonal to the (x,y)-plane. The result is referred to as "a flat Earth approximation". That is, the (x,y)-plane represents the Earth's surface (here the ocean patch) and the area z>0 is the space outside the Earth's masses.

Consistent with the flat-earth approximation are the following equations:

$$\frac{1}{r}\frac{\partial}{\partial \varphi} = \frac{\partial}{\partial y} \qquad \frac{1}{r\,\cos\varphi}\frac{\partial}{\partial \lambda} = \frac{\partial}{\partial x} \qquad \frac{\partial}{\partial r} = \frac{\partial}{\partial z}$$

$$F = -\frac{\partial h}{\partial z} \qquad u \approx -\frac{1}{\gamma}\frac{\partial h}{\partial y} \qquad v \approx -\frac{1}{\gamma}\frac{\partial h}{\partial x}$$

(outside pensum)

Flat earth approximation and boundary value problem

After flat-Earth approximation, the residual disturbing potential h, which is defined in the neighborhood of the ocean patch, is a harmonic function for z>0, i.e., it fulfils Laplace's equation:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0, \quad z > 0$$

$$F = -\frac{\partial h}{\partial z} \qquad u \approx -\frac{1}{\gamma}\frac{\partial h}{\partial y} \qquad v \approx -\frac{1}{\gamma}\frac{\partial h}{\partial x}$$

F is residual gravity

$$-\gamma\frac{\partial F}{\partial z} = \frac{\partial u}{\partial y} + \frac{\partial v}{dx}$$

(outside pensum)

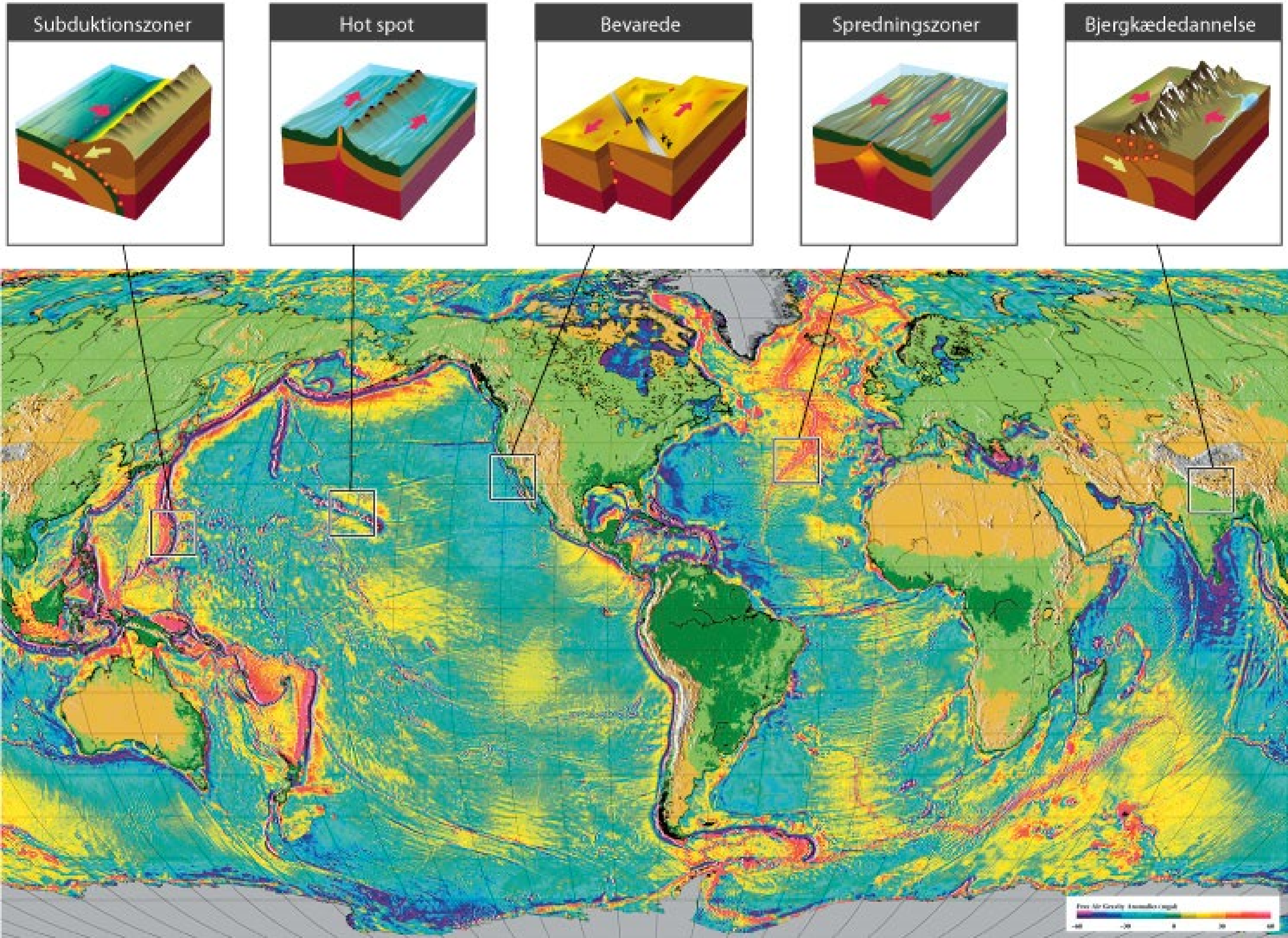
Solution in Fourier Domain

$$\hat{F}(\boldsymbol{k}, z) = 2\pi|\boldsymbol{k}|e^{-2\pi|\boldsymbol{k}|z}\,\hat{h}(\boldsymbol{k}, z) \Rightarrow \hat{F}(\boldsymbol{k}, z=0) = 2\pi|\boldsymbol{k}|\,\hat{h}_0(\boldsymbol{k})$$

$$\hat{u}(\boldsymbol{k}) = -\frac{i\,2\pi k_y}{\gamma}\,\hat{h}_0(\boldsymbol{k}),$$

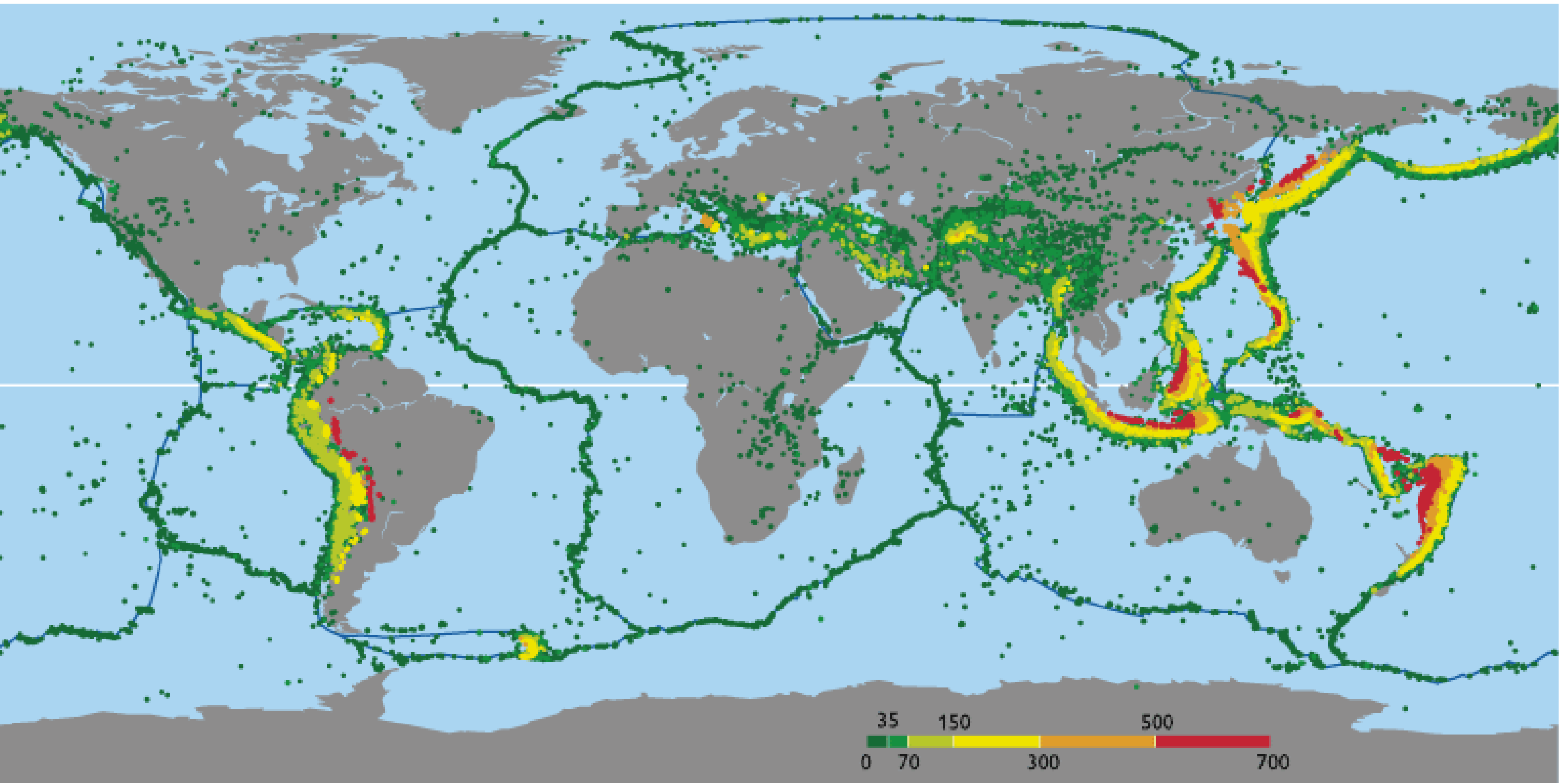
$$\hat{v}(\boldsymbol{k}) = -\frac{i\,2\pi k_x}{\gamma}\,\hat{h}_0(\boldsymbol{k}).$$

$$\hat{F}(\boldsymbol{k}, z=0) = \frac{i\,\gamma}{|\boldsymbol{k}|}\Big(k_x\hat{v}(\boldsymbol{k}) + k_y\hat{u}(\boldsymbol{k})\Big)$$

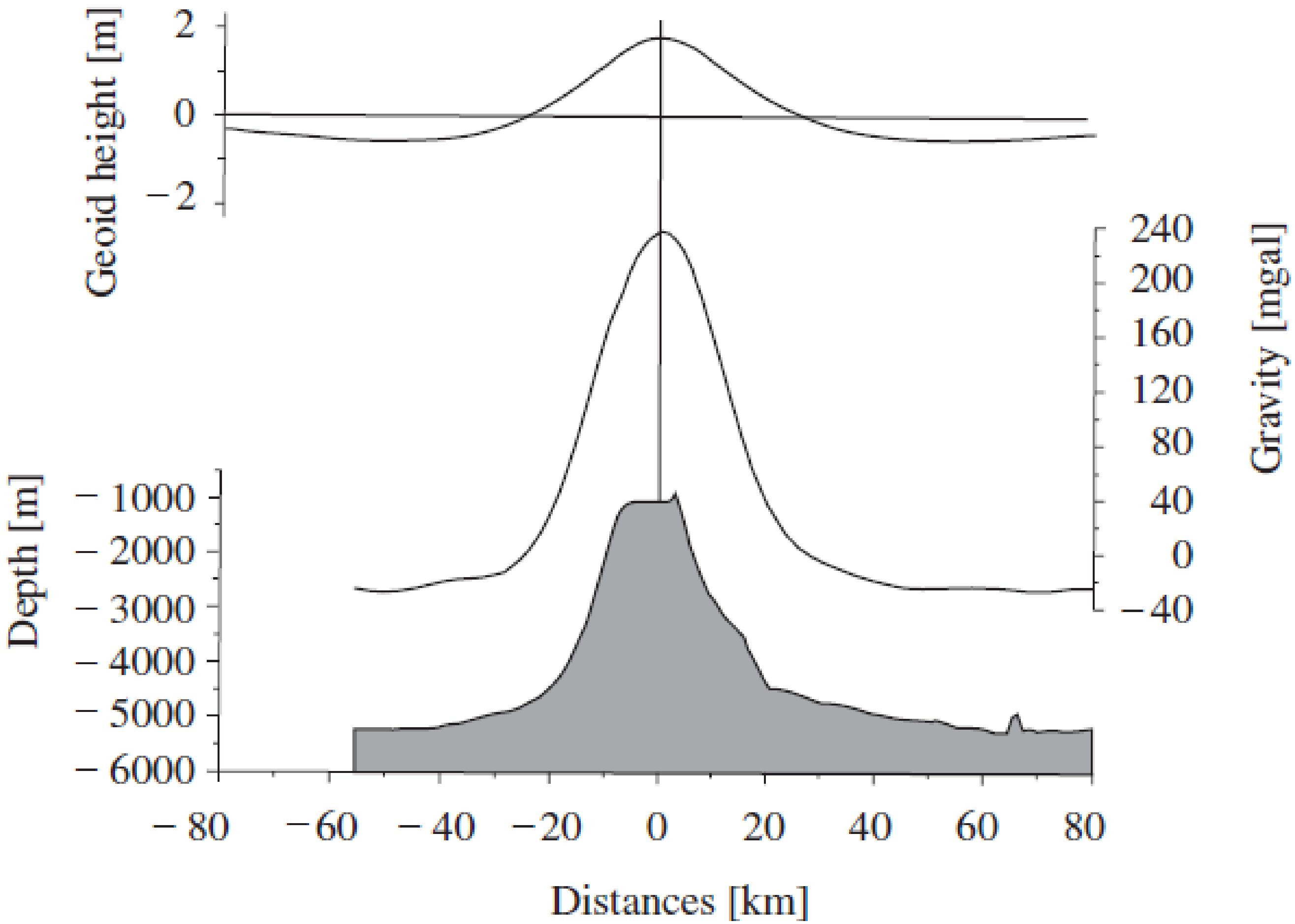


Subduktionszoner
Hot spot
Bevarede
Spredningszoner
Bjergkædedannelse

Gravity -> Bathymetry -> Plate Tectonics -> Earthquakes.



Geoid to Gravity to Bathymetry.



Predicting Bathymetry

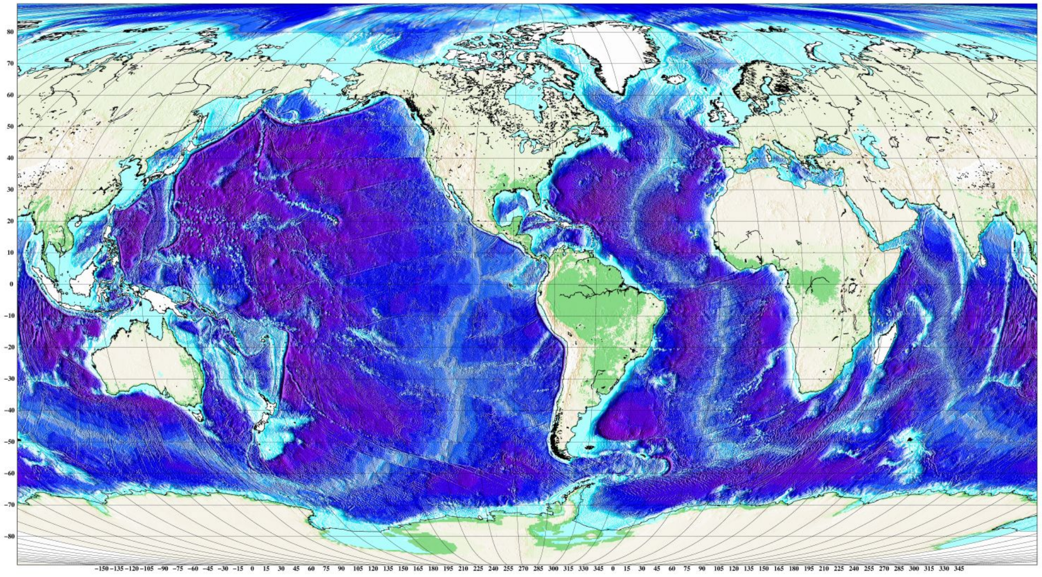
- Density of Rock is roughly 3 times density of sea water.
- Normally this is determined using inversion treating the Spatial variation in bathymetry.

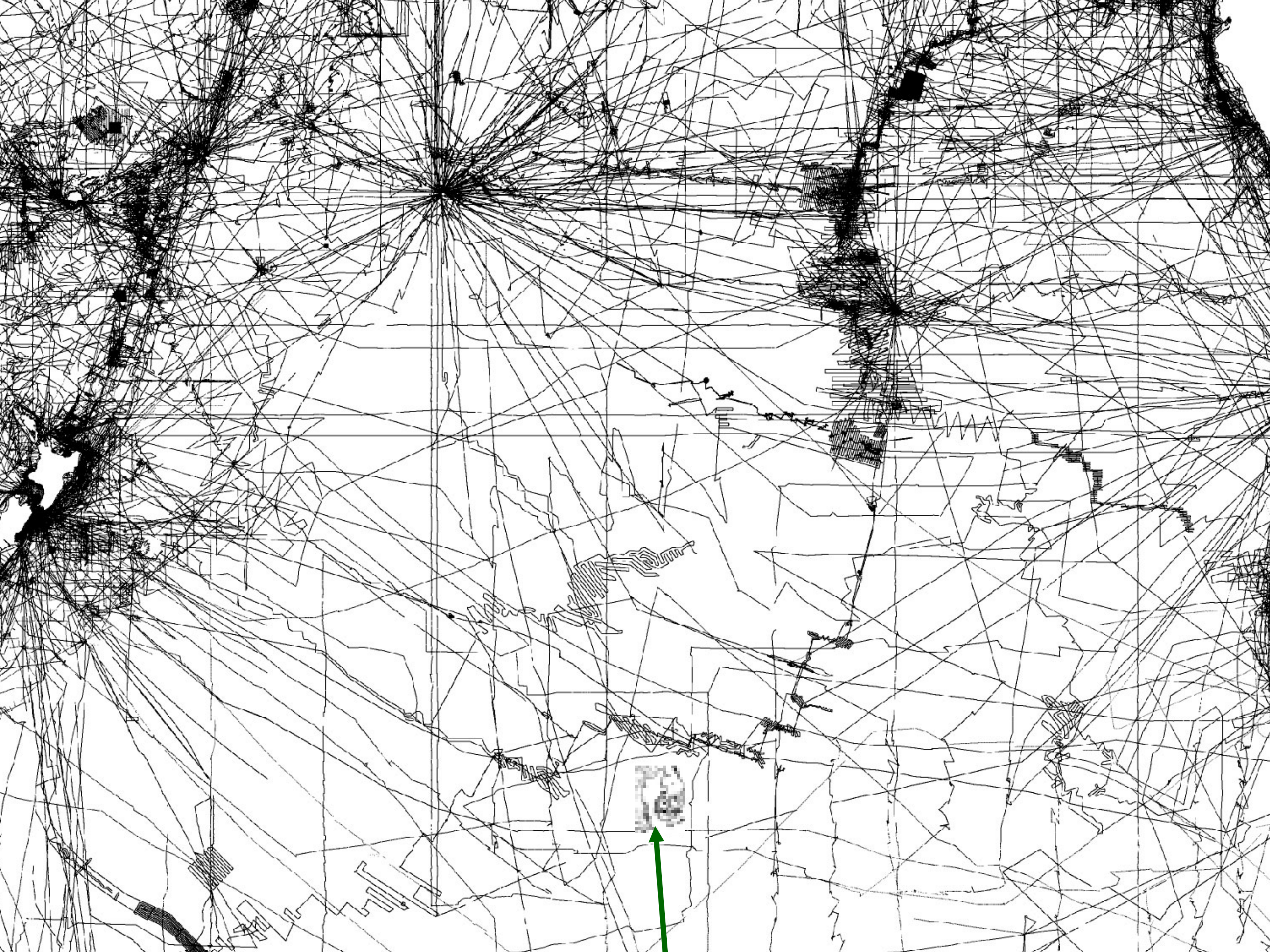
A first rough estimation on bathymetry changes can explore a scaling based on the local density where an infinite Bouguer plate approximation is used.

BOOUGUER APPROXIMATION

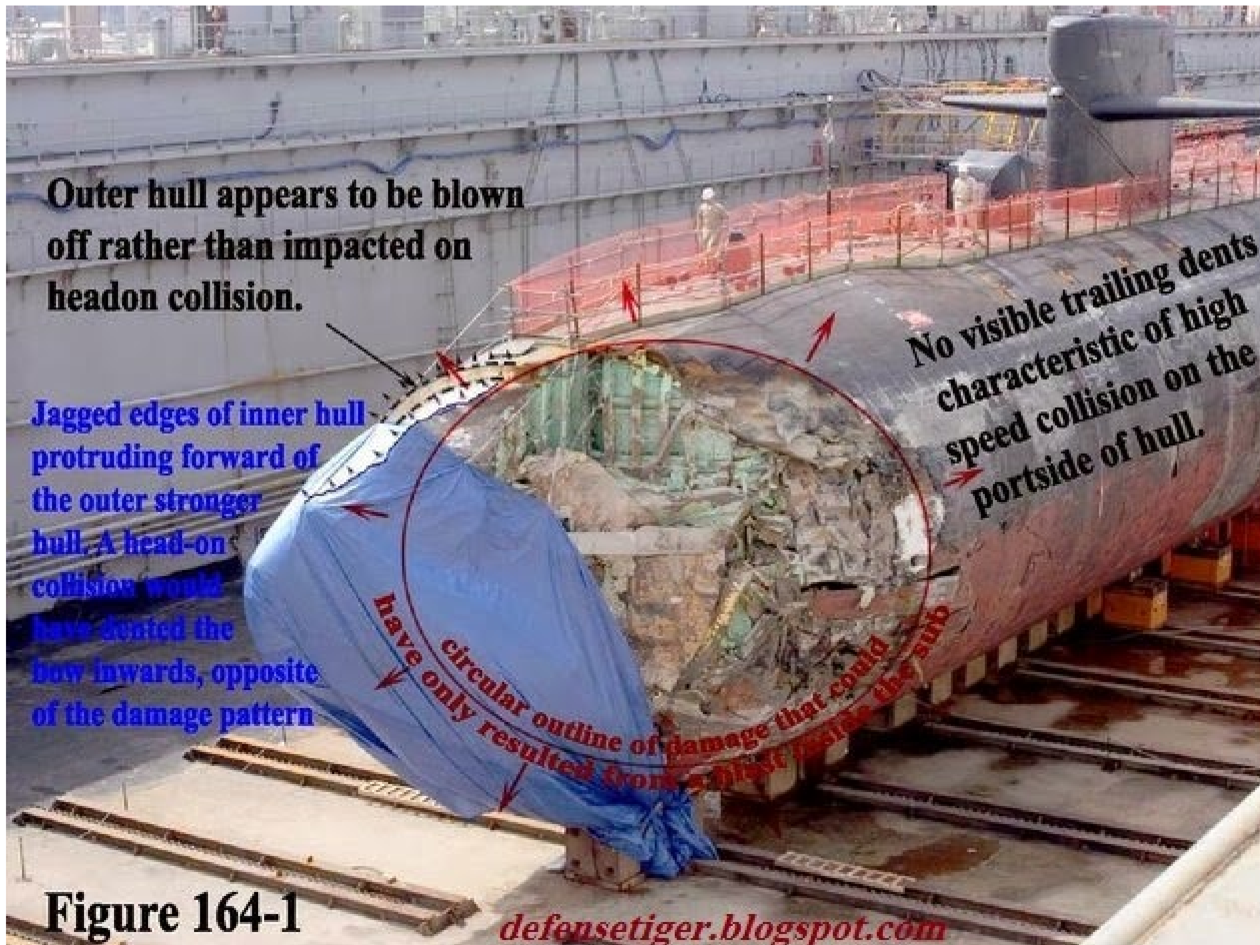
Here Δg (mgal) ~ 0.11 x depth (meters) for typical density of 2.5 g/cm3.

So 10 meters of bathymetry gives 1 mGal.

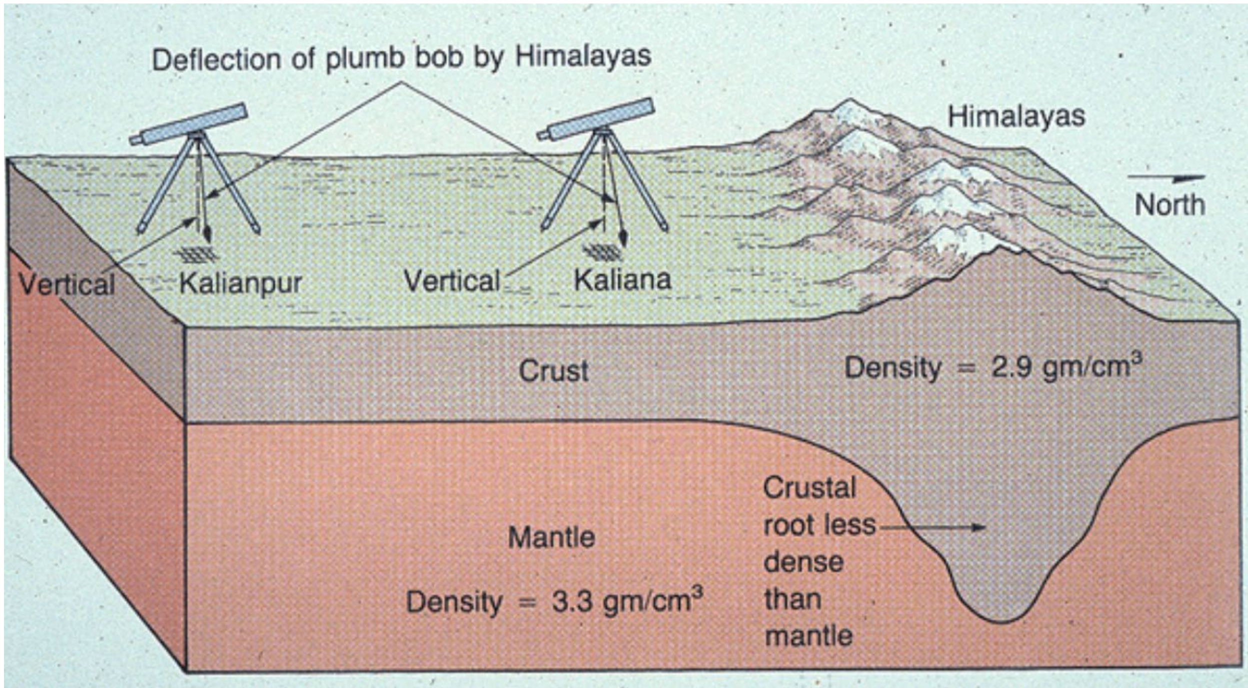




Google "USS San Francisco crash GUAM"



Predicting Bathymetry only works for short wavelength because of isostasy



Another concept that often comes up in this context is that of *isostasy*, which can be described as the buoyant behavior of rocks caused by differences in density. An example is mountains "floating" on denser material underneath (like ice floating on water). The higher the mountains, the deeper they penetrate. The mountains have roots that extend down to the mantle. This causes the gravity anomalies from mountains to be somewhat compensated, i.e. there is less gravity that you might expect. First discovered in 1850 in the Himalayas when plumb bobs were not deflected as much as expected.

Content:

Introduction
- The Mean sea surface, Geoid and Mean Dynamic Topography

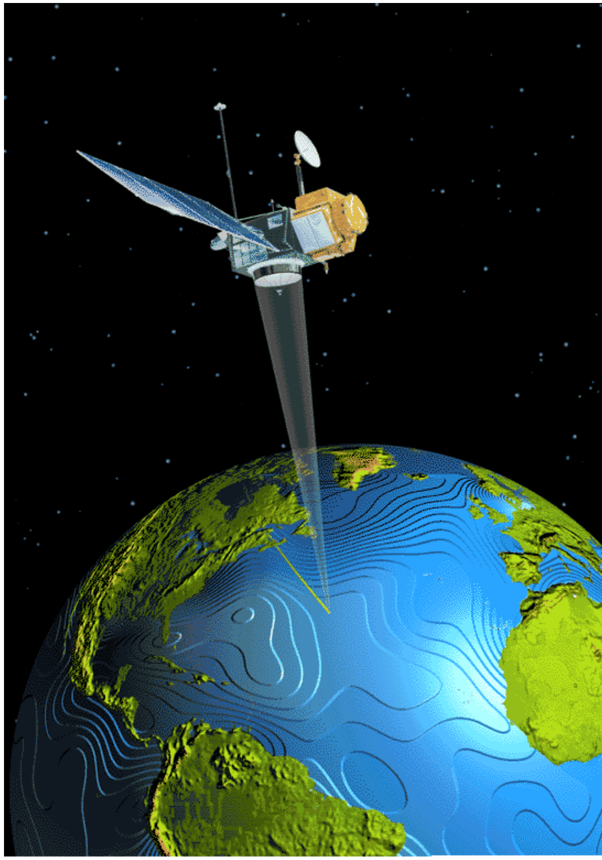
Background
- The reference Potential of the Earth
- Normal gravity and Earth Rotation.
- The geoid.
  - Disturbing Potential and Bruns formula
  - Height and Height systems.

Computing the MSS
- Repeated Tracks
- Geodetic Tracks (crossover adjustment)

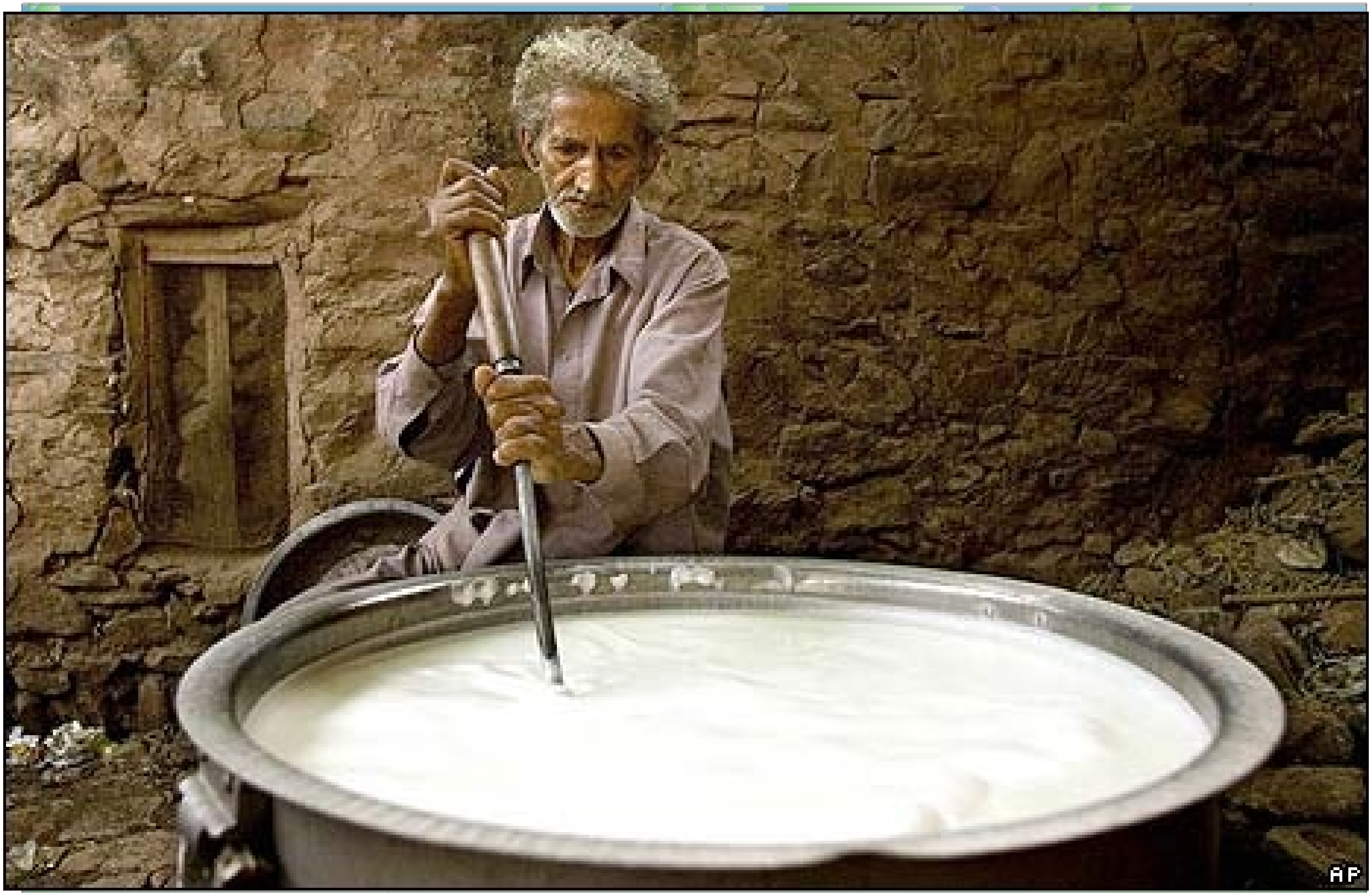
Gravity

Bathymetry and Plate Tectonics

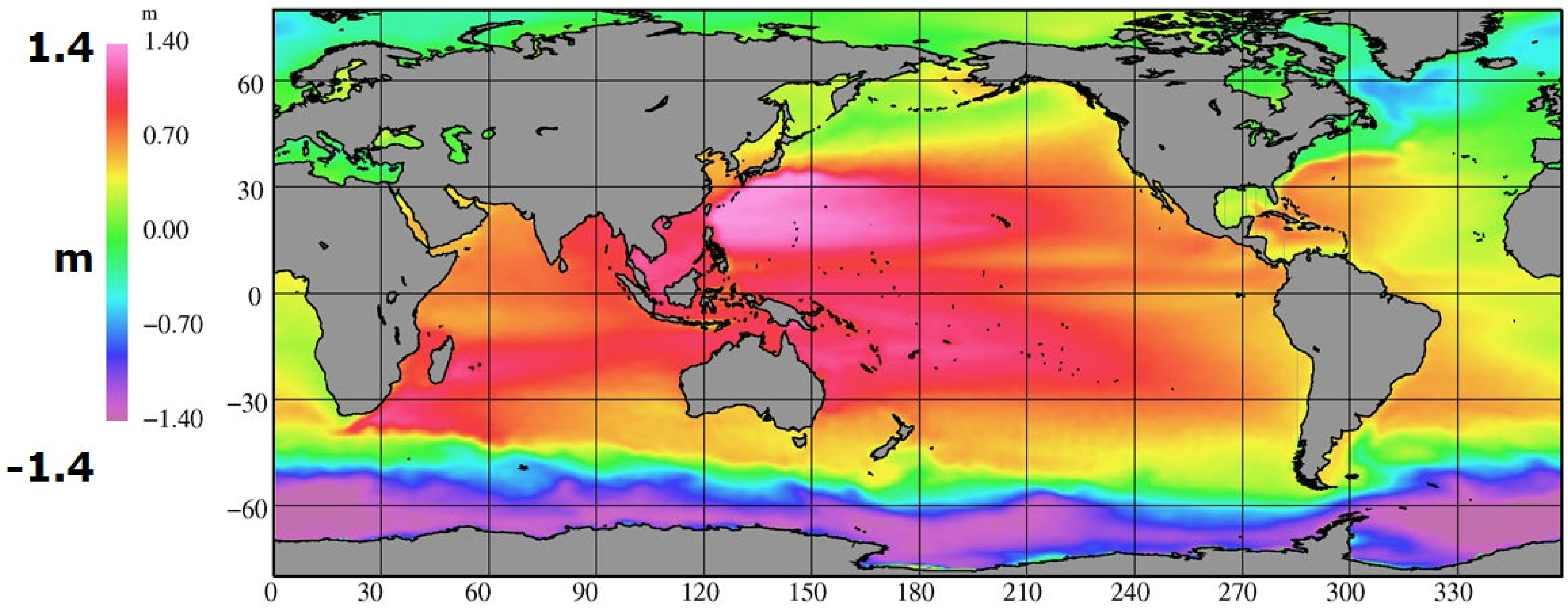
Mean Dynamic Topography and Ocean Currents



Mean Dynamic Topography and ocean currents



The Mean Dynamic Topography.



$$MSS = N + MDT$$

The MDT is a consequence of currents and salinity and temperature.

Contribution from temperature and salinity expansion are determined.

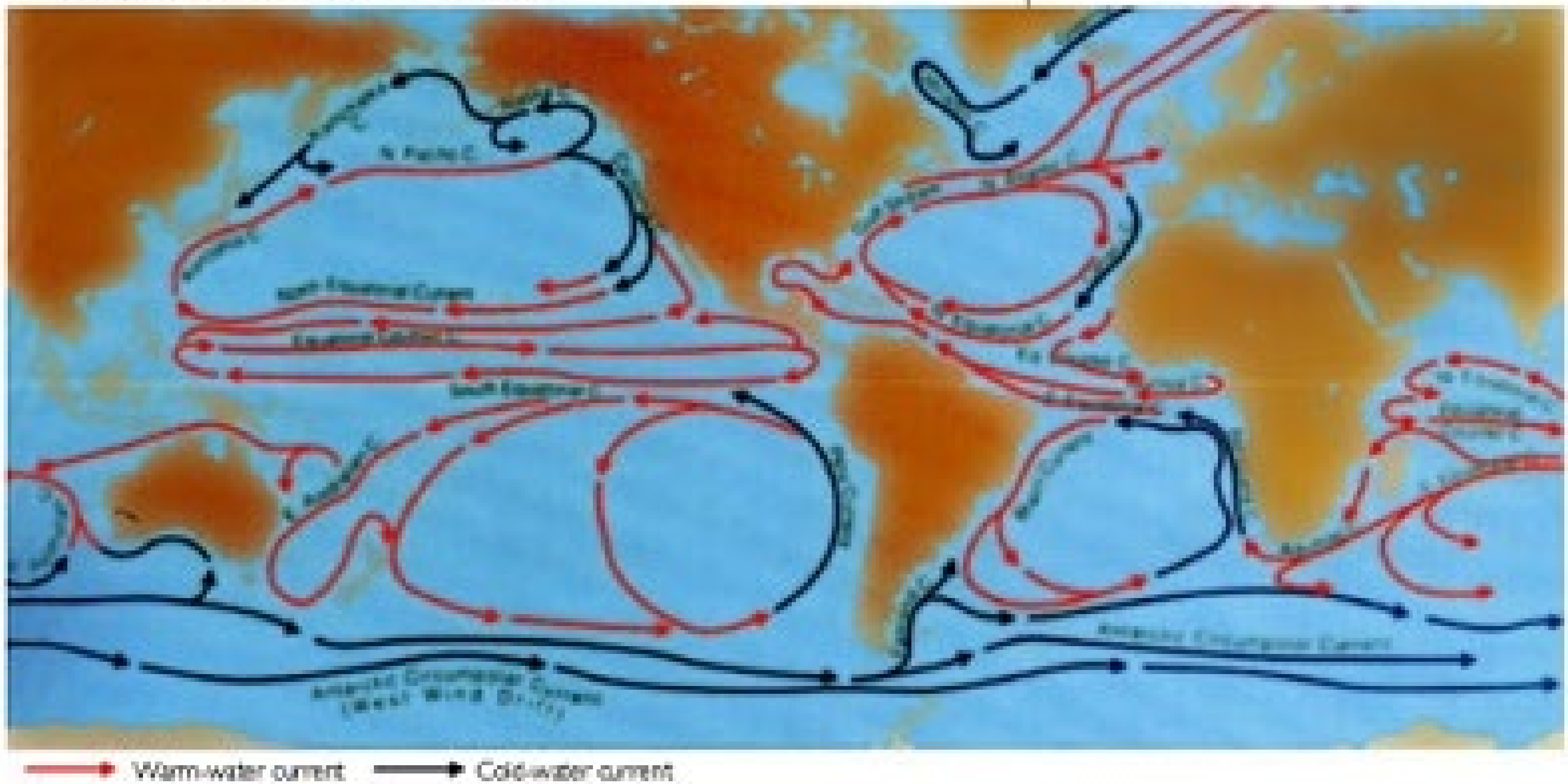
So can we used the MDT to determine major Currents in the world.

Long term change in the MDT will be due to climate change..

Current are important to climate

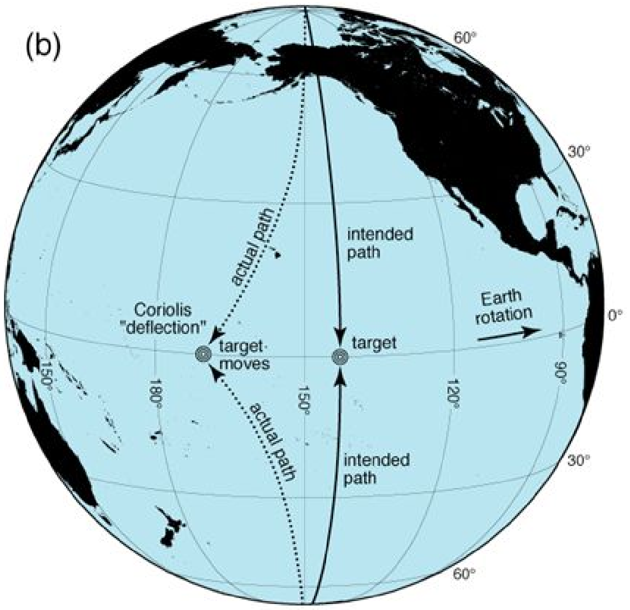
- Fundamental to climate of i.e. NorthEast Europe.
- Transport HEAT NORTHWARDS…………
- If no currents it would be nearly as cold as Siberia…….

FIGURE 3:
The major ocean currents of the world.



Coriolis is due to the rotation of the Earth

Coriolis force



Talley SIO 210 (2014)

$f = 2\,\Omega \sin\varphi$ is the "Coriolis parameter"

$2\,\Omega = 1.414 \times 10^{-4}$/sec

At equator ($\varphi=0$, $\sin\varphi=0$):
$f = 0$

At 30°N ($\varphi=30°$, $\sin\varphi=0.5$):
$f = 0.707 \times 10^{-4}$/sec

At north pole ($\varphi=90°$, $\sin\varphi=1$):
$f = 1.414 \times 10^{-4}$/sec

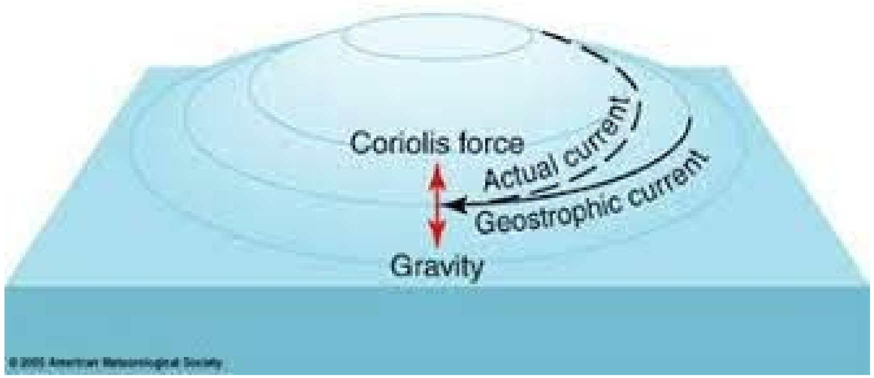
At 30°S ($\varphi=-30°$, $\sin\varphi=-0.5$):
$f = -0.707 \times 10^{-4}$/sec

At north pole ($\varphi=-90°$, $\sin\varphi=-1$):
$f = -1.414 \times 10^{-4}$/sec

Geostrophy

- Sea water wants to move from a location of high pressure (or high sea level) to low pressure (or low sea level).

- Because of Earth rotation the water moves to the right on Northern Hemisphere and left on Southern Hemisphere.
- Water moves along lines of Equal pressyre (isobars) like winds



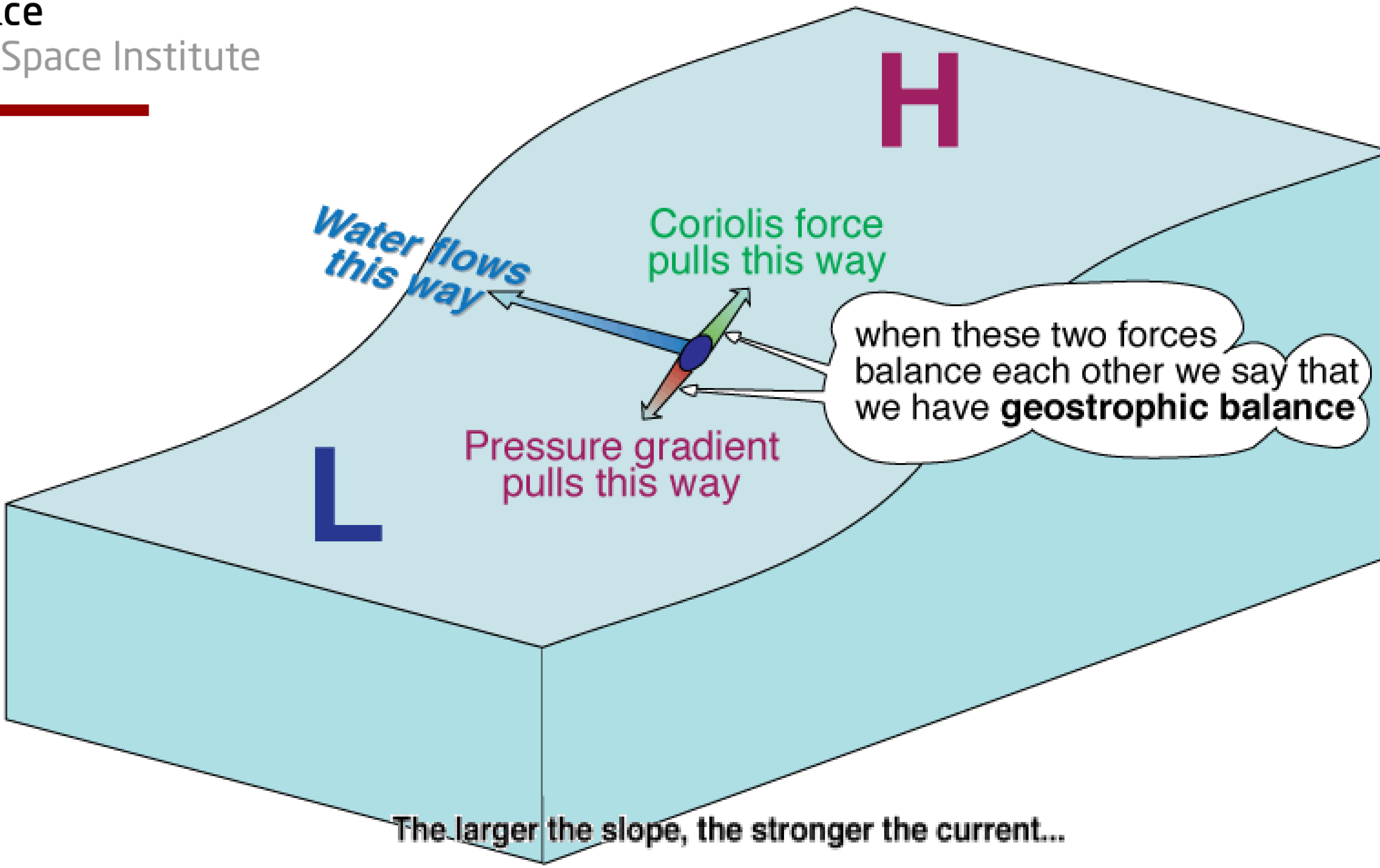
- Wikipedia offers good explanation(Geostrophic currents)

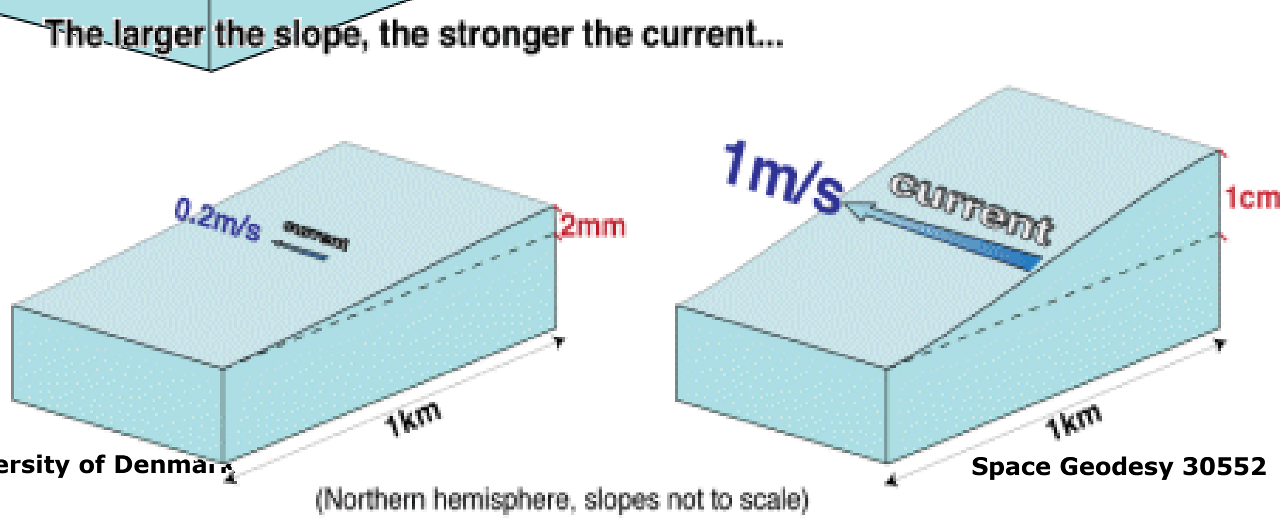
Surface Geostrophic Current

- Surface current is related to surface elevation:

$$u = -\frac{g}{f}\frac{\partial h}{\partial y}, \qquad v = \frac{g}{f}\frac{\partial h}{\partial x}$$

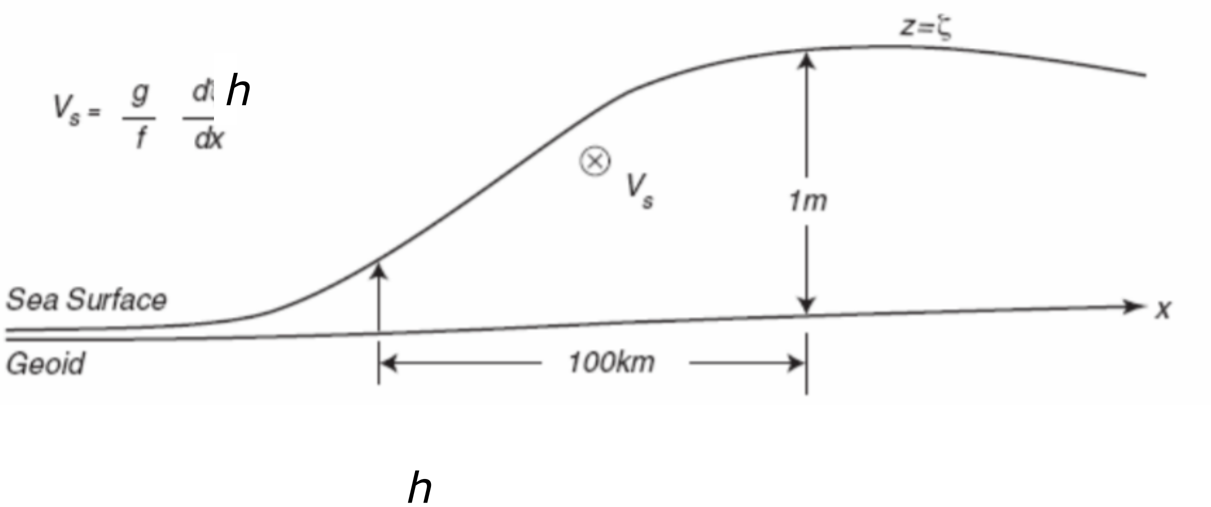
- From satellite data we can get the surface currents from the surface topology


H
L
Water flows this way
Coriolis force pulls this way
when these two forces balance each other we say that we have **geostrophic balance**
Pressure gradient pulls this way

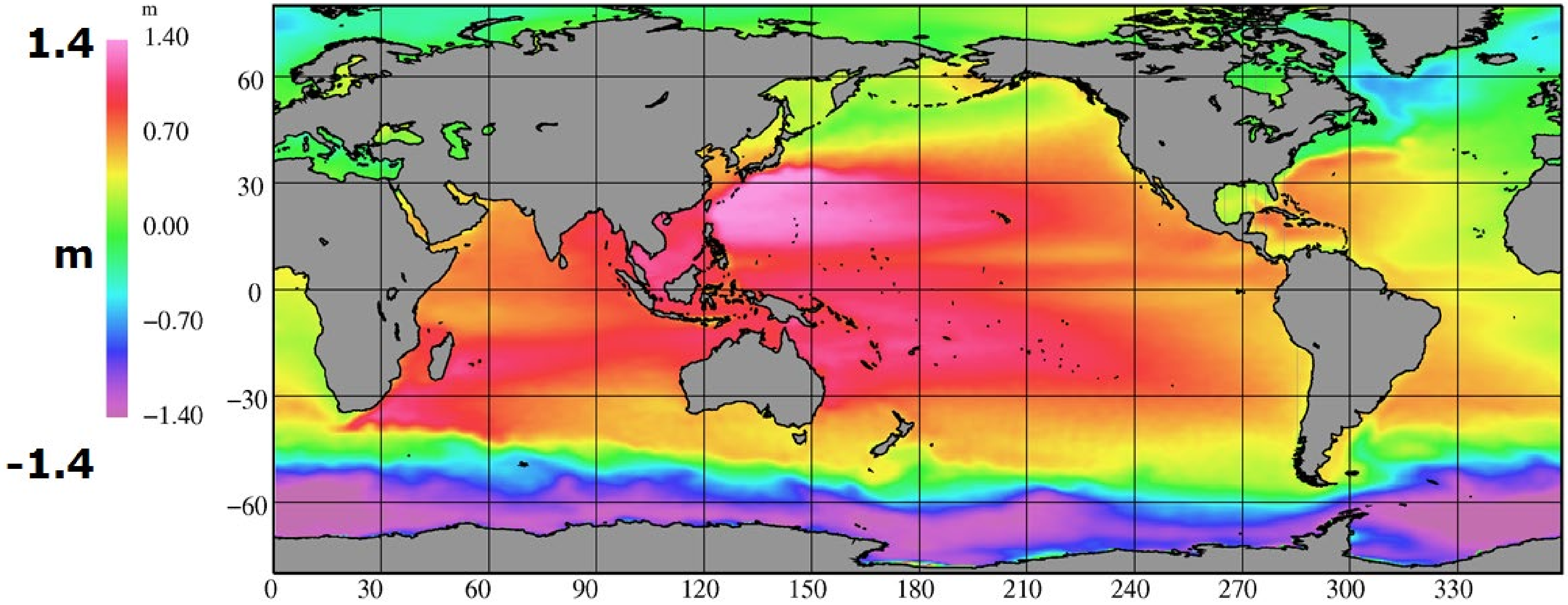

The larger the slope, the stronger the current...
0.2m/s
2mm
1km
1m/s
current
1cm
1km
(Northern hemisphere, slopes not to scale)

Satellite Measurement of Currents

- The height of the sea surface h is the height of the sea surface relative to a particular level surface coincident with the ocean at rest. (This surface is the *Geoid)*
- Since we can measure the surface topography from satellites we can thus calculate the slope of the surface and in turn the surface geostrophic currents.



The Mean Dynamic Topography.



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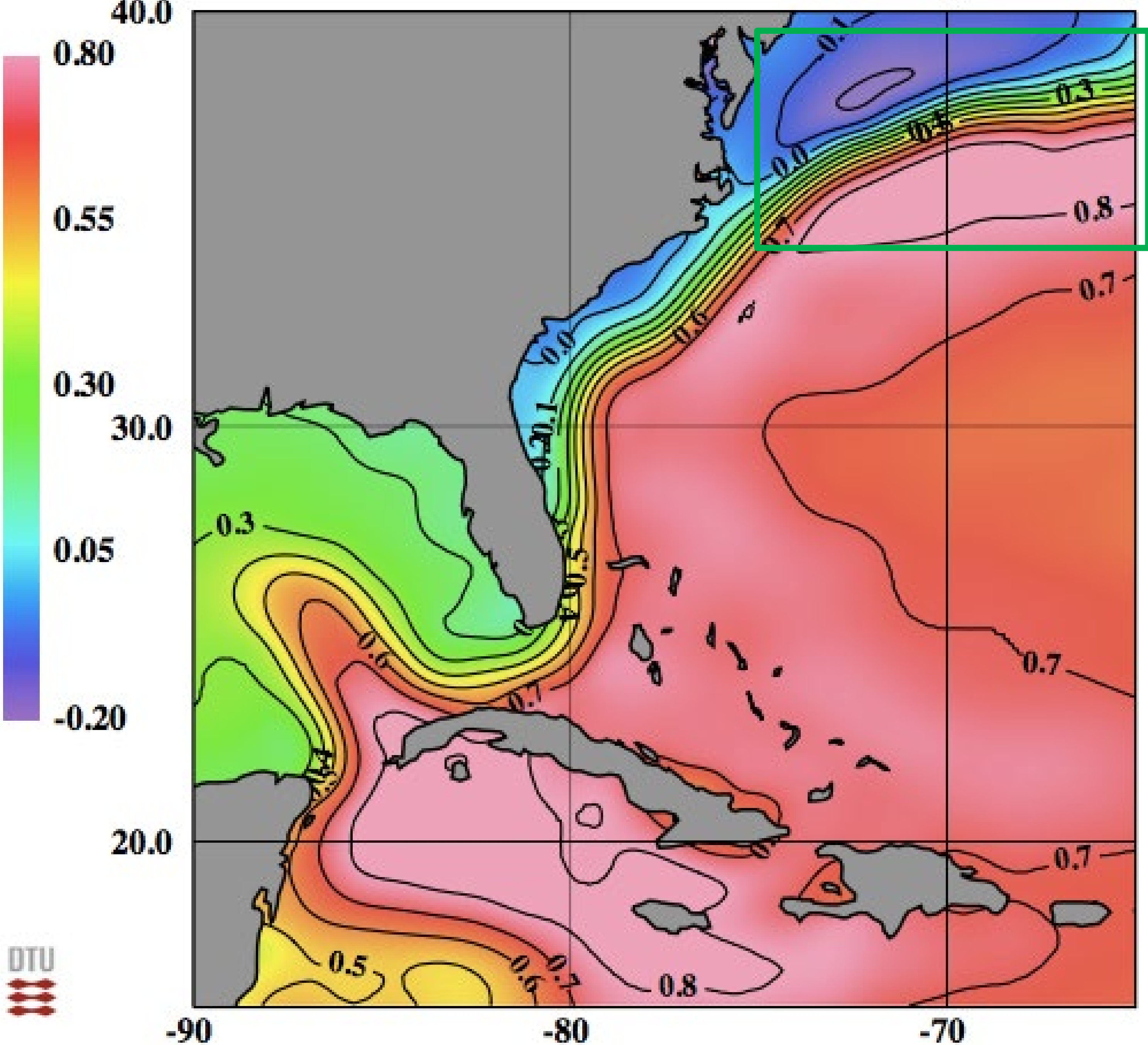
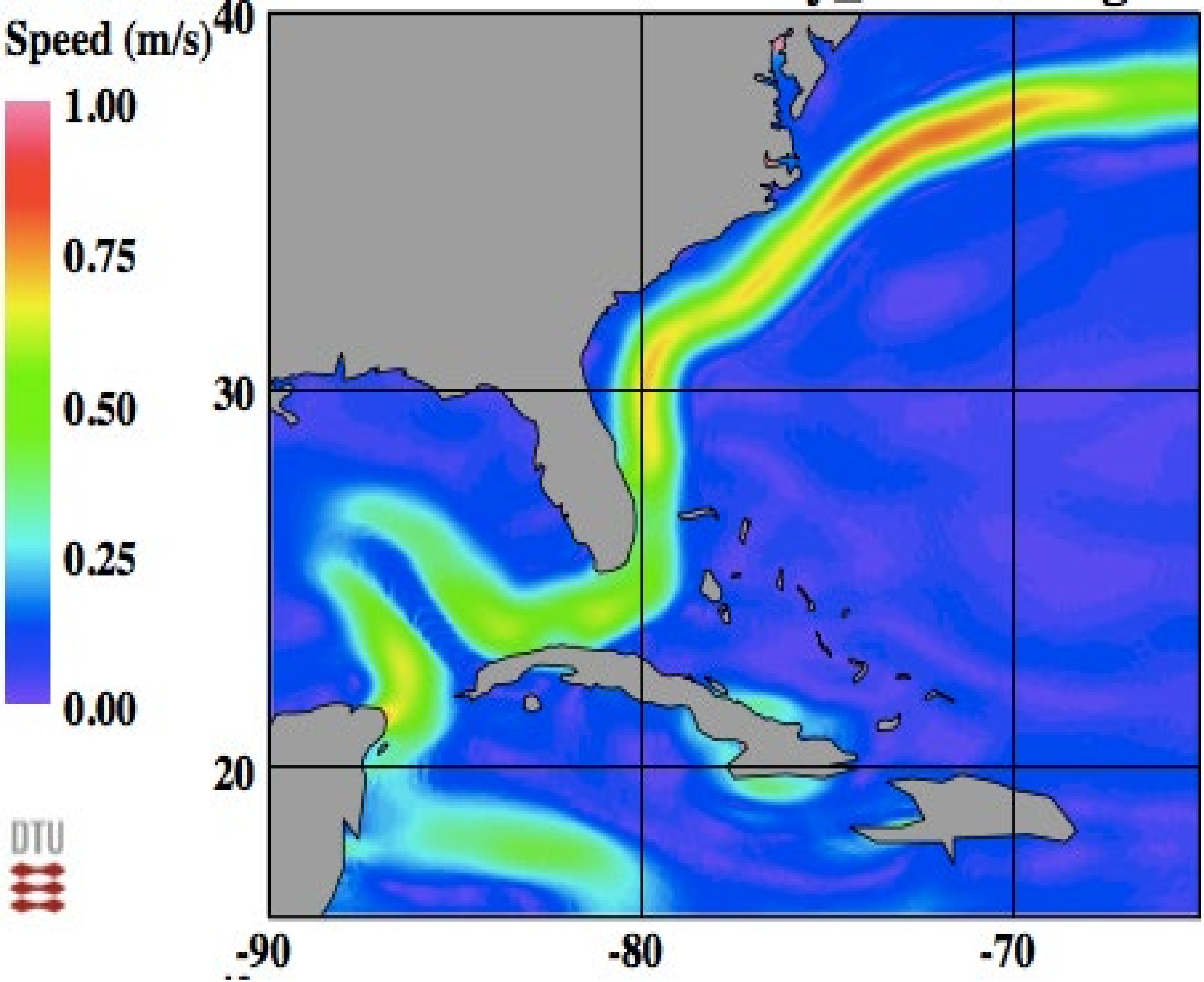
Results

DTU17MDT >

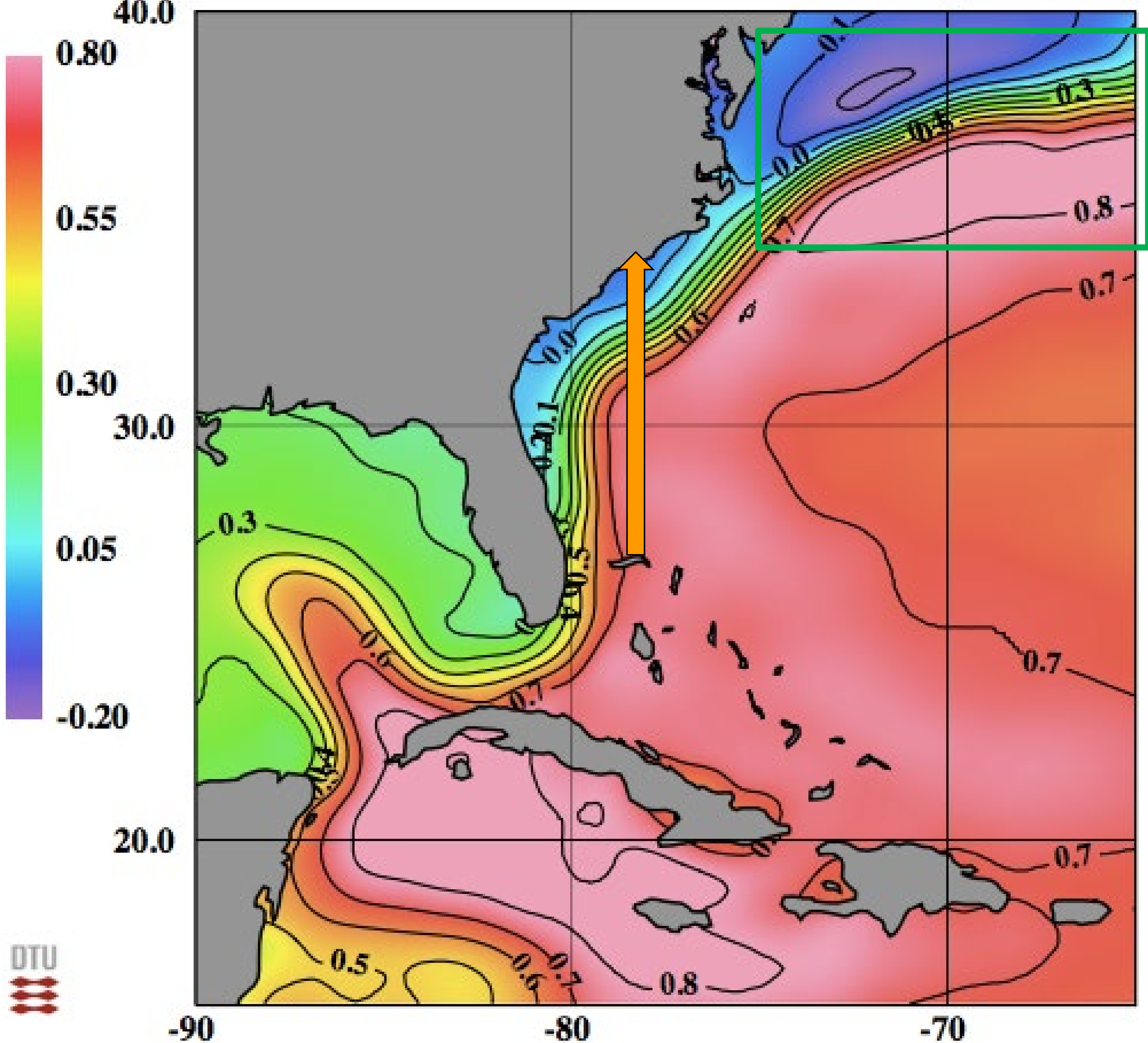


Results

DTU17MDT



**We checked it
Using GPS**



Mount GPS on a boat.

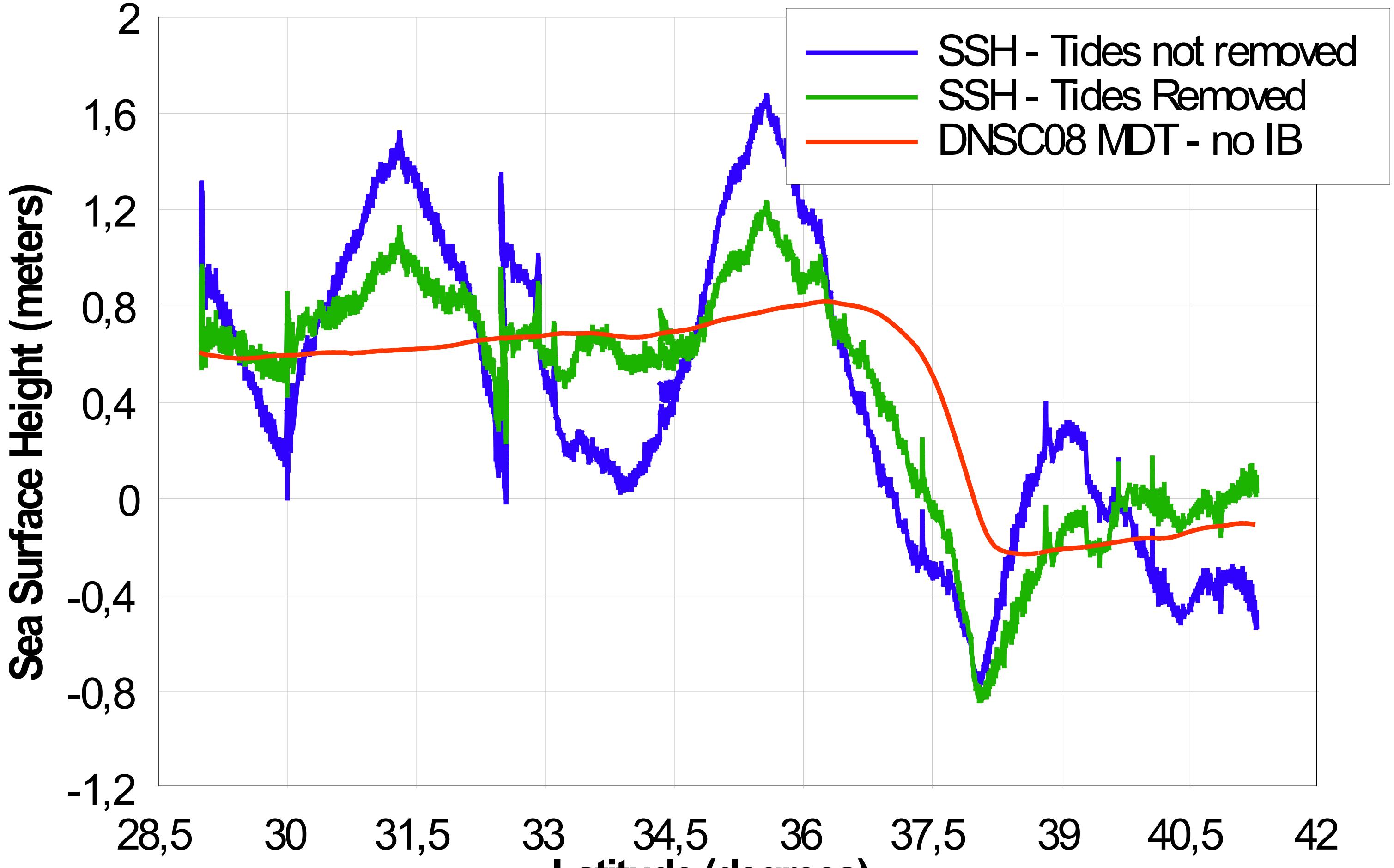
Here GALATHEA.

Crossing the Gulf Stream

Will the height of the ship

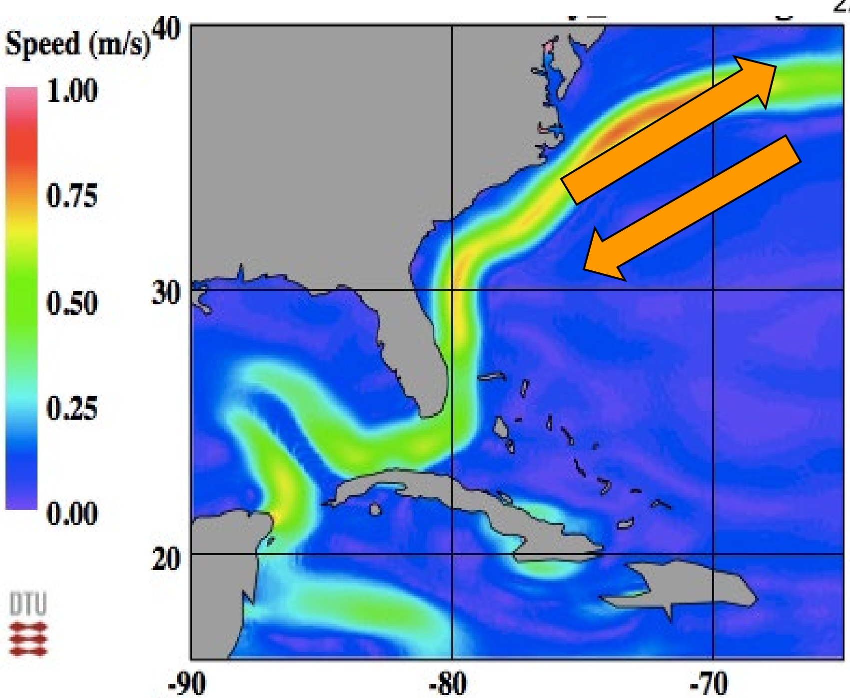
Reflect the Gulf Stream.

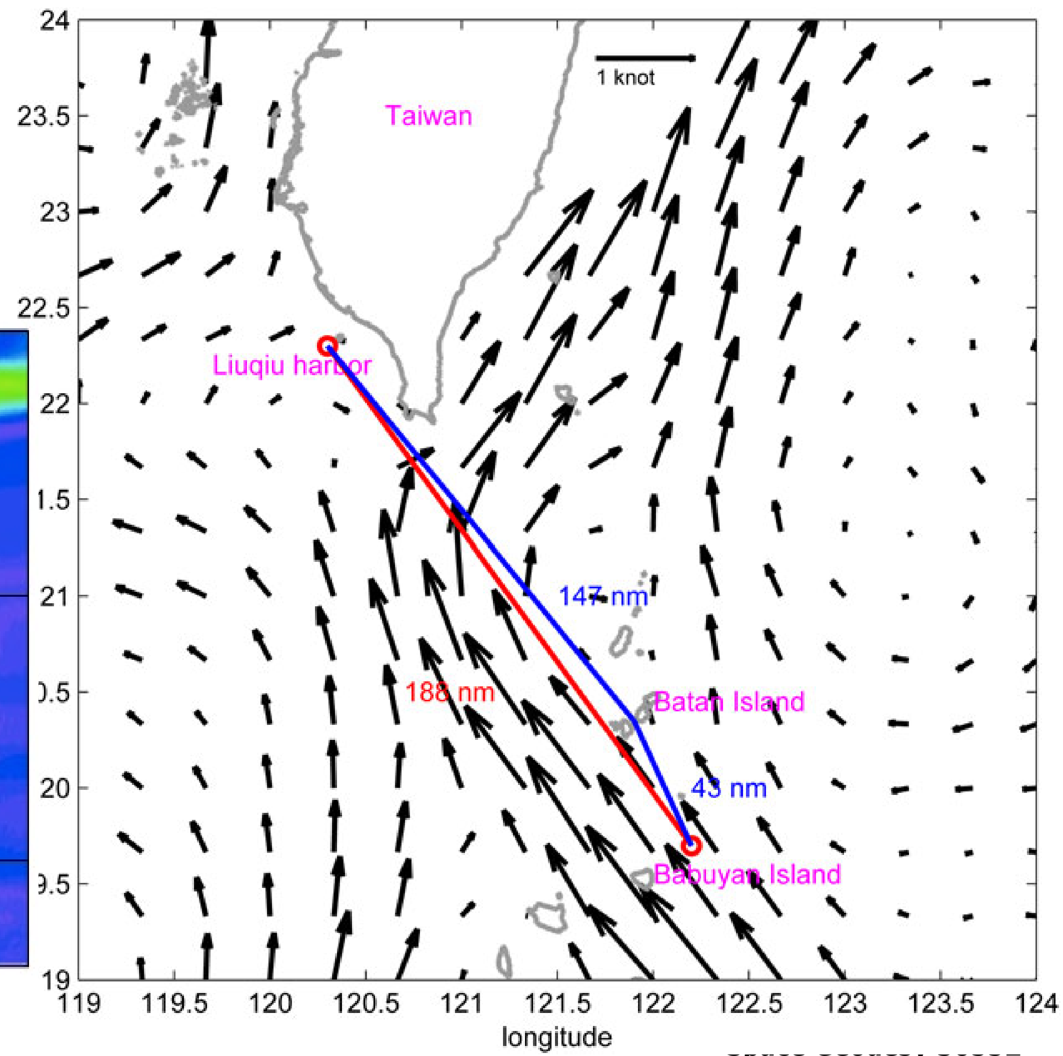
Observations from GALATHEA ....



Currents and Route Optimization.

- Research into this.





Introducing the EXERCISE (5B)

Part 5B:
Use altimetry in the Northsea to investigate the MSS, Geoid and MDT

First try to plot locations and quantiies

Then try to evaluate what you seee..

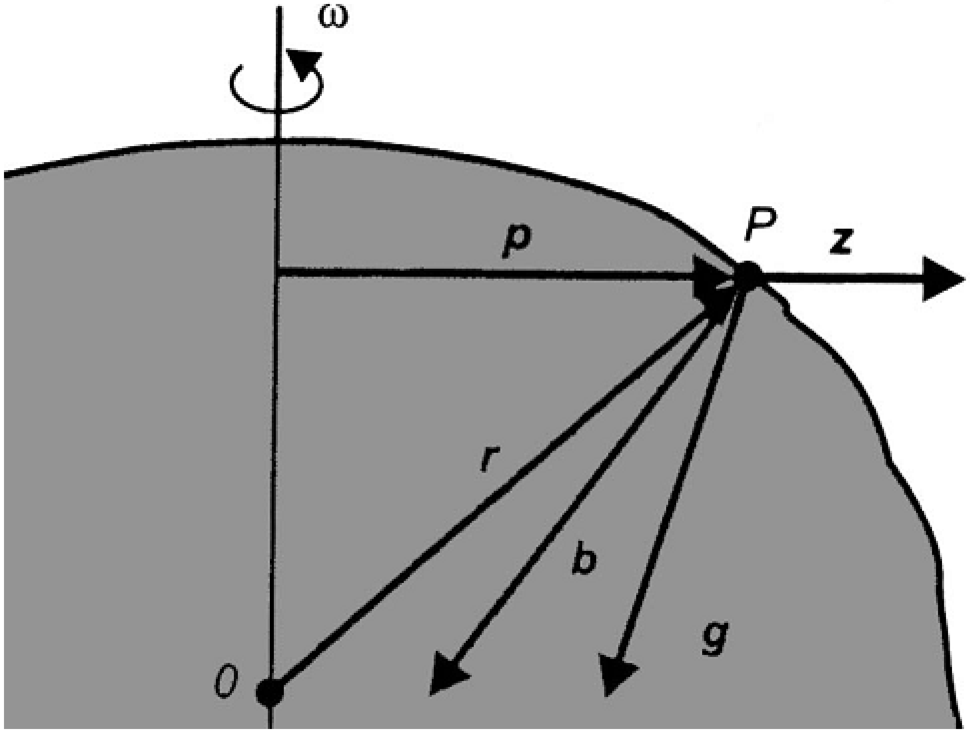
End of today.

Spare slides

The effect of Earth Rotation

A mass on the Earth's surface experiences two accelerations, the gravitational acceleration *b* and an apparent acceleration from the Earth's rotation.

The *centrifugal force* acts on an object of mass on the Earth's surface as the result of the rotation of the Earth about its axis:



The effect of Earth Rotation

The rotation rate of the Earth is: $\omega = 7.292115 \times 10^{-5}\ \text{rad s}^{-1}$

At the equator the centrifugal acceleration is : $z = 3.4\ \text{gal}$

At $\phi = 40^{\circ}$ **the centrifugal acceleration is** $z = 2.6\ \text{gal}$

At $\phi = 90^{\circ}$ **the centrifugal acceleration is** $z = 0\ \text{gal}$

So, now we can define the gravity acceleration (or gravity) in terms of the gravitational acceleration (b) and the centrifugal acceleration (z):

$$\vec{g} = \vec{b} + \vec{z}$$

If you weighed 100 pounds at the north pole on a spring scale, at the **equator** you would weigh 99.65 pounds, or 5.5 ounces less.

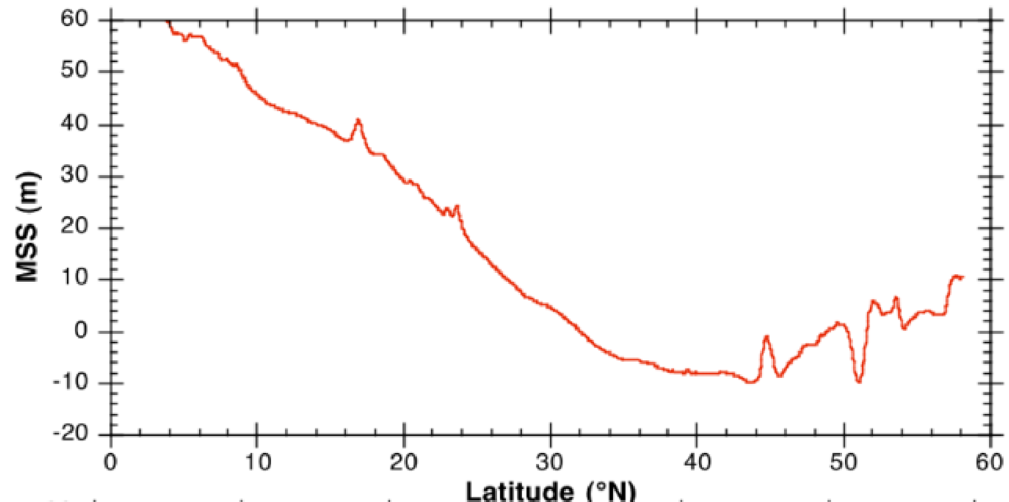
Mean sea surface (MSS) is not flat over a bin

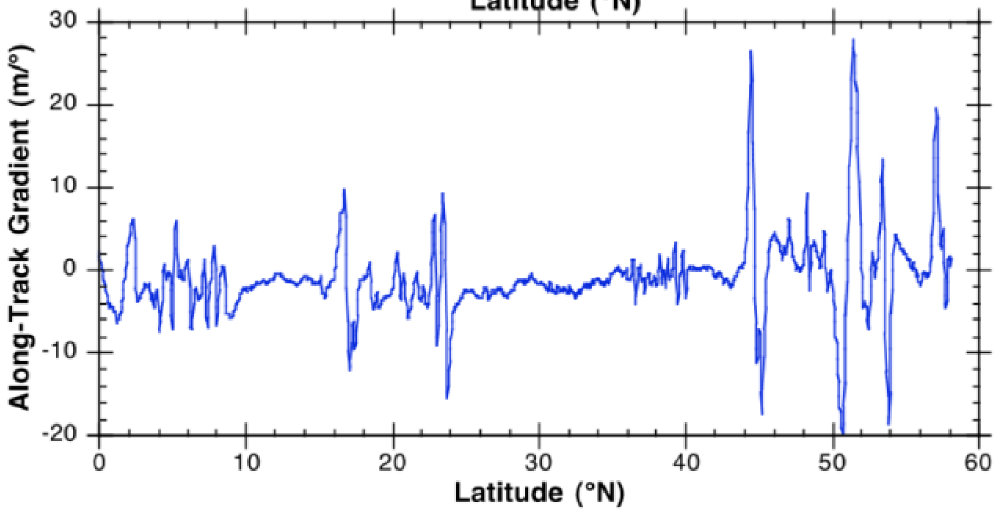
The task is to compute the MSS at the bin center using all measurements within the bin.

However, we cannot assume that the ellipsoidal height of the MSS is constant over a bin. The main contributor to ellipsoidal height variations come from the (time-invariant) geoid.

Ellipsoidal heights of the MSS may change as large as 20 cm/km over trenches and sea mounts; the average gradient is 2 cm/km.

Even with no temporal changes of the sea surface, a different sampling of a bin would provide a different value of the MSS at the bin center when we would just average.



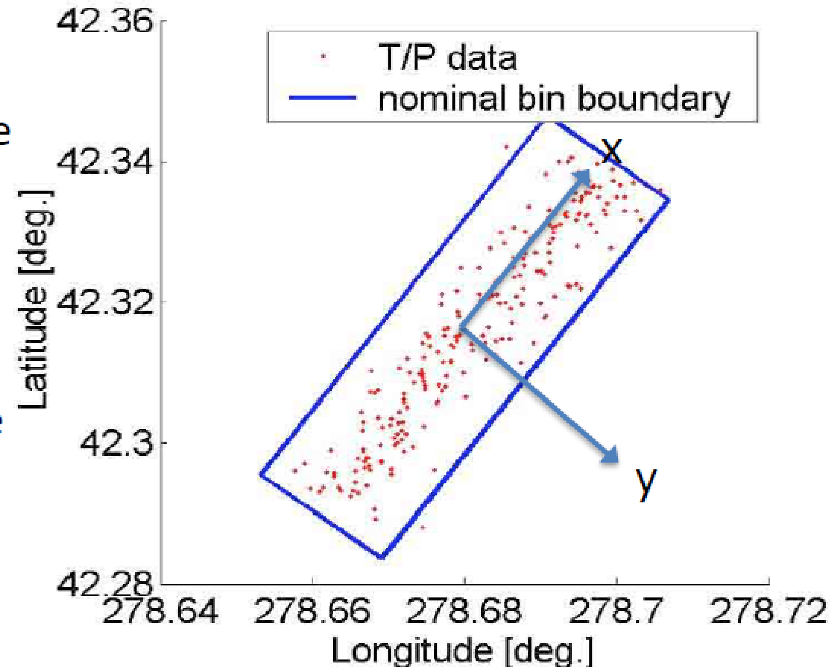


Model of spatial variations for data inside a bin

Assume for the time being that there are no temporal variations in the measured SSH over a bin. Then, the sea surface coincides with the mean sea surface (MSS). Howevei the MSS is not flat over the bin, but a spatially varying function. As the bin size is small, we use a simple model to describe this spatial variability:

$$SSH = a + b\,dx + c\,dy$$

- a = height of the plane at the bin centre (i.e., MSS at the bin centre) after gradient correction
- b = along-track sea surface gradient
- c = cross-track sea surface gradient
- dx = along-track displacement from the bin centre
- dy = cross-track displacement from the bin centre



- By estimating it this way a is the MSS at the bin center.