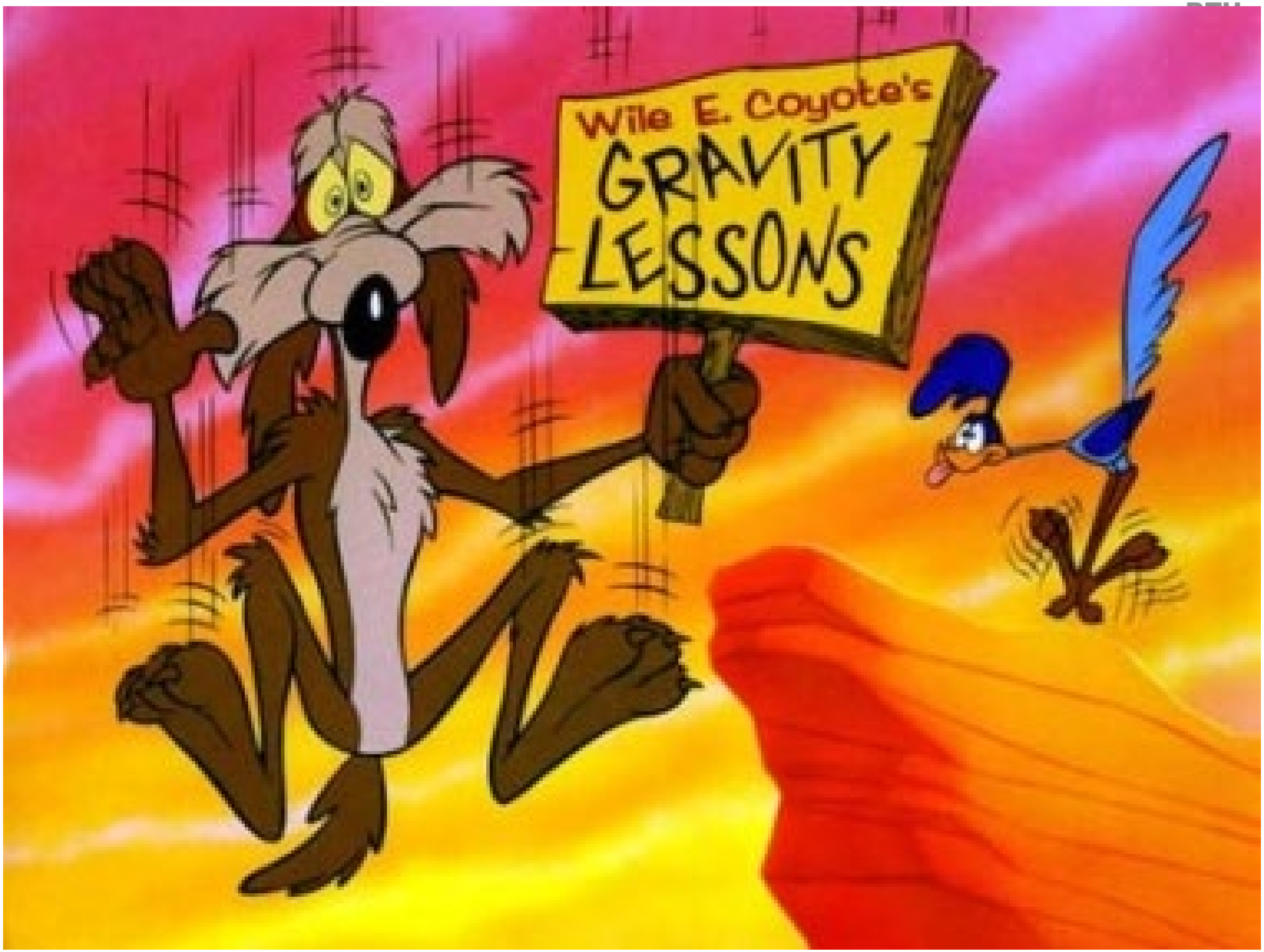
30552 – Lecture 12 .

Satellite, tracing and gradiometry (CHAMP, GRACE and GOCE).

Prof Ole B. Andersen,
DTU Space,
Geodesy and Earth Observation

DTU Space
National Space Institute

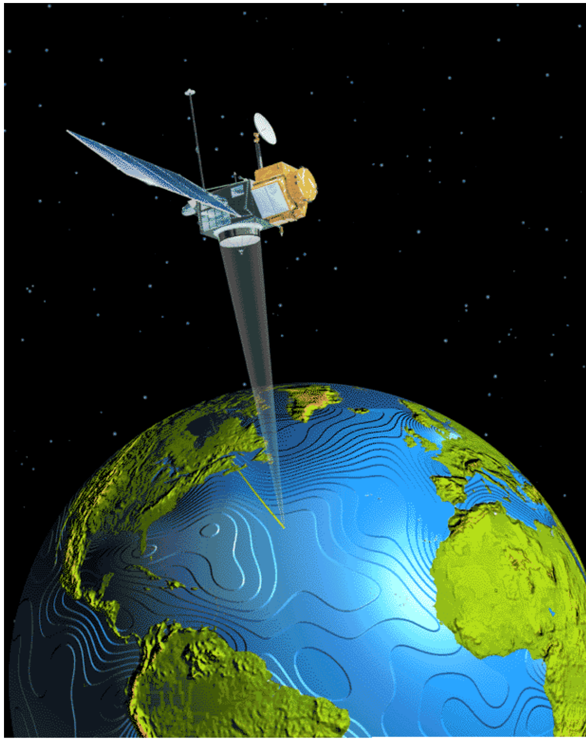


- Thank to Steve Nerem (University of Boulder)

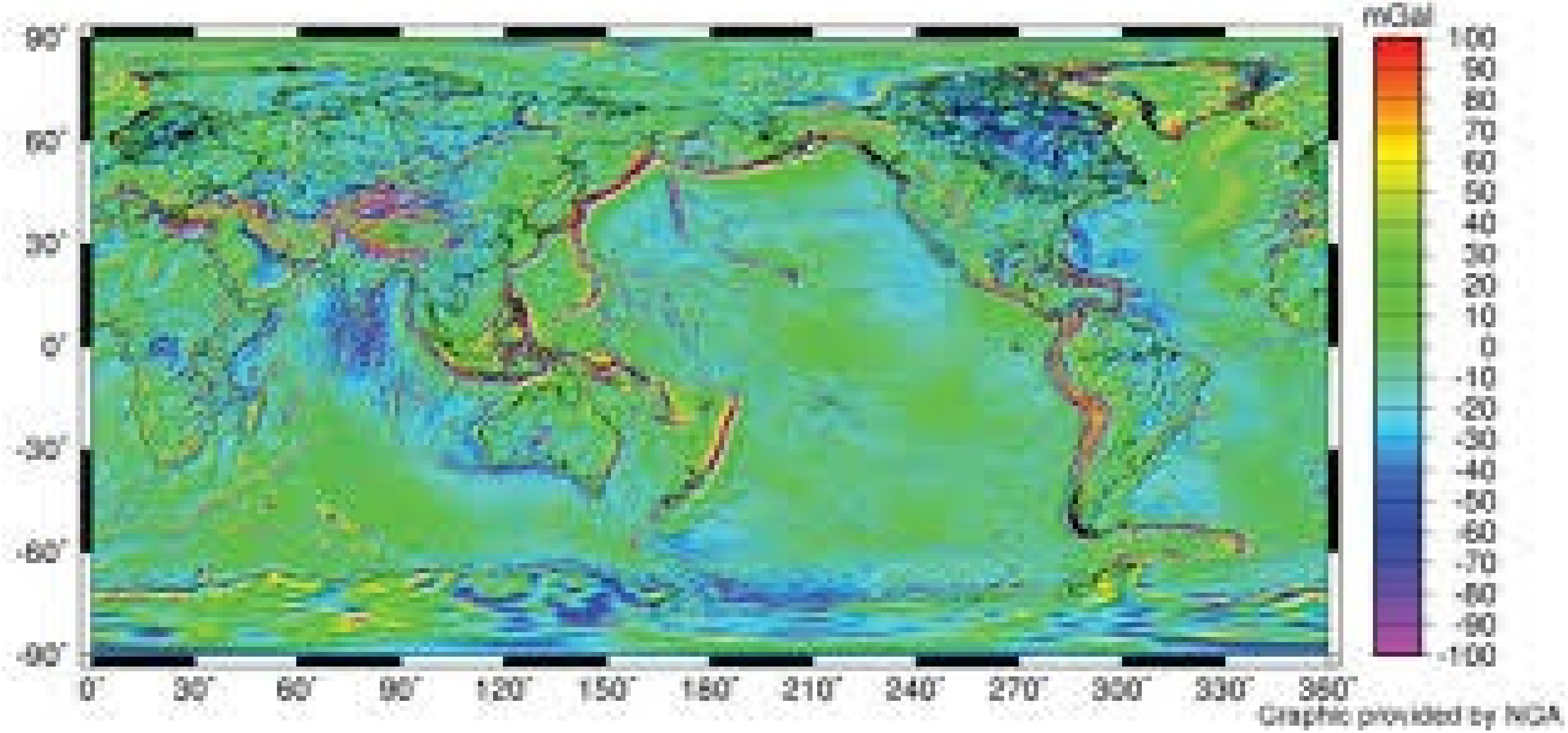
Content

- Purpose: Building global gravity field models.
- Spherical harmonic functions (repetition)
- Degree Variance.
- Least Squares Estimation (repetition).
- Building global gravity field models.
- Upward Continuation / Attenuation/Limits.
- Satellite Tracking (high -low)
    - CHAMP
- Satellite to Satellite Tracking (low – low)
    - GRACE + GRACE FO
- Satellite gradiometry
    - GOCE

Litterature Seeber 469-484



Map the gravity field of the Earth (EGM 2008)



Surveying and other non-science applications require accuracies of about 1 mgal. In geodesy, we would like accuracies of around 10 μgal, because moving 3 cm radially outward from the Earth decreases gravity by about 10 μgal.

Introduction.

- Where are we going? We want find a complete formula for the **gravitational potential and gravity field** for a nearly-spherical body.
- We will derive the form of the potential that is a solution in spherical coordinates. A complete solution will be in the form of spherical harmonics.
- Spherical harmonic are very convenient and compact.

- Spherical Harmonics are not the only way to represent the gravity field of the Earth
  - Mass concentration blocks (mascons)
  - Wavelets
  - Geographical grids (Problems at high latitudes)
  - More….

Spherical Harmonic functions

$$V = \frac{GM}{r} \sum_{n=0}^{\infty} \left(\frac{r_E}{r} \right)^n \sum_{m=0}^{n} P_{nm}(\sin\phi)[C_{nm}\cos m\lambda + S_{nm}\sin m\lambda]$$

The Degree is called n.

The Order is called m.

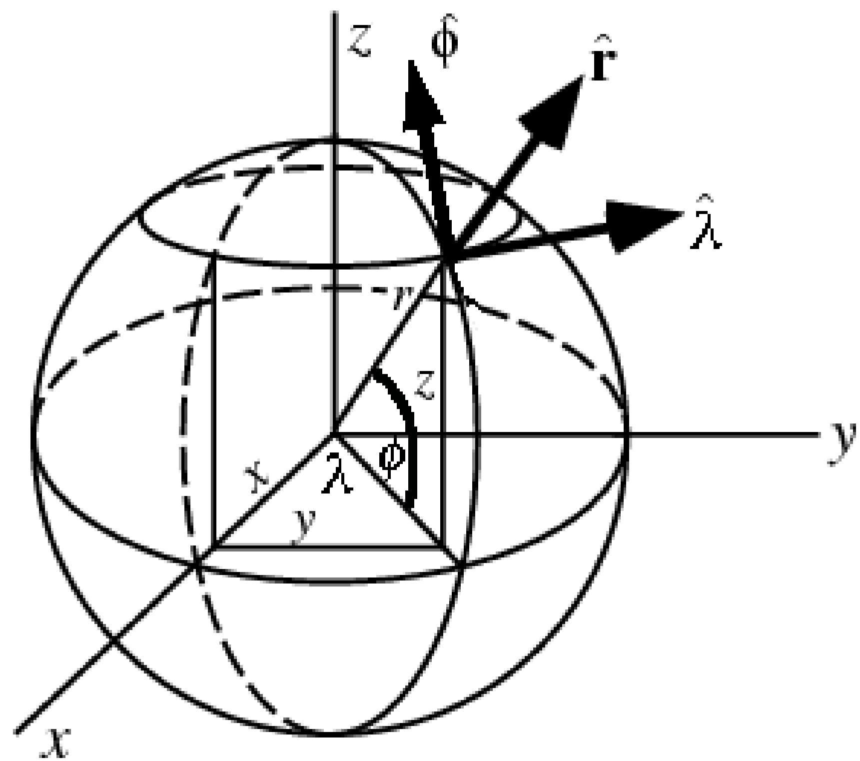
C_{nm} and S_{nm} are the coefficients.

The terms

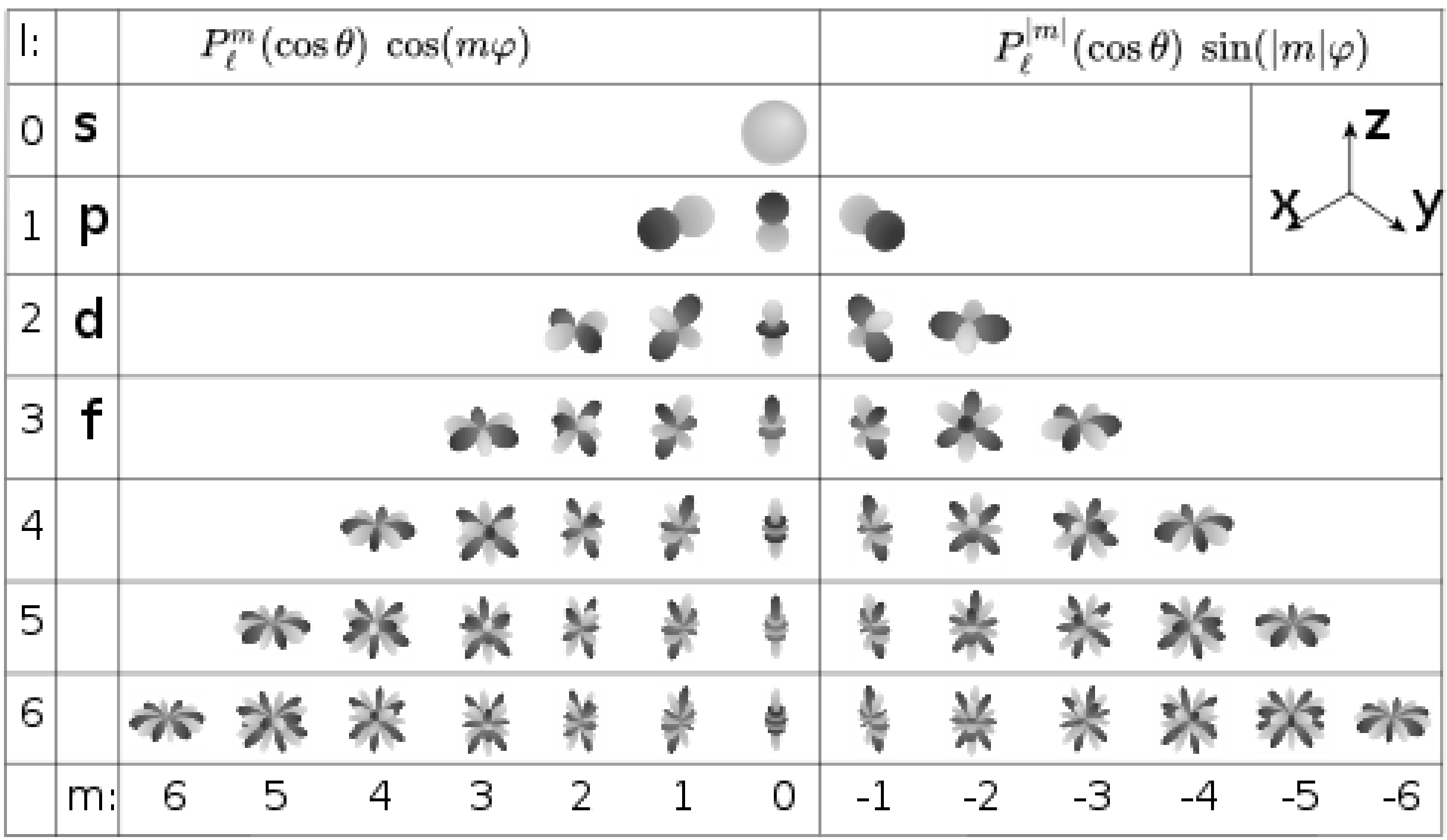
$$P_{nm}(\sin\phi)\cos m\lambda$$

$$P_{nm}(\sin\phi)\sin m\lambda$$

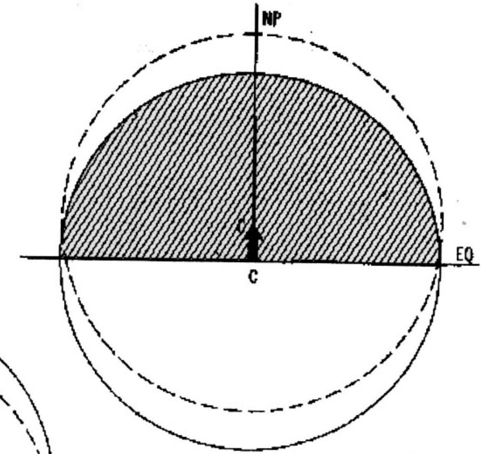
are the *spherical harmonics*.



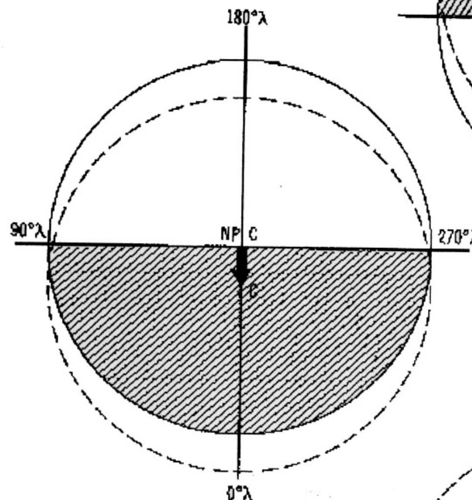
Number of Coefficients.
For degree n its 2n+1



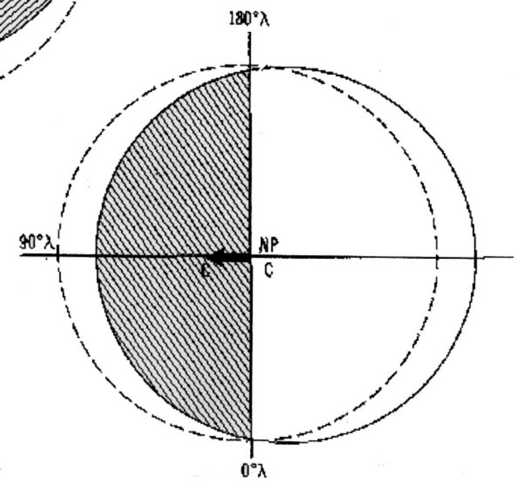
N = 0 and N = 1



$P_{10}(\sin\phi)$



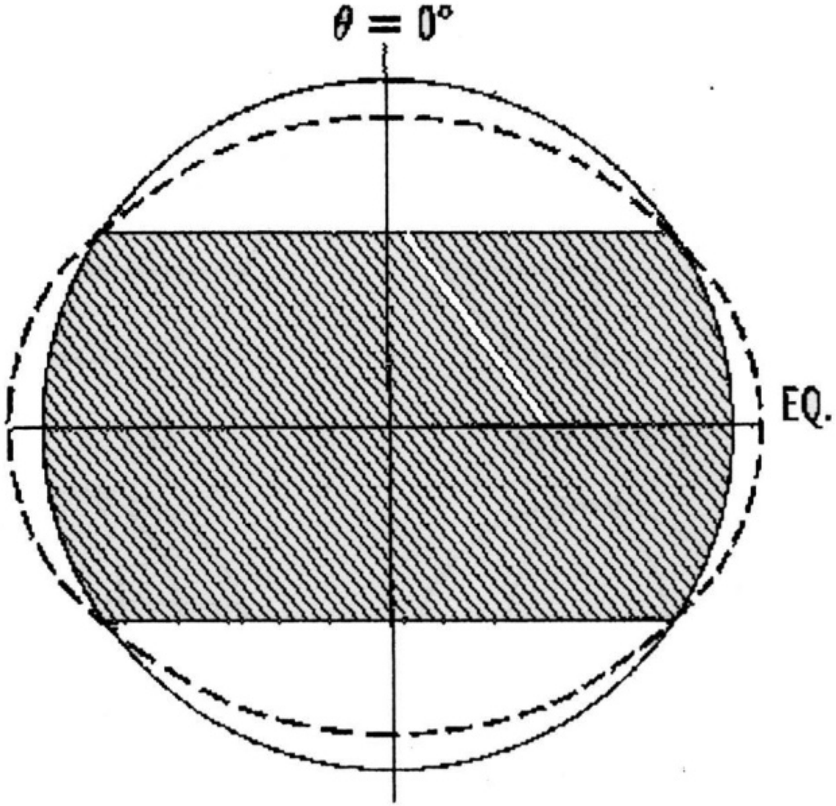
$P_{11}(\sin\phi)\cos\lambda$



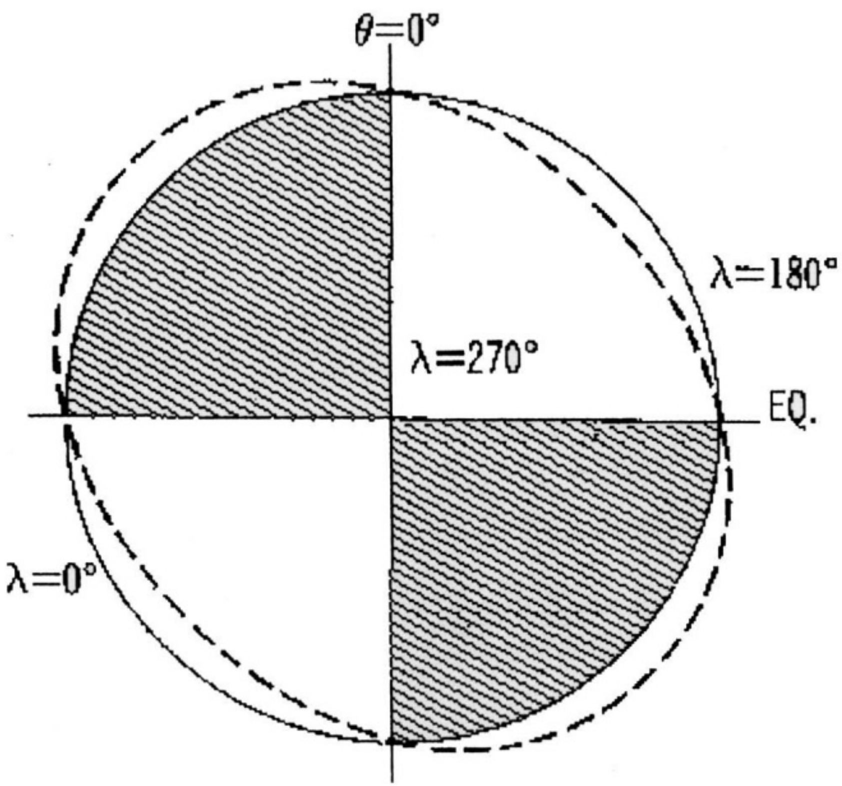
$P_{11}(\sin\phi)\sin\lambda$

- N = 0 Defines the size of the Earth.
- N = 1 Defines the center of Mass

N = 2

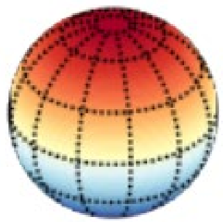


$P_{20}(\sin\phi)$

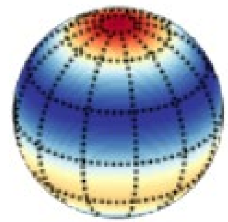


$P_{21}(\sin\phi)$

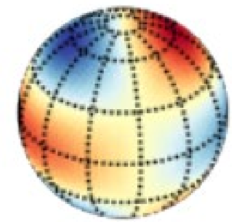
Spherical Harmonics



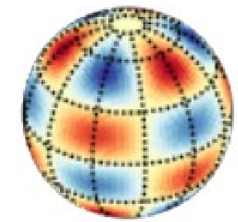
$m = 0,\ n = 1$



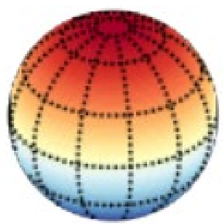
$m = 1,\ n = 1$



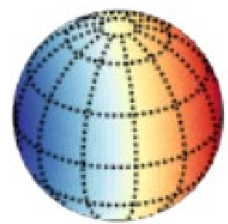
$m = 2,\ n = 2$



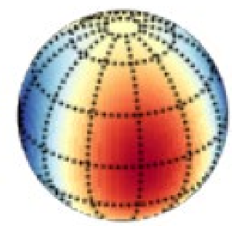
$m = 4,\ n = 5$



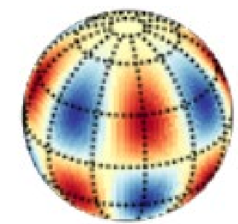
$m = 0,\ n = 2$



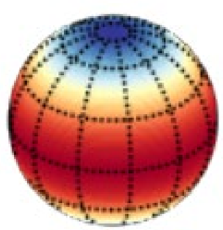
$m = 1,\ n = 2$



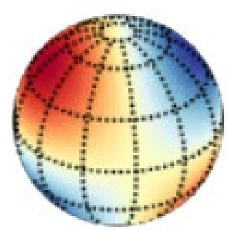
$m = 2,\ n = 3$



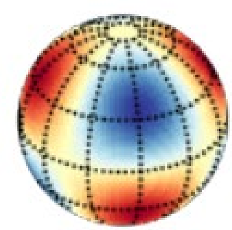
$m = 5,\ n = 7$



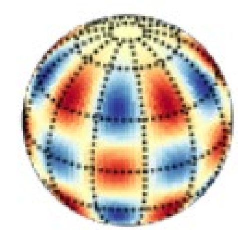
$m = 0,\ n = 3$



$m = 1,\ n = 3$



$m = 3,\ n = 6$



$m = 6,\ n = 10$

Zonal Harmonics (m=0). No longitude dependence as cos(mλ = 0)

- **When the order $m = 0$, the spherical harmonics are called *zonal harmonics*.**

$$P_{n0}(\sin\phi) = P_n(\sin\phi) \equiv \text{Legendre polynomials}$$

$$P_{00}(\sin\phi) = 1$$

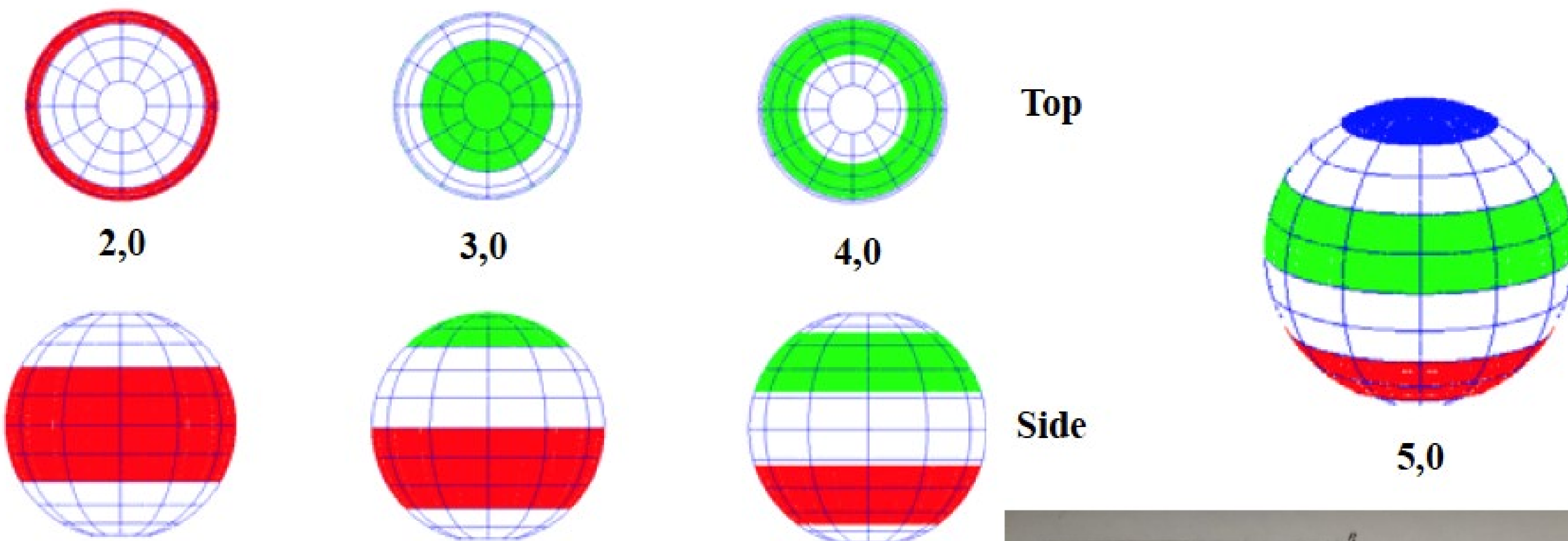
$$P_{10}(\sin\phi) = \sin\phi$$

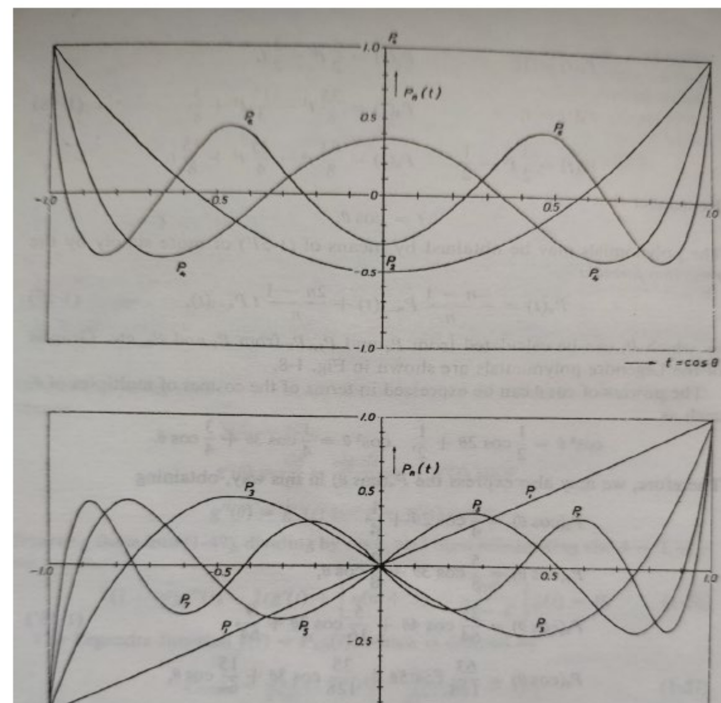
$$P_{20}(\sin\phi) = \frac{1}{2}\left(3\sin^2\phi - 1\right) \equiv \text{oblateness}$$

$$P_{30}(\sin\phi) = \frac{1}{2}\left(5\sin^3\phi - 3\sin\phi\right)$$

- $P_{n0}(\sin\phi)$ **have n zeros from pole to pole [–90° to 90°]**
- **Even degree zonals are symmetric about the equator.**
- **Odd degree zonals are asymmetric about the equator.**

Zonal harmonics





Major Terms.

- **For the Earth:**

$$J_2 = -C_{20} = 0.0010826$$

$$\overline{C}_{20} = -0.484 \times 10^{-3}$$

$$\overline{C}_{30} \sim 1 \times 10^{-5}$$

- **Other values of spherical harmonic coefficients are on the order of 1E-8, 1E-9 when getting to degree > 5; thus, C_{20} dominates**

Degree Variance

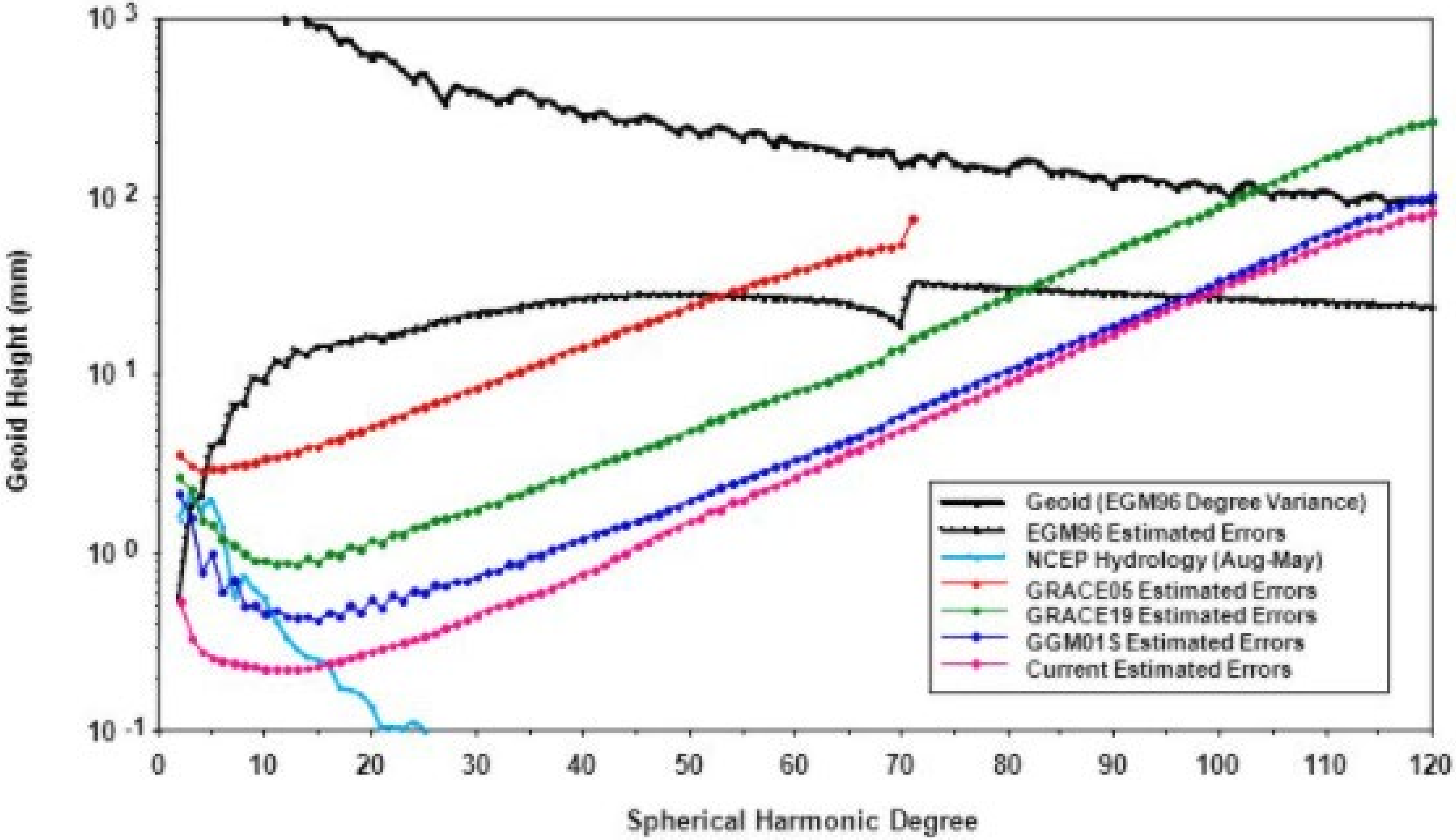
Degree variance tells you the average power there is in the gravity field or potential for each degree *n*:

$$\mathrm{RMS}_n = \sqrt{\frac{\sum_{m=0}^{n}\left(\bar{C}_{nm}^2 + \bar{S}_{nm}^2\right)}{2n+1}}$$

We can also compute the degree error variance by using the errors in each of the coefficients:

$$\sigma\mathrm{RMS}_n = \sqrt{\frac{\sum_{m=0}^{n}\left(\sigma\bar{C}_{nm}^2 + \sigma\bar{S}_{nm}^2\right)}{2n+1}}$$

The way to use Degree variances



Important property for Geodesy.

$$V = \frac{GM}{r}\sum_{n=0}^{\infty}\left(\frac{r_E}{r}\right)^n \sum_{m=0}^{n} P_{nm}(\sin\phi)[C_{nm}\cos m\lambda + S_{nm}\sin m\lambda]$$

- If we approximate this for a spherical Earth $r_E = r$
- *Then* $V = \frac{GM}{r} \sum_{n=0}^{\infty} 1 \sum_{m=0}^{n} P_{nm}(\sin\phi)[C_{nm}\cos m\lambda + S_{nm}\sin m\lambda]$
- *If we pick the center of mass as reference.*
- *Then* $V = \frac{GM}{r} \sum_{n=2}^{\infty} \sum_{m=0}^{n} P_{nm}(\sin\phi)[C_{nm}\cos m\lambda + S_{nm}\sin m\lambda]$
- *Using Bruns theorem.*
- *Then* $N = \frac{GM}{\gamma r} \sum_{n=2}^{\infty} \sum_{m=0}^{n} P_{nm}(\sin\phi)[C_{nm}\cos m\lambda + S_{nm}\sin m\lambda]$
- *And the formula for gravity*
- *Then* $\Delta g = \frac{GM}{r^2} \sum_{n=2}^{\infty}(n+1) \sum_{m=0}^{n} P_{nm}(\sin\phi)[C_{nm}\cos m\lambda + S_{nm}\sin m\lambda]$

Wavelength of spherical harmonics.

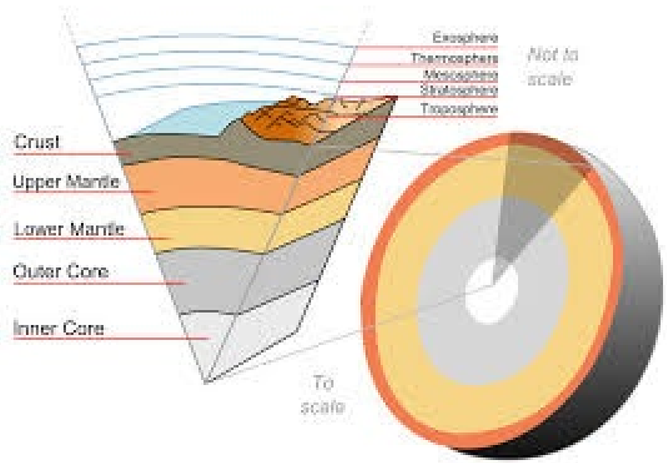
The values of *n* and *m* tell us something about the spatial wavelengths of the spherical harmonics.

$$P_{nm}(\sin\phi)\cos m\lambda$$

$$P_{nm}(\sin\phi)\sin m\lambda$$

Table 10.1. Typical subdivision of the gravity field expansion

Subdivision	long	mean	short	very short
wavelength, λ [km]	> 8000	> 1000	> 200	< 200
degree and order, N, of the representation	< 5	< 36	< 200	> 200
block size, S [degrees] (mean anomalies)	> 10	> 5	> 1	< 1



Long wavelength = Low Degree/order = deep interior

Shorter wavelength = Higher D/O = crustal changes)

Building an EARTH geopotential model (EGM)

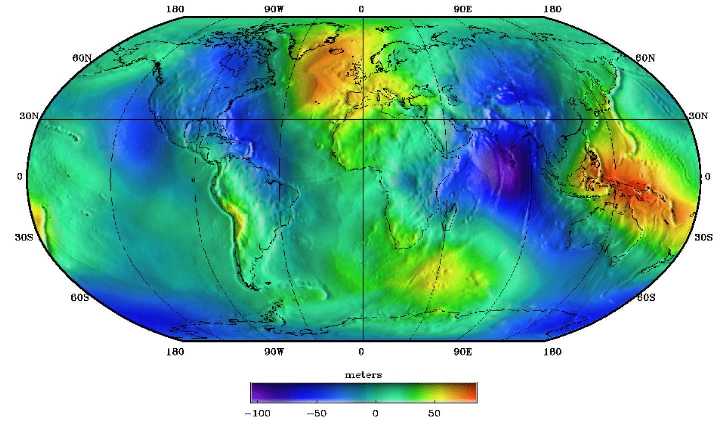
- Low-degree models (S or SATELLITE ONLY MODELS)
  - Satellite-tracking data or satellite gradiometry

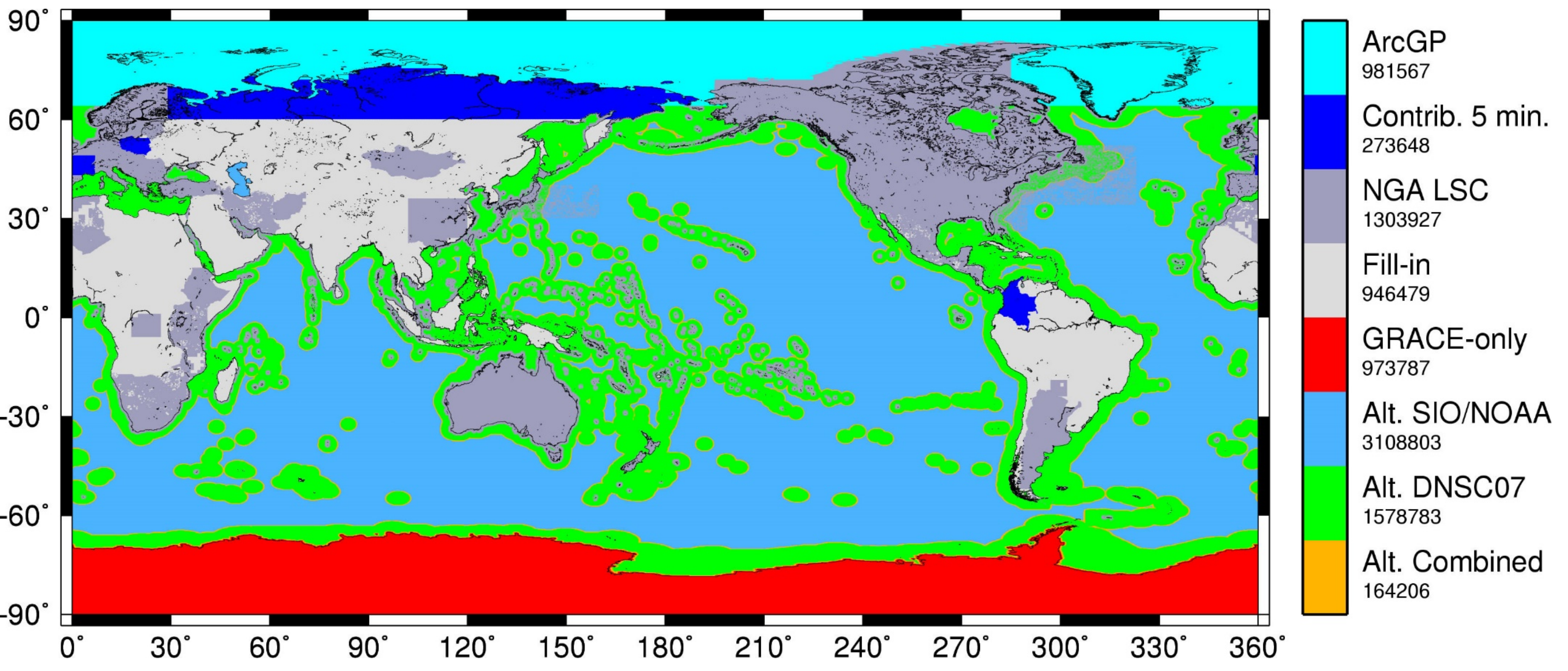
- High degree models (C or COMBINATION MODELS) includes
  - Topographic and surface gravity databases
  - Altimetry-derived gravity anomalies

  Data are weighted and are used to find a least-squares solution
  - Errors can be estimated from the covariance matrix
  - Errors can also be estimated by a model's ability to reduce in radial orbit error

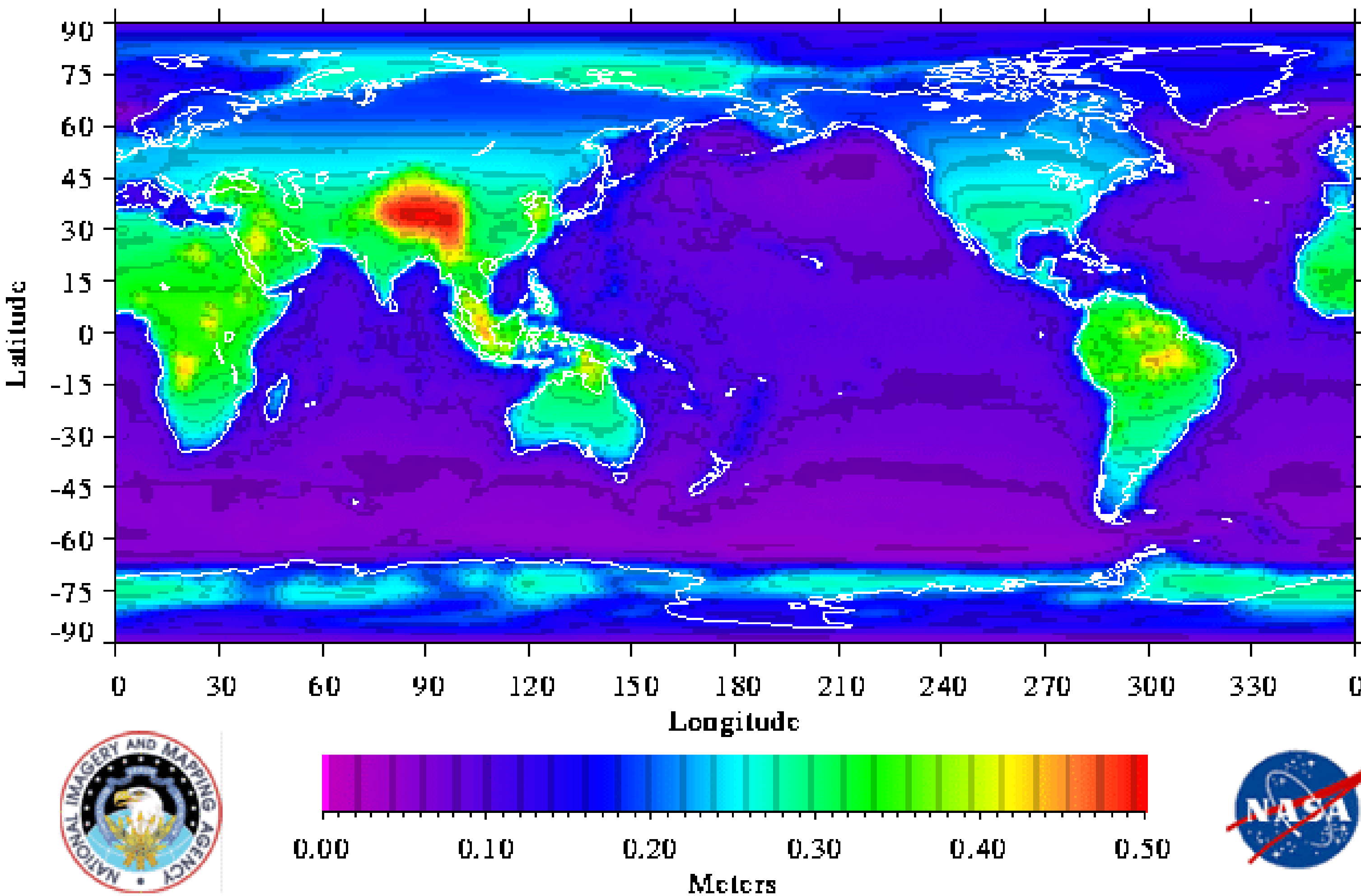
  - Need an independent method to judge orbits
  - DTU IMPORTANT GLOBALLY
    - Fly All of Africa/Mongolia/Indonesia/A
    - Recently even Antarctica
    - -> Rene Forsberg

The Global model combines all sources of data. Satellite and ground. (EGM2008).



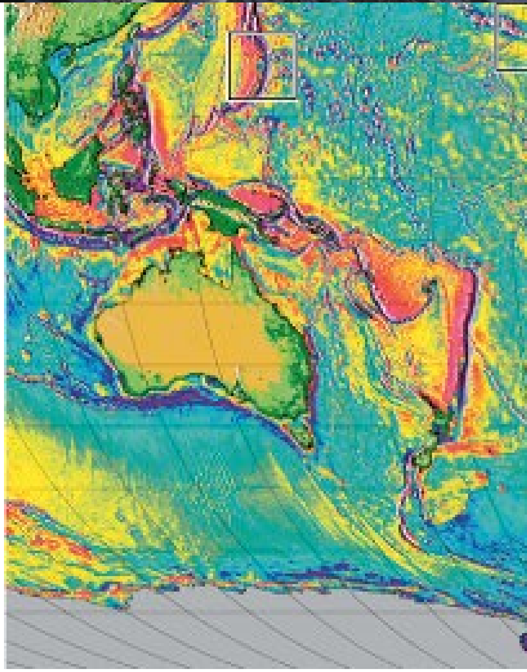


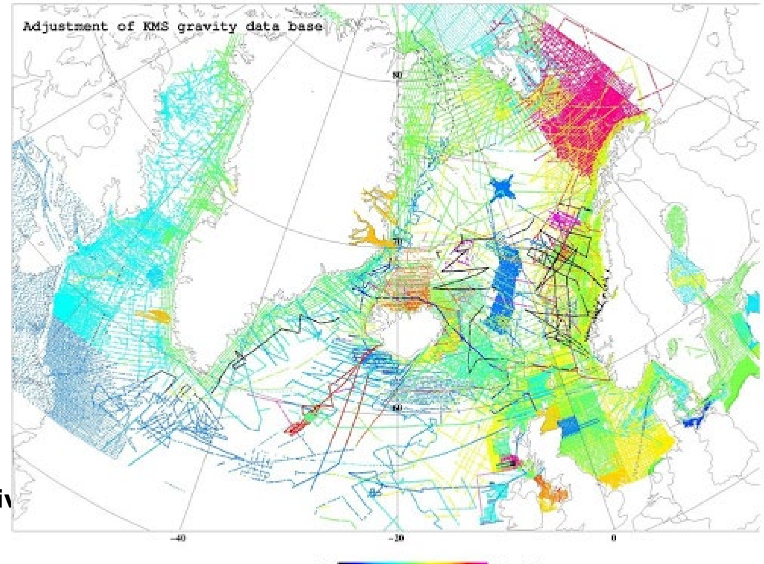
EGM96 Error – NEED FOR UPDATE



EGM08 is d/o = 2159
Wl = 40000/2159=15 km

- **Satellite (orbit perturbation and gravity)**
- Satellite altimetry.
- Airborne.
- Marine and land borne observations.





What we want is to build a geopotential model

Single Satellite:

$$A_i \hat{x} = B_i$$

Multiple Satellites:

$$\left(\sum_i A_i \right) \hat{x} = \left(\sum_i B_i \right)$$

$$\hat{x} = \left(\sum_i A_i \right)^{-1} \left(\sum_i B_i \right)$$

$$\hat{x} = \begin{bmatrix} r_x \\ r_y \\ r_z \\ v_x \\ v_y \\ v_z \\ C_D \\ C_R \\ J_2 \\ C_{2,2} \\ \vdots \end{bmatrix}$$

Least squares adjustment

- Given the observation equation:

$$\mathbf{l} = \mathbf{A}\mathbf{x} + \mathbf{v}$$

- We want to find the best solution according to the least squares principle, namely the solution which minimises the residual (or the influence of the noise and errors in our observations):

$$\min\left[\mathbf{v}^T\mathbf{v}\right]$$

- The estimate of **x** providing the minimum sum of the squared residuals is then given as:

$$\hat{\mathbf{x}} = \left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T\mathbf{l}$$

- The weights are introduced in a weight matrix, P, so:

$$\hat{\mathbf{x}} = \left(\mathbf{A}^T\mathbf{P}\mathbf{A}\right)^{-1}\mathbf{A}^T\mathbf{P}\mathbf{l}$$

- Where P is a diagonal matrix:

$$\mathbf{P} = \begin{bmatrix} P_{11} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & P_{nn} \end{bmatrix}$$

Break

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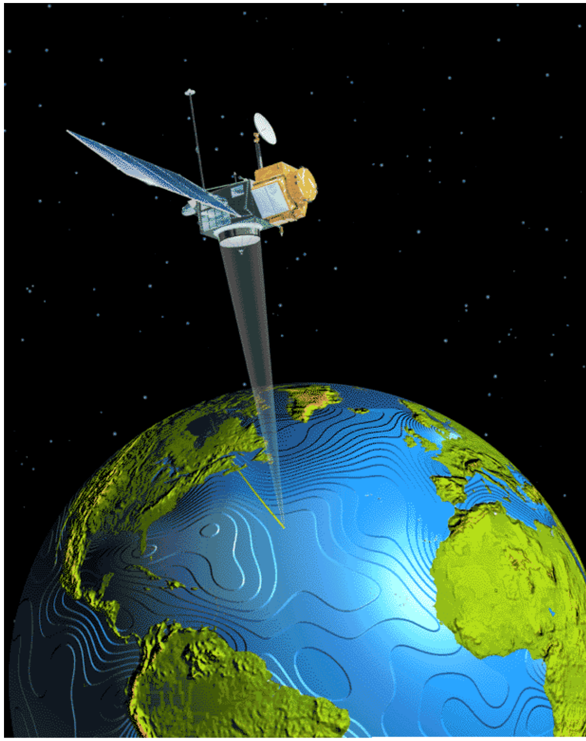

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Attenuation and downward continuation

- At a height h above the Earth's surface, a constituent of potential V with harmonic degree l is reduced (*attenuated*) by a factor of $[r_E/(r_E + h)]^{(n+1)}$.
- Similarly, gravity anomalies are attenuated by $[r_E/(r_E + h)]^{(n+1)}$
- Gravity gradients are attenuated by $[r_E/(r_E + h)]^{(n+2)}$

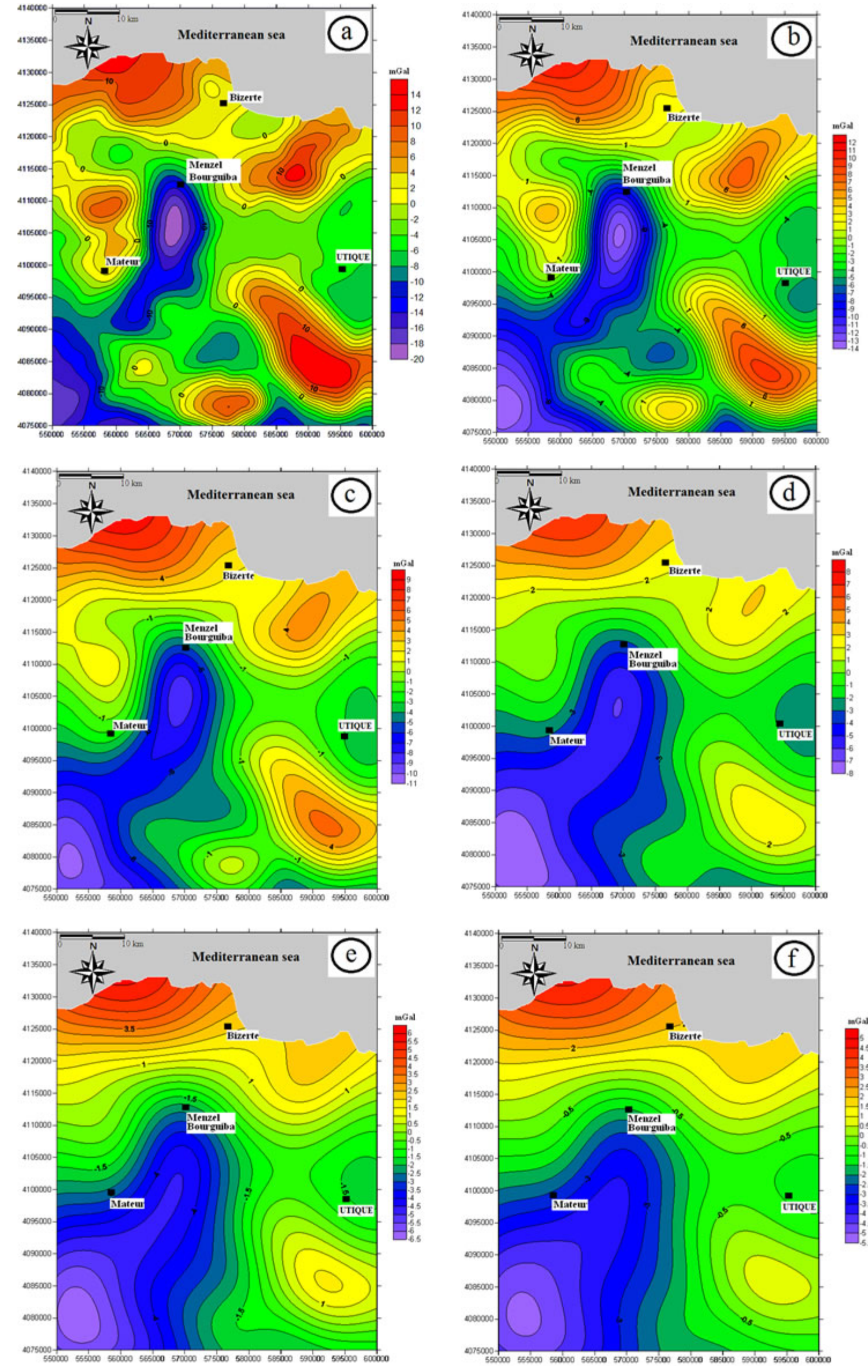
- Measurements made by satellites must be *downward continued* to estimate the potential and gravity at the surface.
  - Downward continuation amplifies the errors by the reciprocals of the attenuation factors.
  - Thus, there is a critical wavelength ($\approx h$) such that wavelengths much longer than the critical can be resolved, much shorter cannot.

Upward continuation.

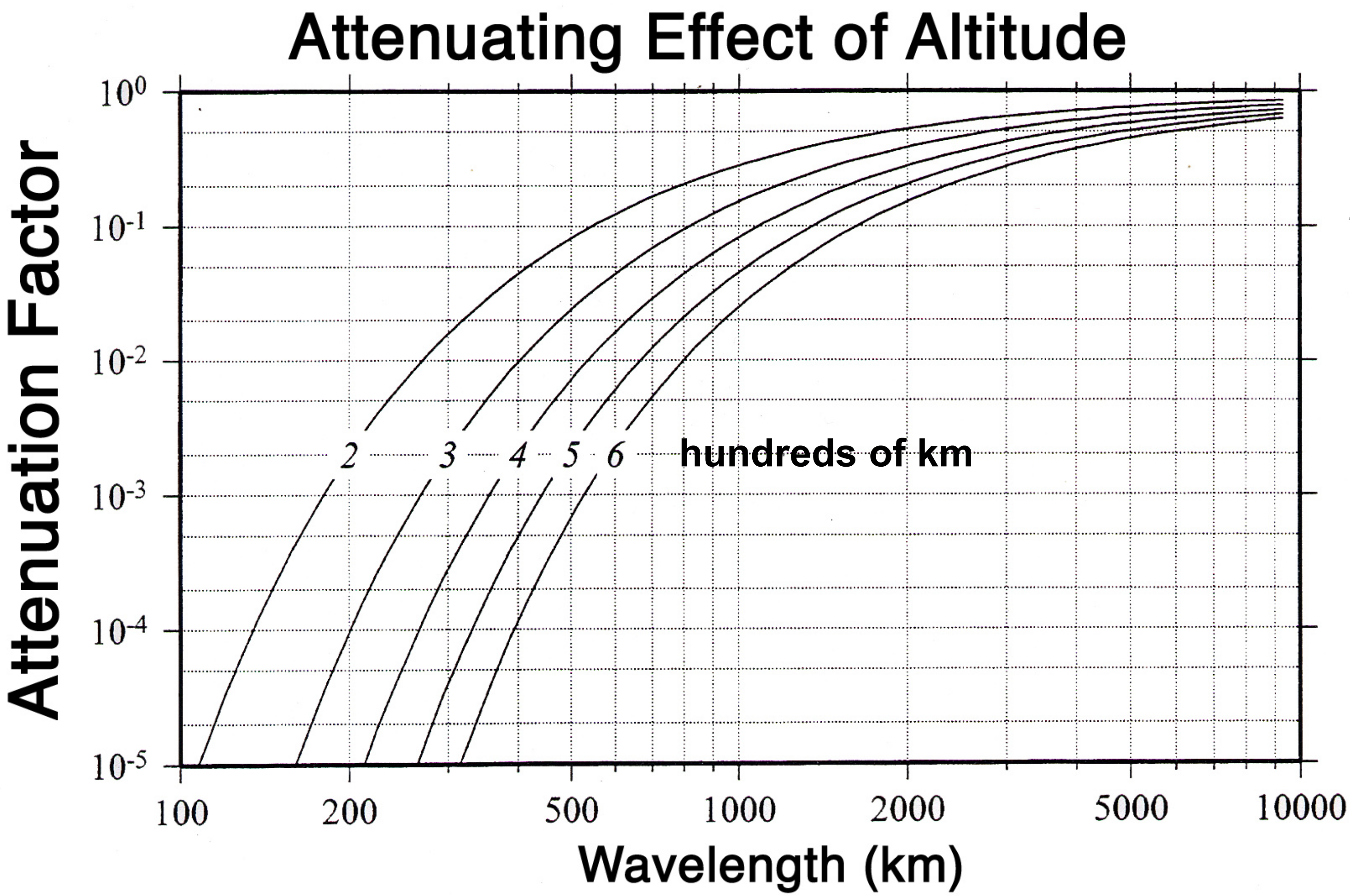
The gravity field becomes
Smoother with height
(until pointmass).

So the higher you fly the less
Detail you se.
So you want to fly LOW

AS LOW AS POSSIBLE
BUT DRAG BECOMES ISSUE



Sensitivity versus altitude and wavelength



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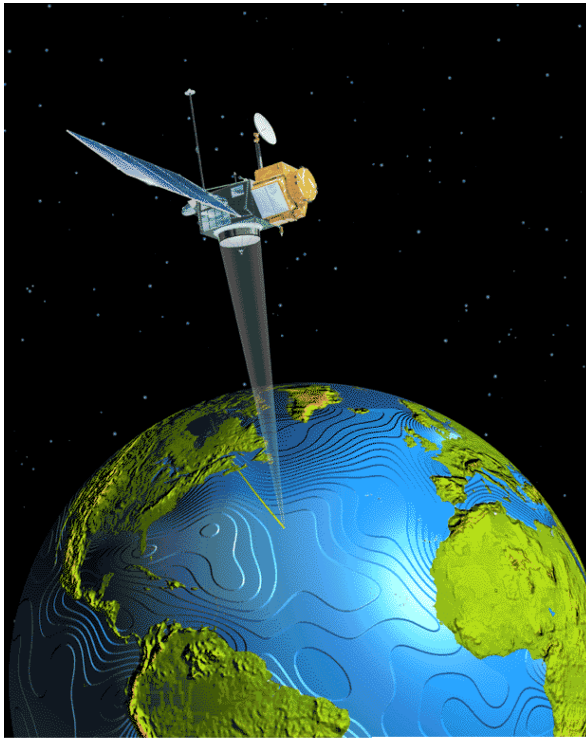

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Gravity field Mission design

IN ALL DESIGNS THE SATELLITE ARE PROBES IN THE GRAVITY FIELD

GPS Tracked accelerometer

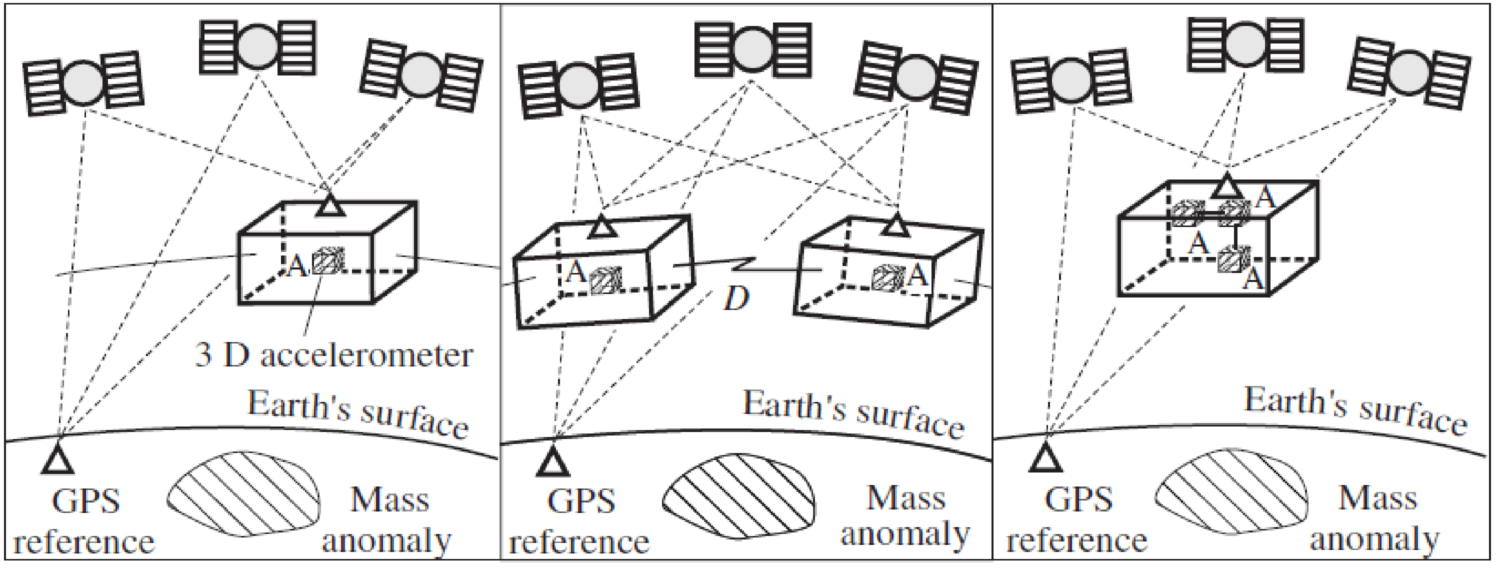
- High-low: CHAMP

Satellite to Satellite Tracking (SST)

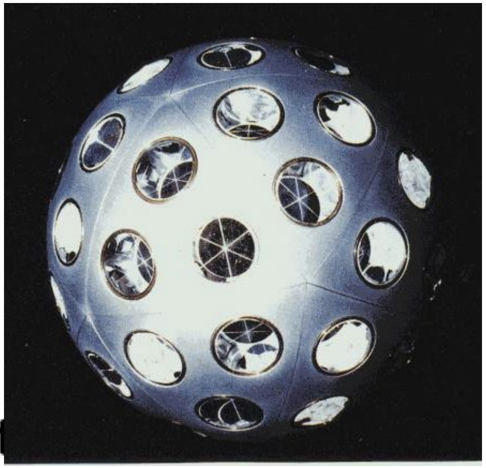
-low-low+GPS accerometers (GRACE + GRACE-FO)

Satellite Gravity Gradiometry (SGG)

-GOCE



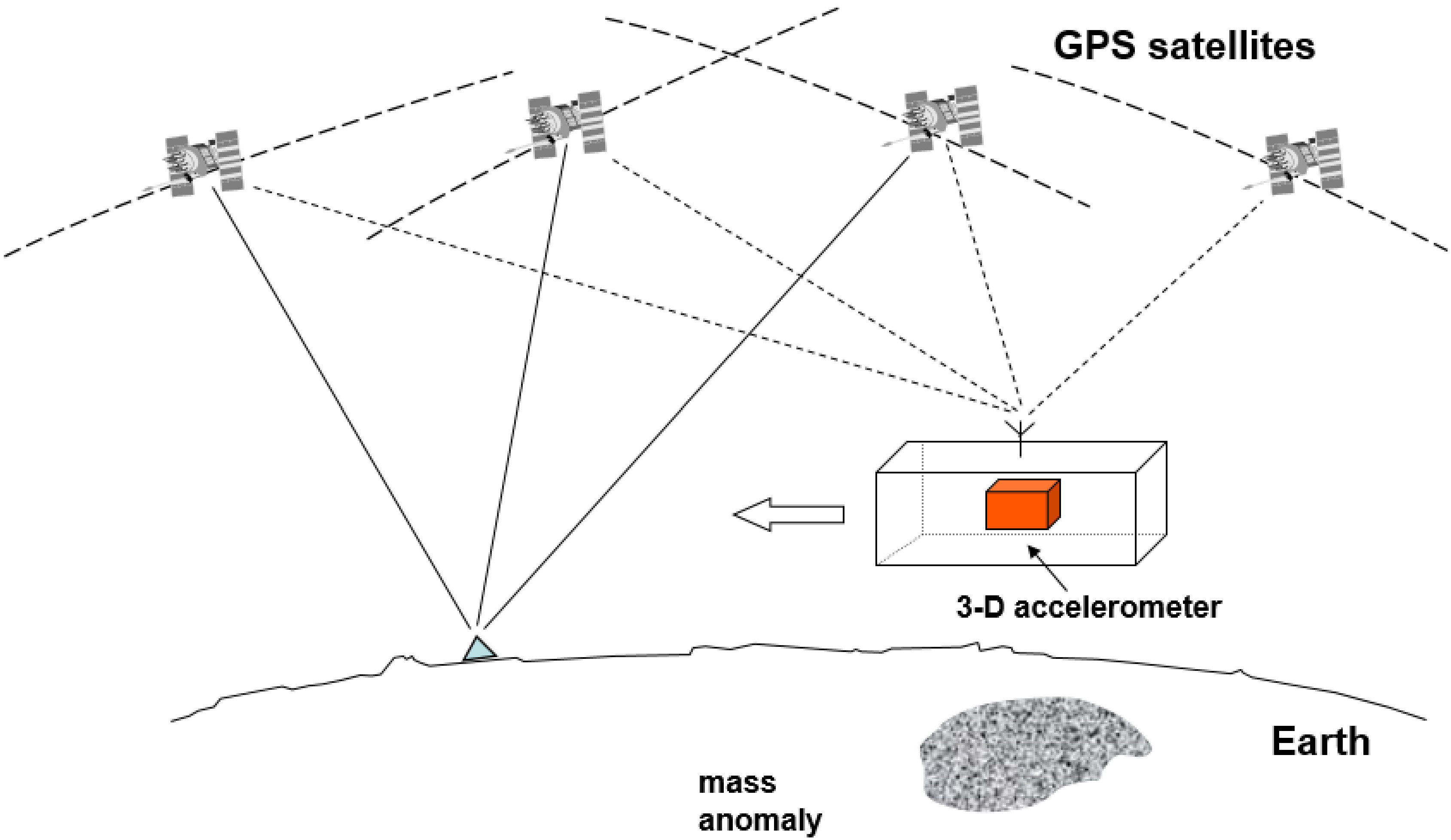
Satellite Missions



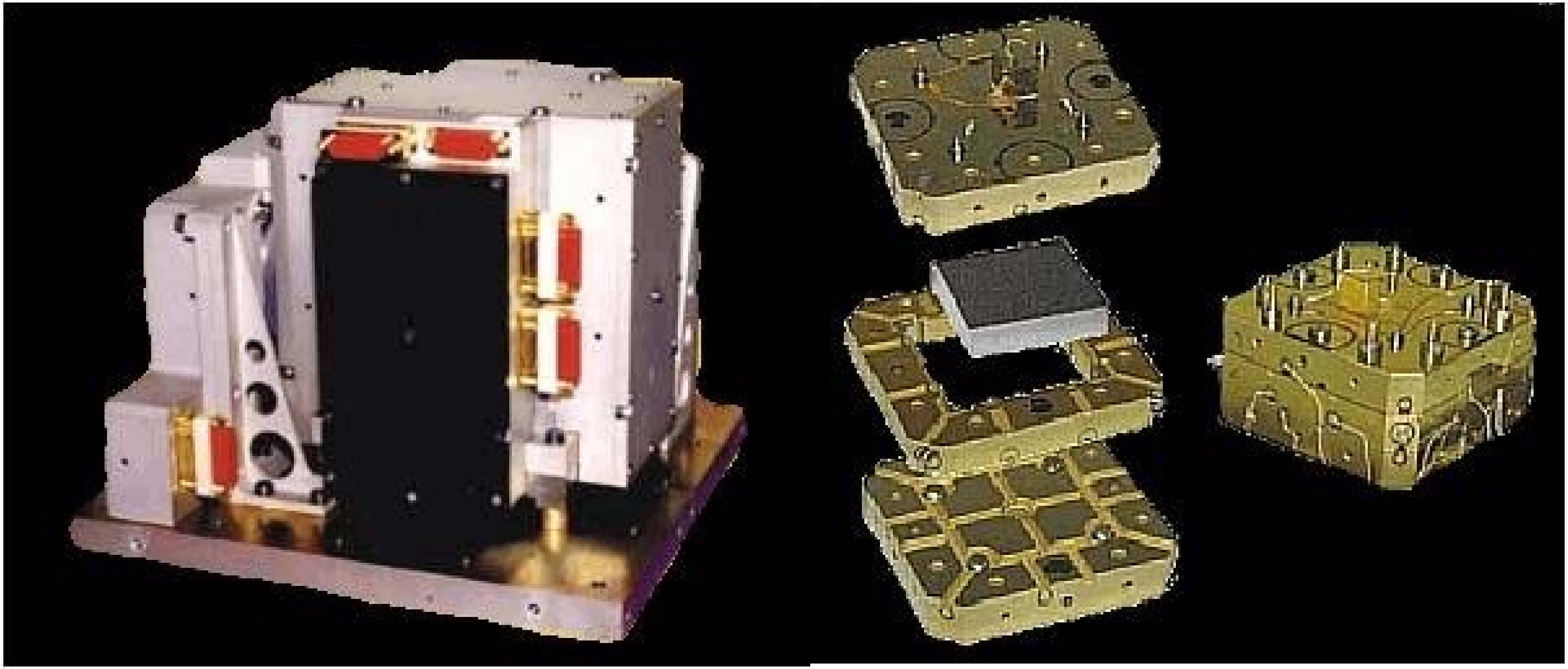
- Geodetic Satellite Laser Ranging
  - Starlette (1975-present)
  - LAGEOS (1976-present)
  - LAGEOS II (1992-present)
- Dedicated Satellite Gravity Missions
  - CHAMP (DLR, 2000, 5 years, GPS/accelerometer)
  - GRACE (NASA & DLR, 2002, 5 years, satellite-to-satellite microwave link/GPS/Accelerometer)
  - GOCE (ESA, 1 year, gradiometer): static field main goal
- Ongoing
  - GRACE Follow-On (NASA, 5-10 year mission, 2017-22)
    - main goal is an extended time variable field

NGGM (Proposed NOW)

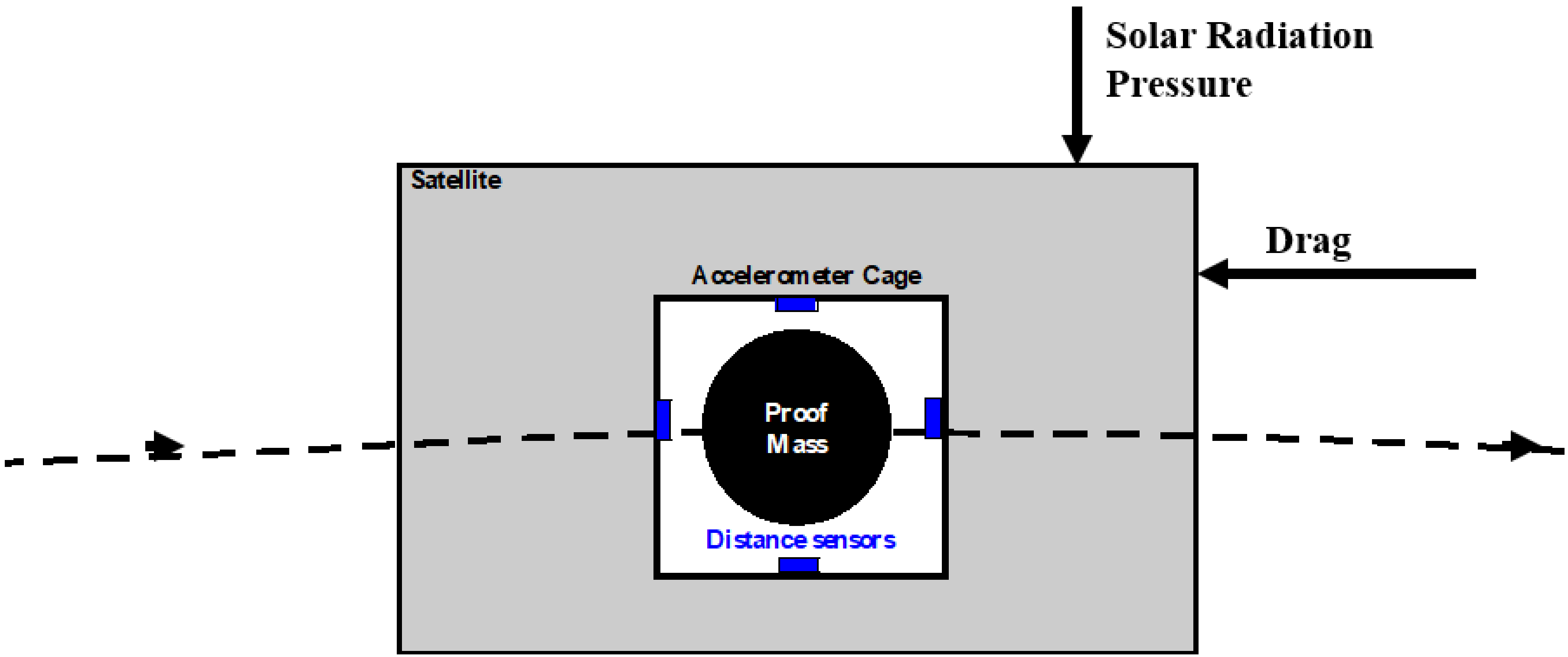
High-low GPS accelerometer (CHAMP)



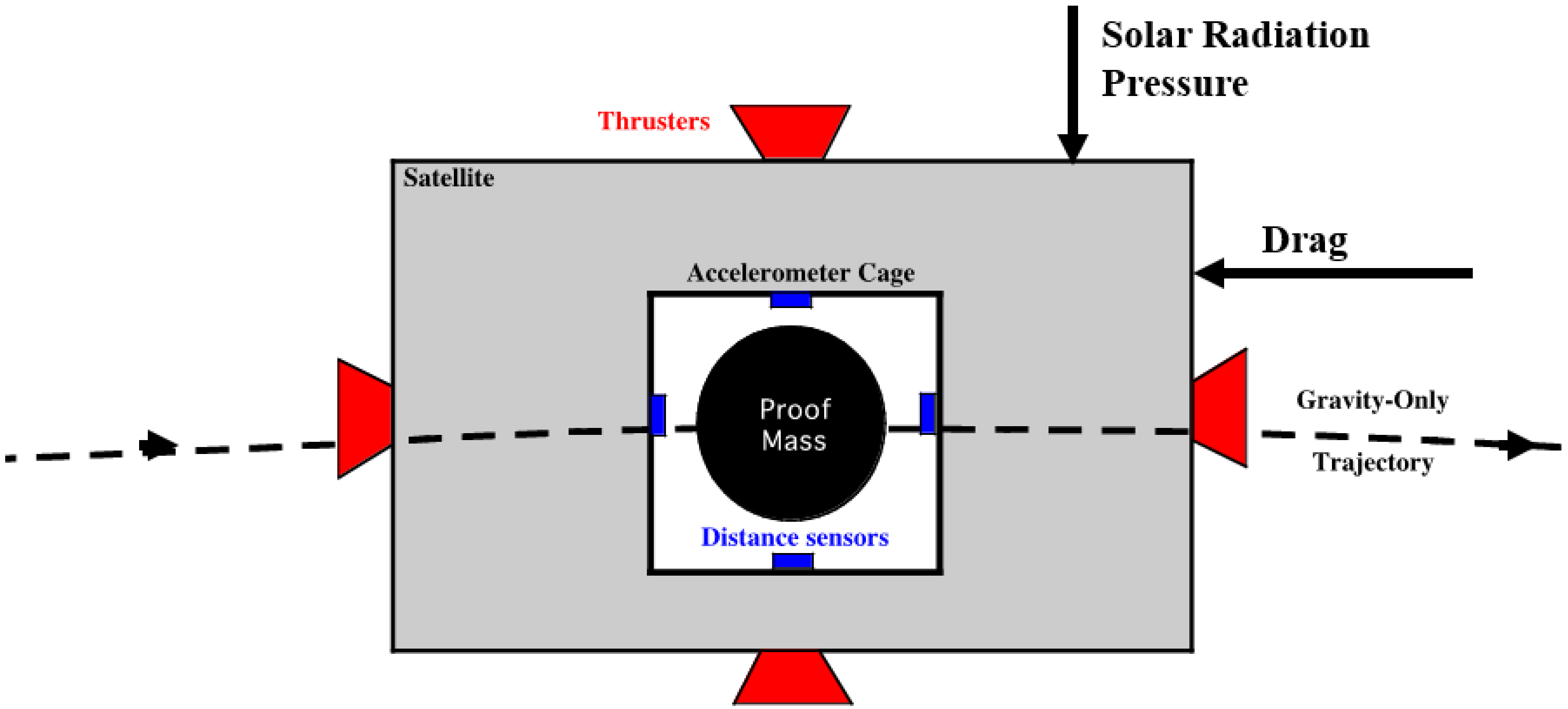
CHAMP has one accelerometer



Accelerometer principle

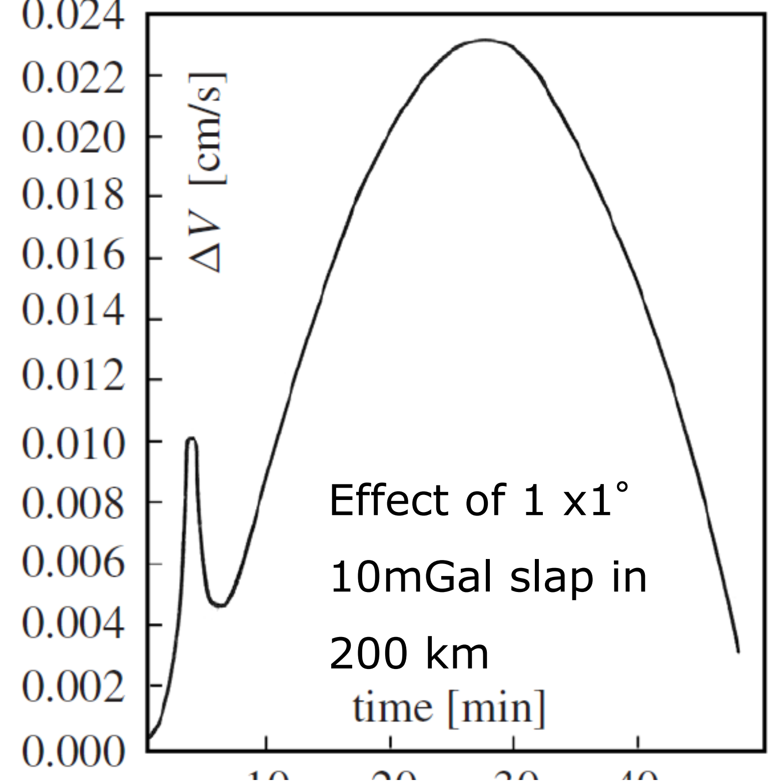


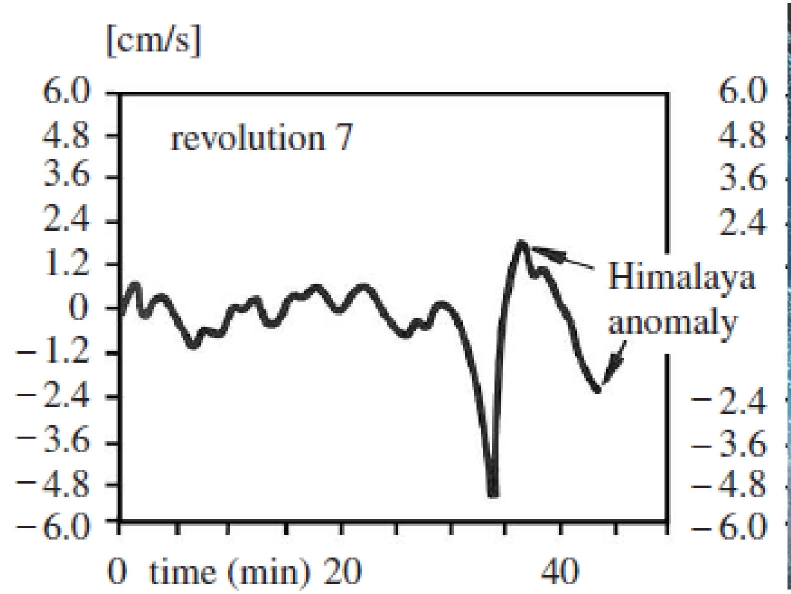
Add Thrusters to make it Drag Free”



CHAMP

- The CHAMP satellite was launched with a Russian COSMOS launch vehicle on 15 July, 2000 into an almost circular, near polar (87) orbit with an initial altitude of 454 km. The design lifetime of the satellite system is 5 years.
  - The 87° inclination is the maximum inclination which can be served from Plesetsk
- GPS-CHAMP high-low satellite-to-satellite and ground based laser tracking
- Nearly cm-accuracy up to a spatial resolution of about 650 km half-wavelength.
- Corresponds to Degree and order 70.
- The long-wavelength geoid obtained with CHAMP then serves as a perfect reference for higher resolution global or regional gravity field modelling.





Content

- Purpose: Building global gravity field models.
- Spherical harmonic functions (repetition)
- Degree Variance.
- Least Squares Estimation (repetition).
- Building global gravity field models.
- Upward Continuation / Attenuation/Limits.
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    - Satellite Tracking (high -low)
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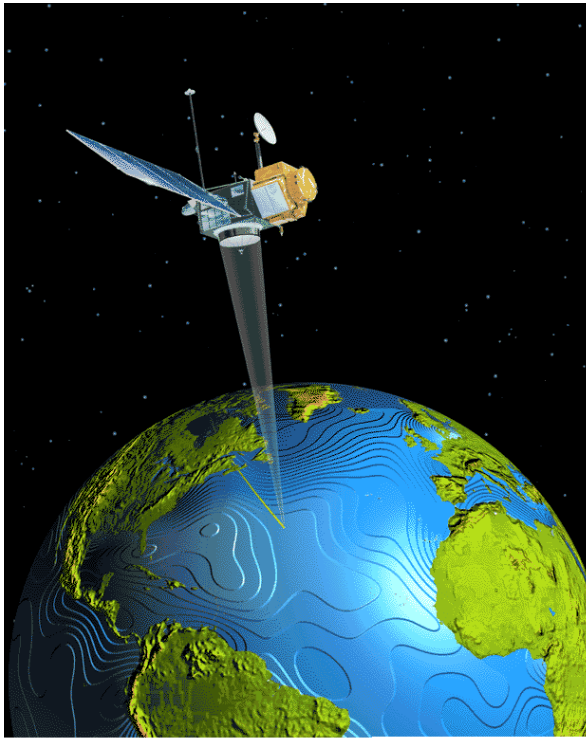
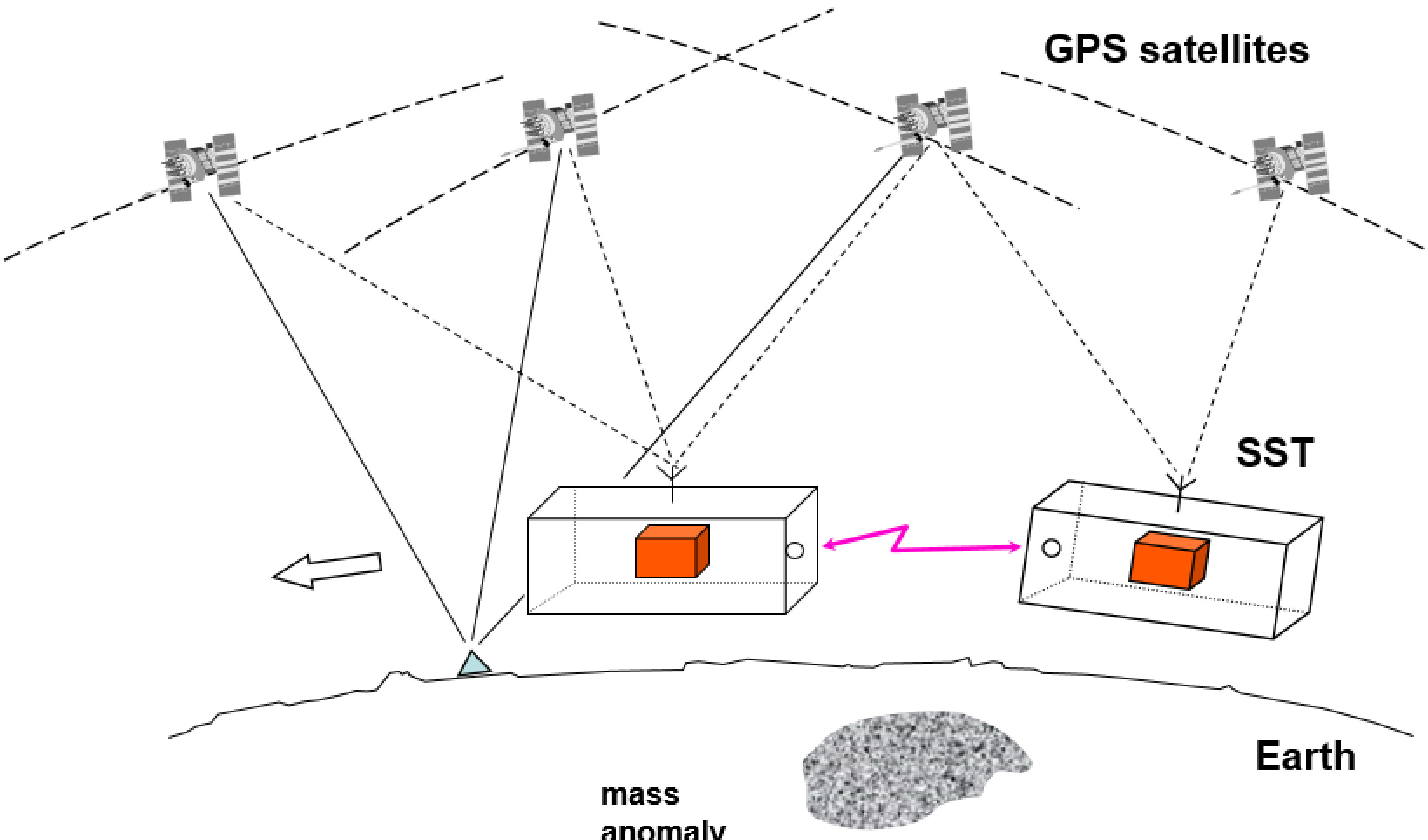
    - GRACE + GRACE FO
    - Satellite gradiometry
    - GOCE



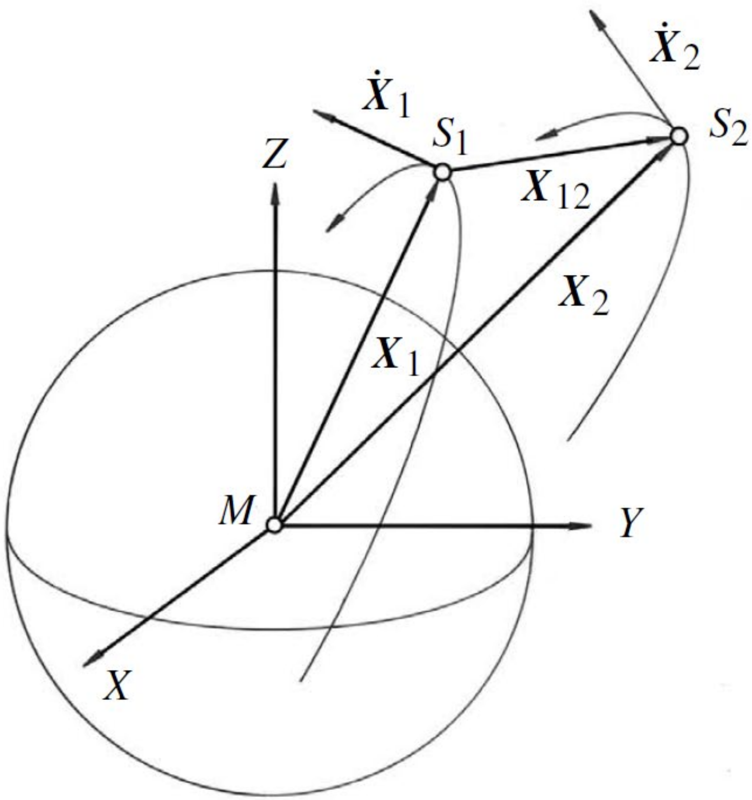
Litterature Seeber 469-484

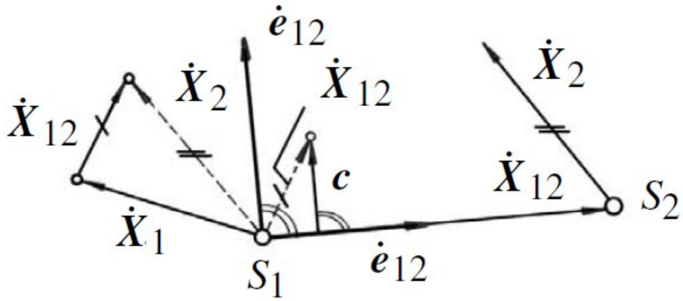
Low-Low: Satellite to Satellite tracking + accelerometers + GPS (low –high)



Satellite to Satellite Tracking (GRACE)

How Satellite to Satellite Tracking works





$$\dot{\rho} = \dot{X}_{12} e_{12}.$$

$$e_{12} = \frac{X_2 - X_1}{|X_2 - X_1|} = \frac{X_{12}}{\rho}$$

Seeber chapter 10.2.1

Fundamental observation is velocity range rate $\dot{\rho}$:

Or velocity change **along $\mathbf{e_{12}}$ called** $\ddot{\rho}$

When one satellite speed's up

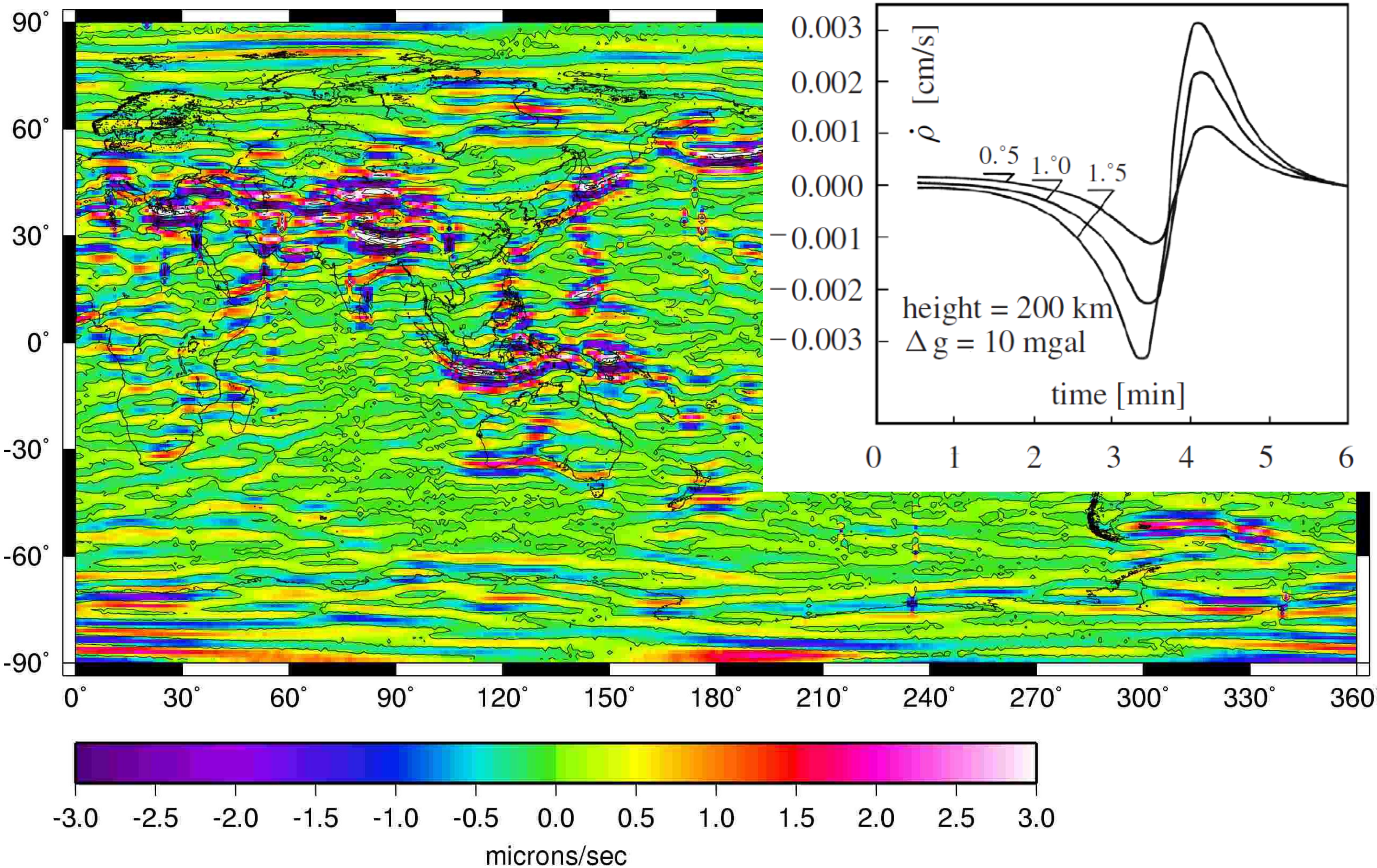
The Range Rate Change

$$\begin{aligned}\ddot{\rho} &= \ddot{\boldsymbol{X}}_{12}\boldsymbol{e}_{12} + \dot{\boldsymbol{X}}_{12}\dot{\boldsymbol{e}}_{12} \\ &= \ddot{\boldsymbol{X}}_{12}\boldsymbol{e}_{12} + \dot{\boldsymbol{X}}_{12}(\dot{\boldsymbol{X}}_{12} - \dot{\rho}\boldsymbol{e}_{12})\rho^{-1} \\ &= \ddot{\boldsymbol{X}}_{12}\boldsymbol{e}_{12} + ((\dot{\boldsymbol{X}}_{12})^2 - (\dot{\rho})^2)\rho^{-1}\end{aligned} \tag{10.4}$$

$$\dot{\boldsymbol{e}}_{12} = \frac{d}{dt}(\boldsymbol{X}_{12}\rho^{-1}) = (\dot{\boldsymbol{X}}_{12} - \dot{\rho}\boldsymbol{e}_{12})\rho^{-1} = \boldsymbol{C}\rho^{-1}. \tag{10.5}$$

Skipping the linearization the fundamental output is the acceleration Vector orthogonal to the interconnecting line between both satellites.

$$\boldsymbol{a} = \ddot{\boldsymbol{X}}_{12} - (\dot{\boldsymbol{X}}_{12}\boldsymbol{e}_{12})\boldsymbol{e}_{12}$$


90°
60°
30°
0°
-30°
-60°
-90°
0° 30° 60° 90° 120° 150° 180° 210° 240° 270° 300° 330° 360°
0.003
0.002
0.001
0.000
−0.001
−0.002
−0.003
$\dot{\rho}$ [cm/s]
0.°5
1.°0
1.°5
height = 200 km
Δ g = 10 mgal
time [min]
0 1 2 3 4 5 6
-3.0 -2.5 -2.0 -1.5 -1.0 -0.5 0.0 0.5 1.0 1.5 2.0 2.5 3.0
microns/sec

Advantage of Satellite to Satellite Tracking (SST)

Accurately determines the effects of the gravity difference between the two satellite positions

The separation distance between the satellites determines which spatial wavelengths of the geopotential that the mission would be most sensitive

SST missions are not very sensitive to wavelengths with lengths equal to the separation distance because those wavelengths would cause the two spacecraft to move in phase with no change in the satellite-to-satellite-range

Limitation is 200 km wavelength

Or degree and order 200

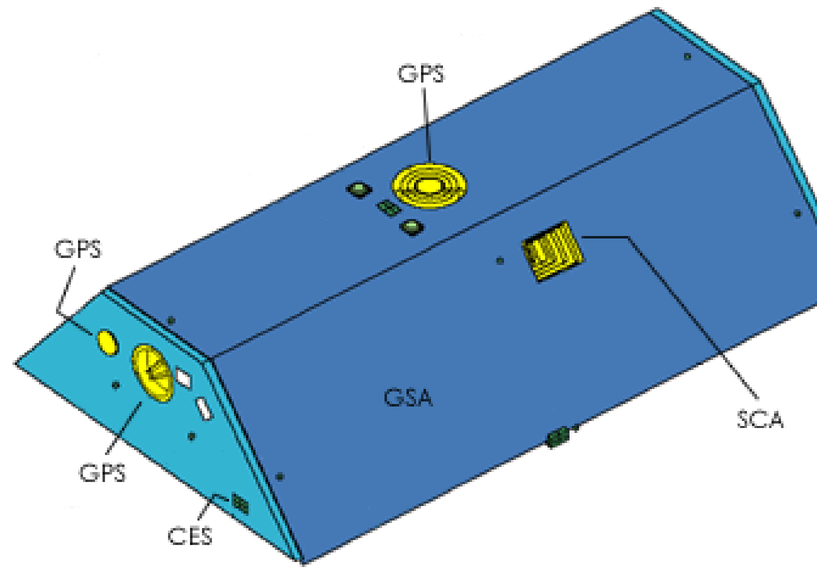
GRACE Formation Flying Satellites

KBR+USO Ranging (JPL/SSL/APL)

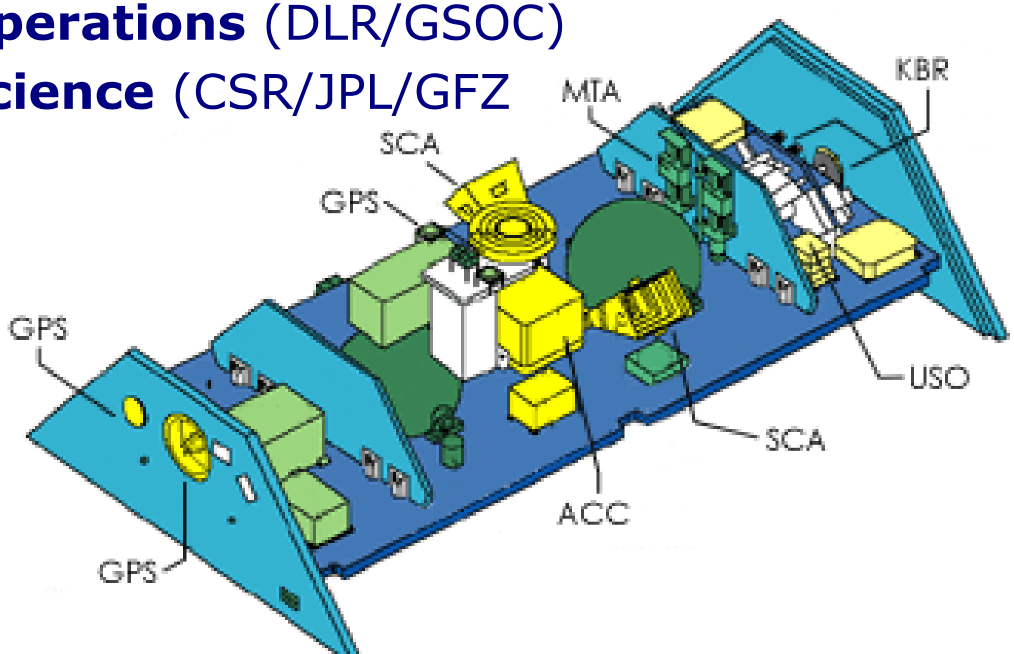
ACC SuperSTAR Accelerometer (ONERA)

SCA Star Cameras (DTU)

GPS Receiver (JPL)



- **Satellite** (JPL/Astrium)
- **Launcher** (DLR/Eurockot)
- **Operations** (DLR/GSOC)
- **Science** (CSR/JPL/GFZ



Orbit

Launched: March 17, 2002

Initial Altitude: 500 km

Inclination: 89 deg

Eccentricity: ~0.001

Distance: ~220 km

Nominal Mission : 5 years

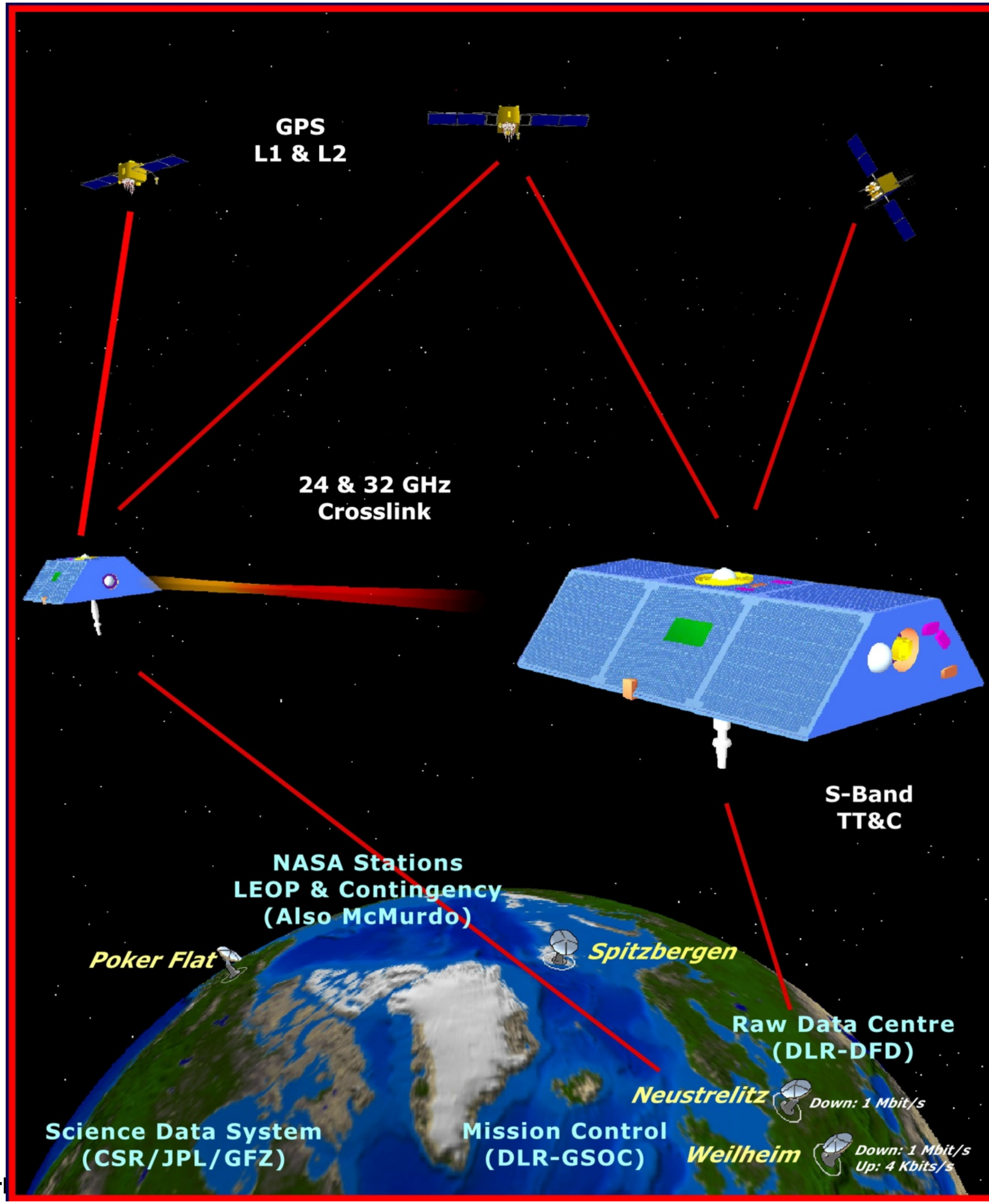
Non-Repeat Ground Track, Earth Pointed, 3-Axis Stable

2005 GRACE 1-2 Orbit Switch

Mission Lifetime

- **5 Years**
- **Extended to 9 Years**

But worked 14 years (2016)



GRACE measurement technique

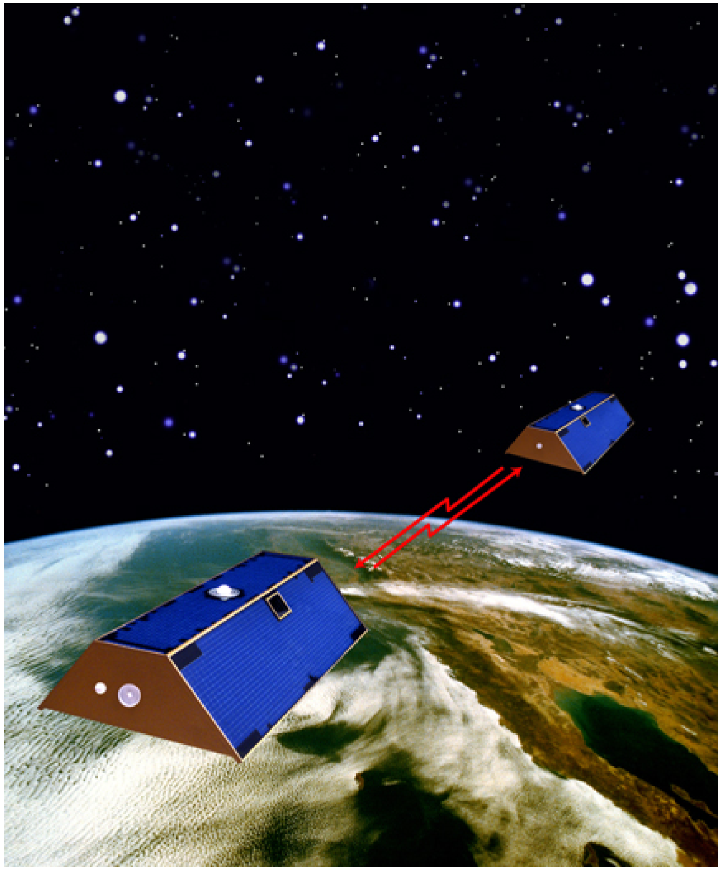
The two satellites act like a very big scale because of ORBIT pertubation……

Orbit pertubation/ranging is measured by KBR ranging

Accuracy currently 2 Micrometer

1/100 thickness of a hair

Better than diameter of blood cell



GRACE and GRACE Follow-on

GRACE FO Launched in 2018 to continue GRACE observations

Similar to GRACE but flies a laser interferometer (measures ranges 100 times better)

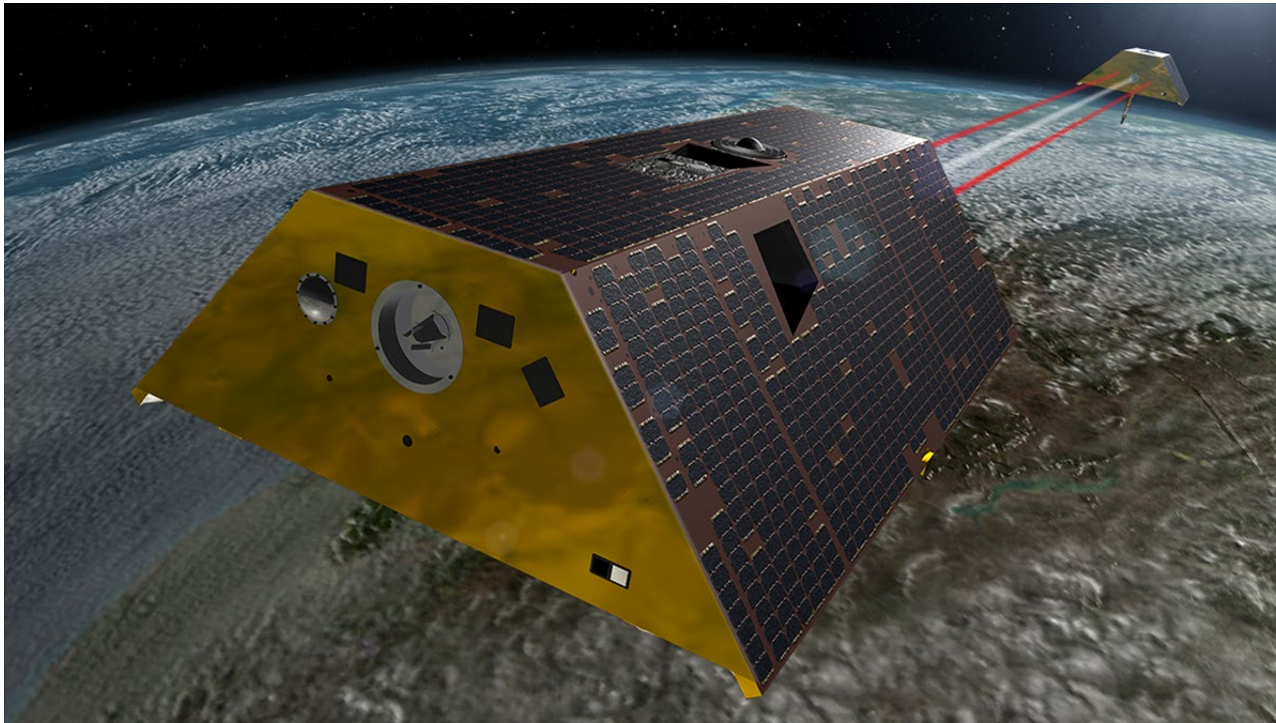
Both pairs flies HIGH (500 km) and resolves limited scales (> 200 km).

It provides Monthly gravity field.

So with GRACE we have fields from 2002 until end in 2016

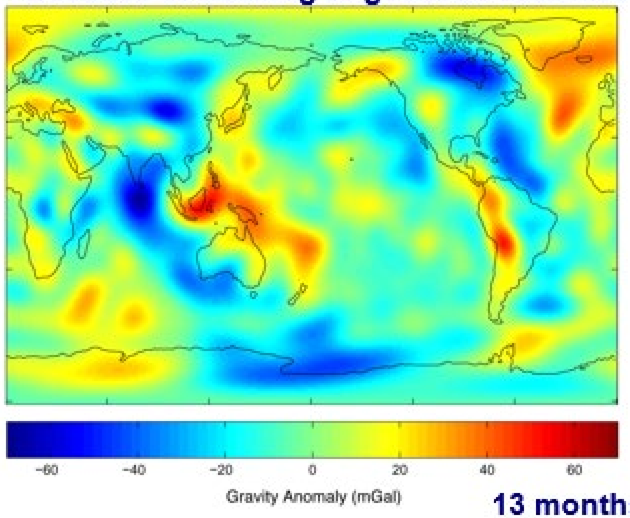
From GRACE follow in we have monthly fields from 2018-2020

Will use it to study the Earth gravity field variations in Lecture 13 next time.

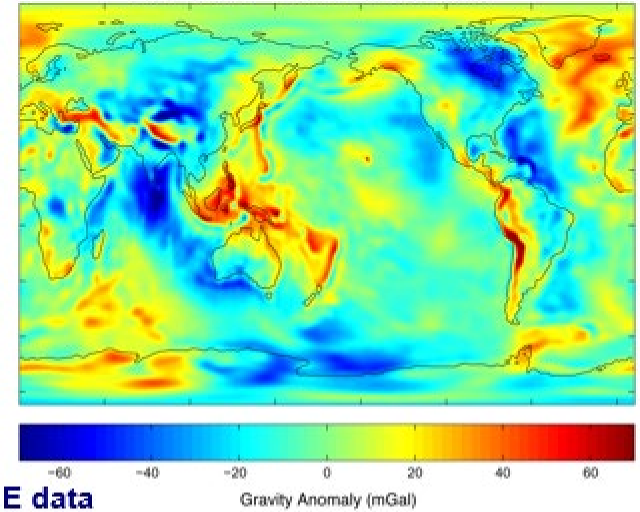


Progress in Gravity with GRACE

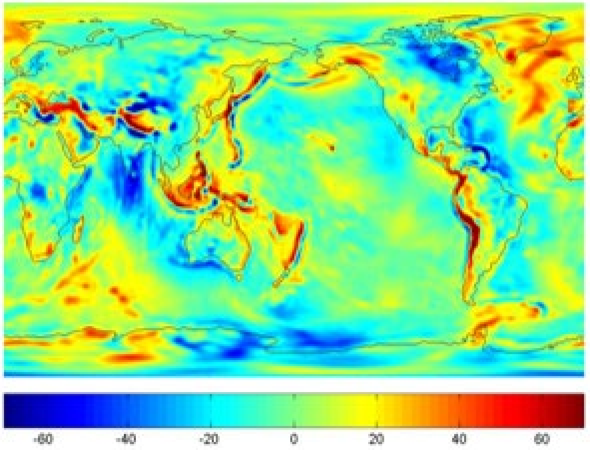
Decades of tracking to geodetic satellites



111 days of GRACE data



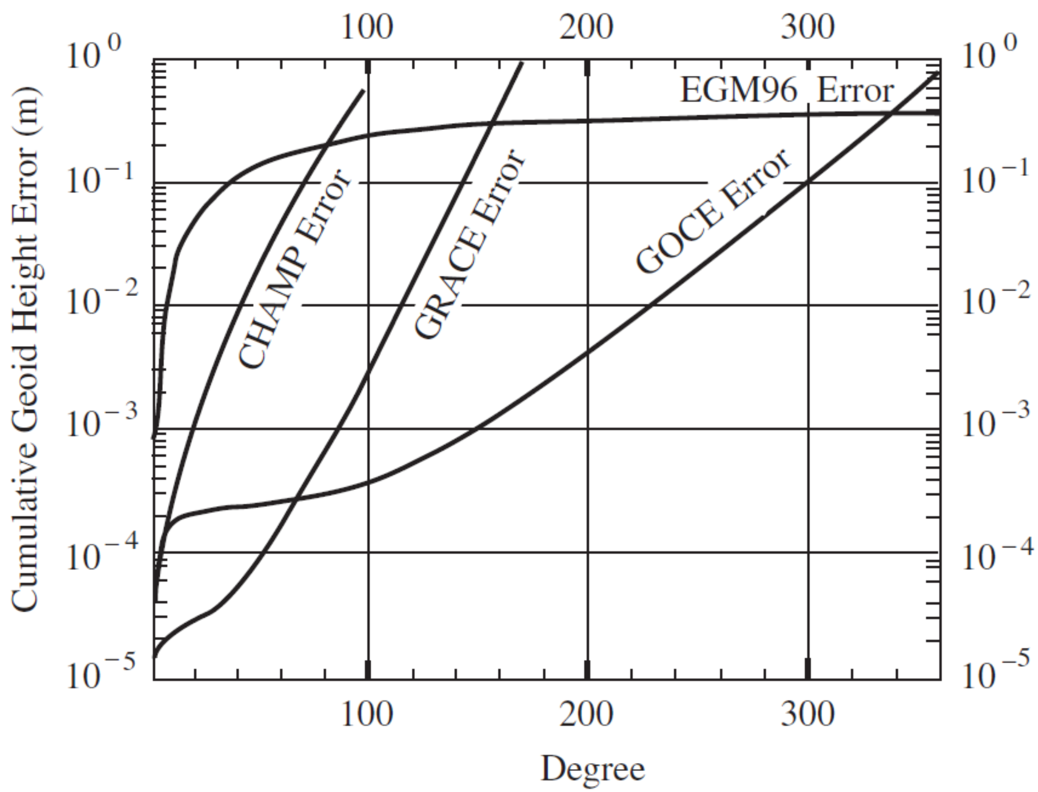
13 months of GRACE data



Detailed geophysical features are being detected by GRACE with no surface gravity inputs and no satellite altimetry

Latest model shows more detail because less smoothing is required to remove artifacts

Limitation of Satellite to Satellite tracking.



Limitation is degree and order 180-200 (200 km)

You could fly lower and closer but drag means that mission will only last a few years.

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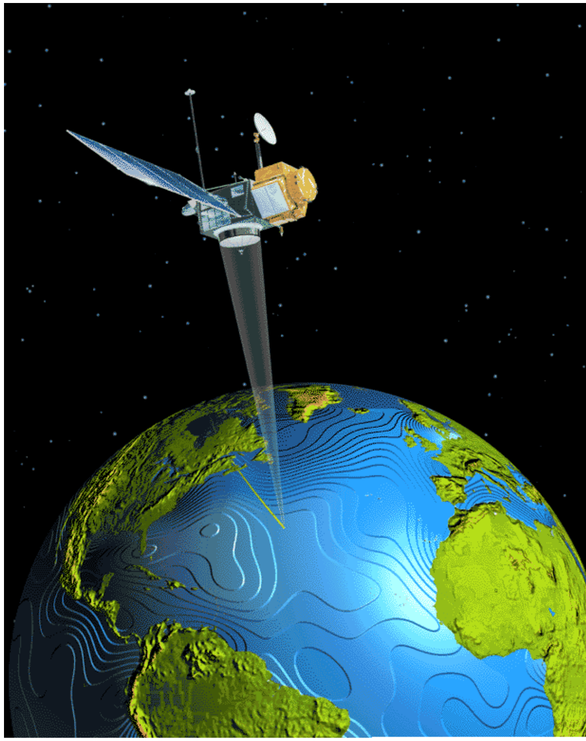
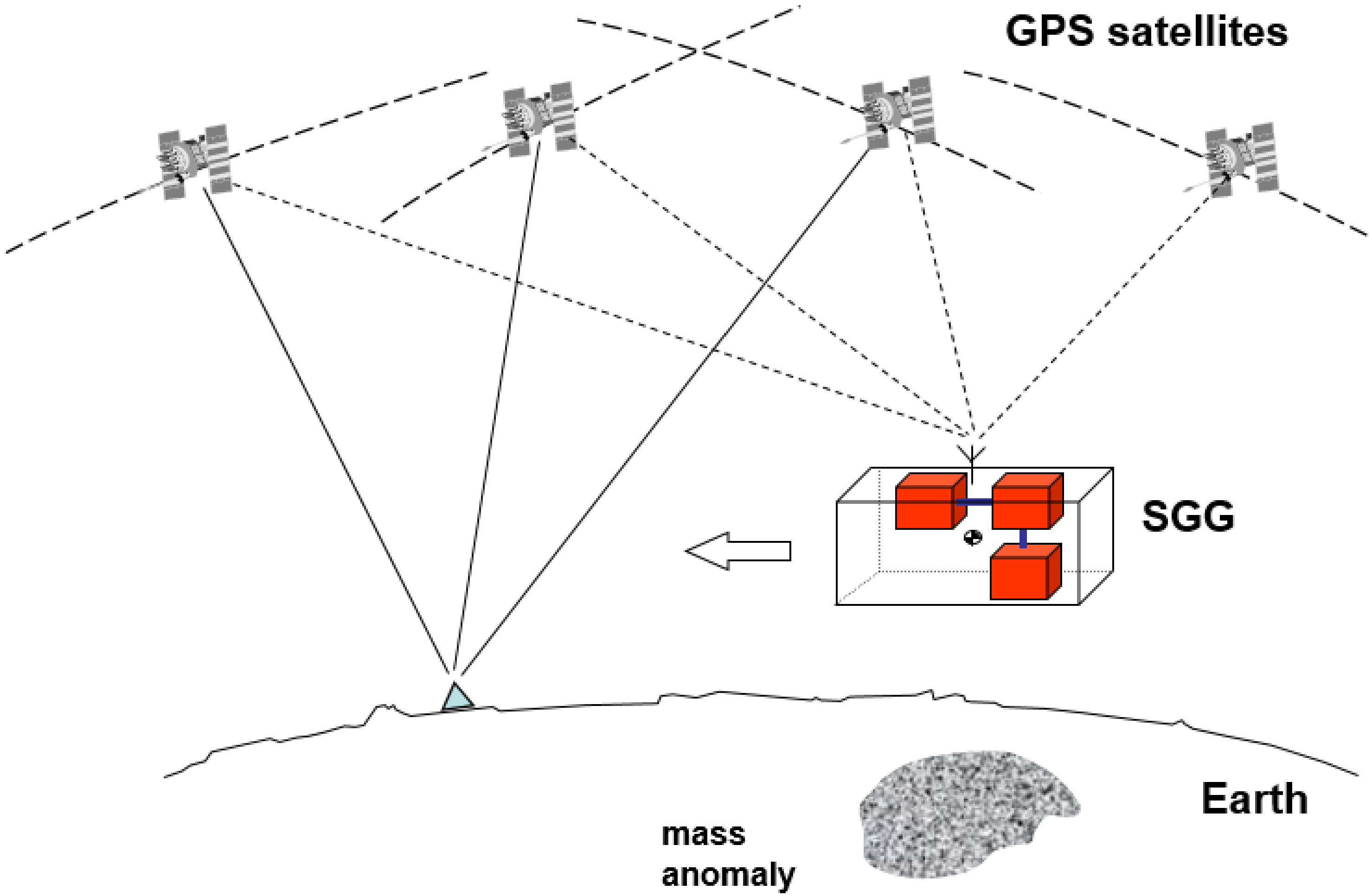
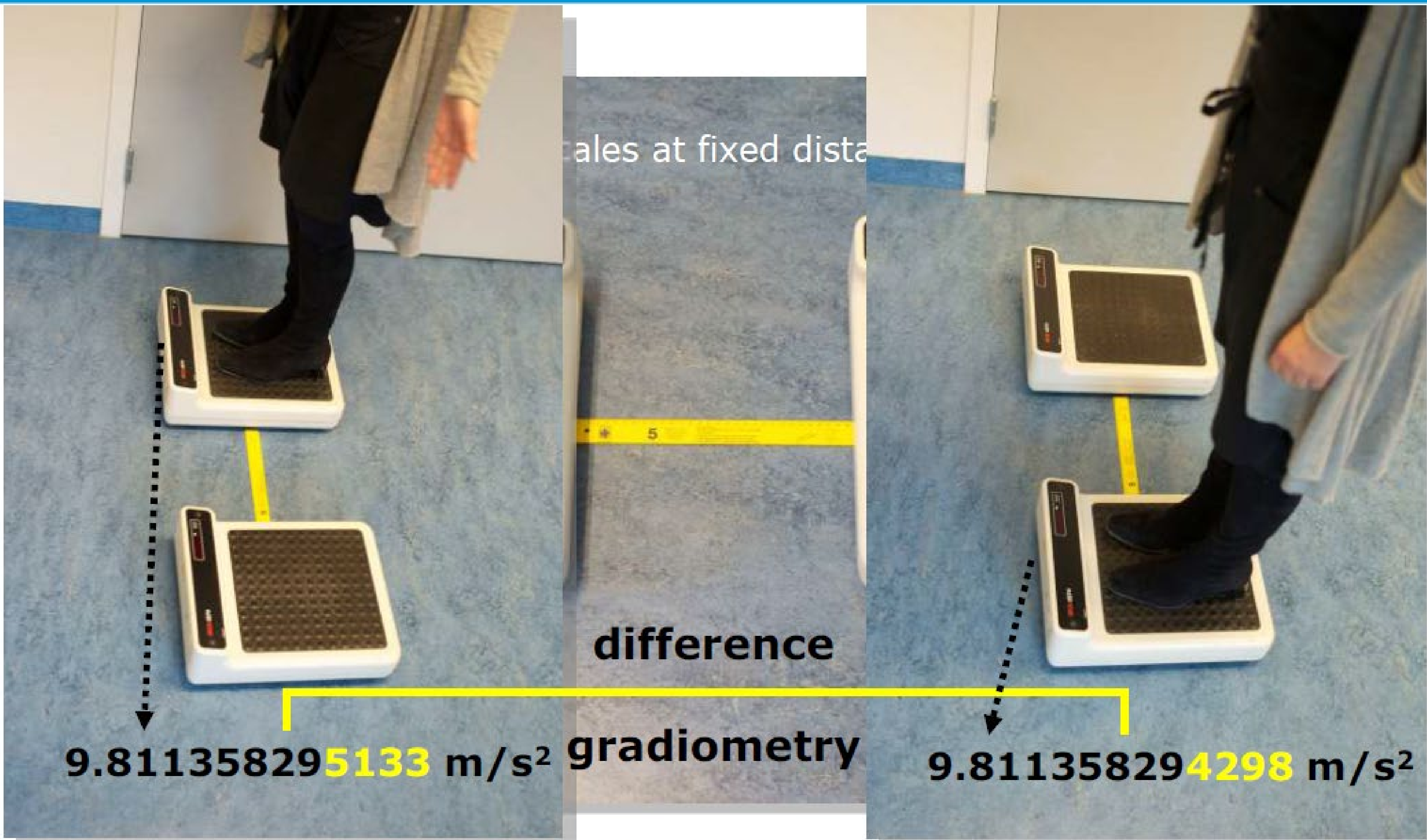


Litterature Seeber 469-484

Spaceborne Satellite Gradiometry (GOCE).



Scale, gravity, gradiometry



Seeber,

10.3.1 Concepts

A gradiometer is a sensor that can measure the change of the gravity acceleration in space, i.e. the gravity gradient. The first derivatives of Earth's gravitational potential $V = V(X, Y, Z)$ are given with the vector, $\boldsymbol{g}$, of the gravity acceleration. A gradiometer is hence capable of measuring the second derivatives. In total, the second derivatives, given by

$$V_{ij} = \frac{\partial^2 V}{\partial i \, \partial j}$$

form a tensor, the *gravity gradient tensor* or *Eötvös-tensor*,

$$V'' = \begin{pmatrix} V_{XX} & V_{XY} & V_{XZ} \\ V_{YX} & V_{YY} & V_{YZ} \\ V_{ZX} & V_{ZY} & V_{ZZ} \end{pmatrix} . \tag{10.9}$$

X, Y, Z is an orthogonal triple. Only five of the 9 elements in the Eötvös-tensor are mutually independent. It holds that

$$V_{XY} = V_{YX}, \quad V_{XZ} = V_{ZX}, \quad V_{YZ} = V_{ZY}, \tag{10.10}$$

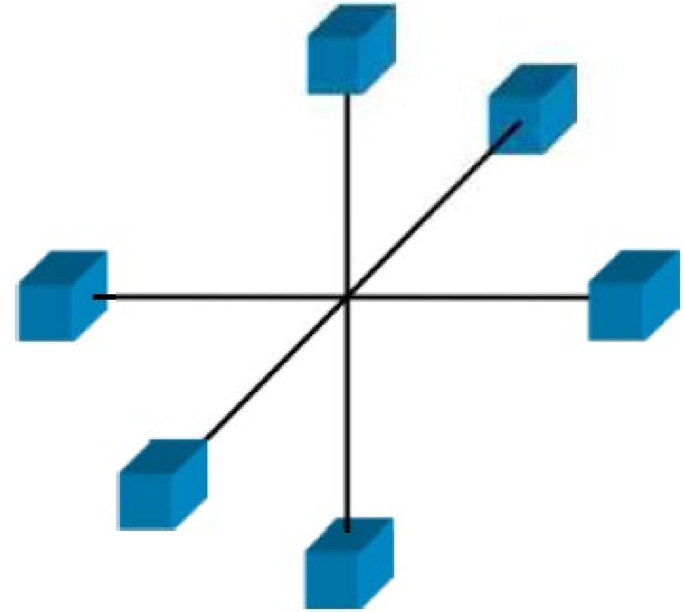
as does the Laplace condition (i.e. a vanishing trace of the tensor):

$$V_{XX} + V_{YY} + V_{ZZ} = 0.$$

Gradiometer working

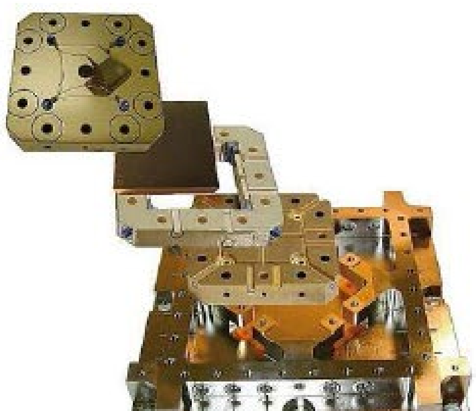
This is GOCE (Gravity and Ocean Circulation Experiment)

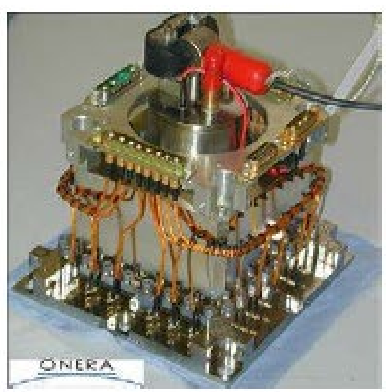
GOCE Gradiometer

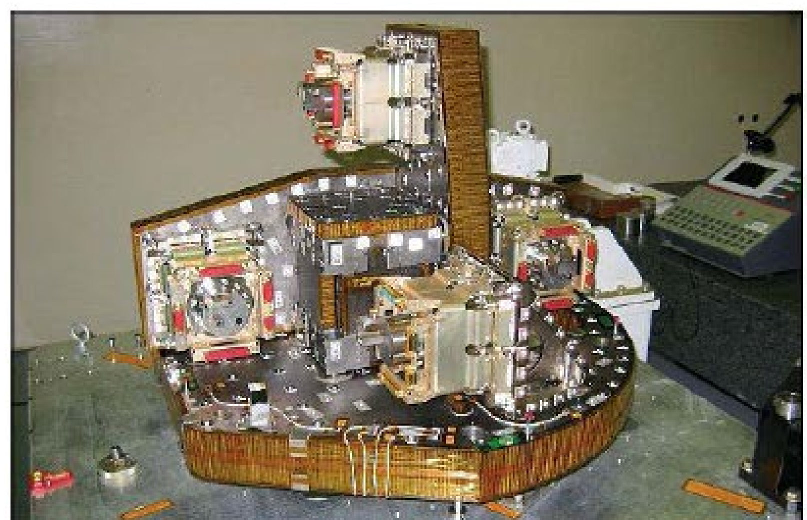


Gradiometer = 6 Accelerometers

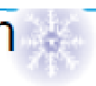
- 100 times more sensitive than any accelerometer previously flown
- Proof-mass is kept levitated at the center of a slightly larger cage by applying control-voltages to electrodes on the inner walls of the cage
- Control-voltages are representative for accelerations experienced by accelerometer
- Dimensions of proof-mass = 4 x 4 x 1 cm
- 2 ultra-sensitive axes + 1 less sensitive axis







Get a feeling for what GOCE can do…….

0.2 gram 

Super-tanker acceleration
due to attracting snowflake:

0.000000000002 m/s^2

smallest acceleration measurable
in space by GOCE

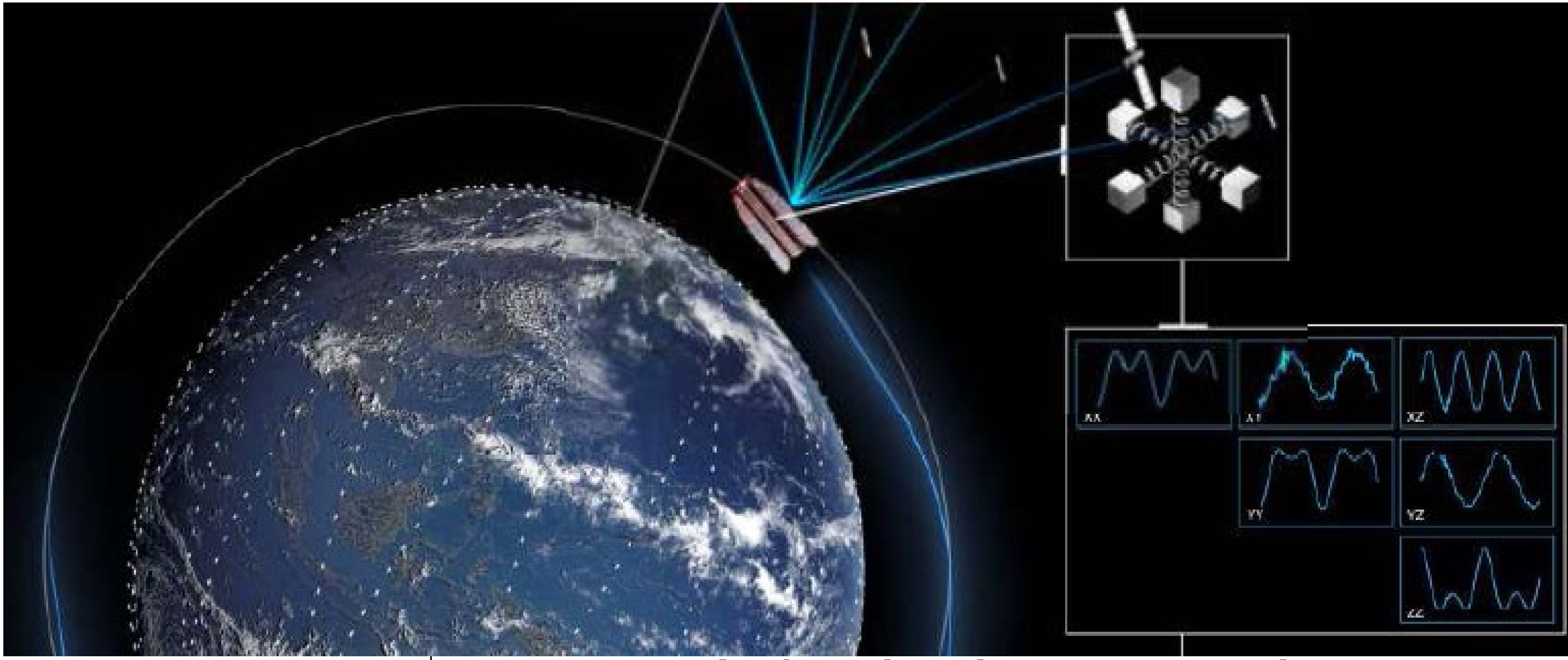
1 000 000 metric tonnes

Attenuation and downward continuation

- At a height h above the Earth's surface, a constituent of potential V with harmonic degree l is reduced (*attenuated*) by a factor of $[r_E/(r_E + h)]^{(n+1)}$.
- Similarly, gravity anomalies are attenuated by $[r_E/(r_E + h)]^{(n+1)}$
- Gravity gradients are attenuated by $[r_E/(r_E + h)]^{(n+2)}$

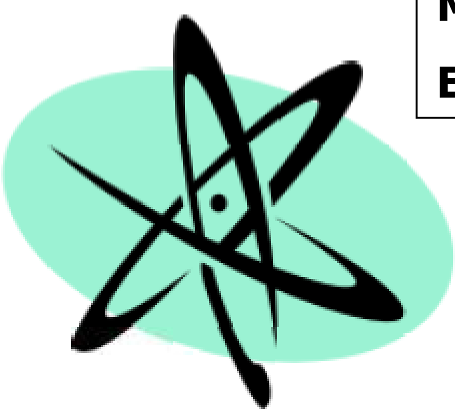
- Measurements made by satellites must be *downward continued* to estimate the potential and gravity at the surface.
  - Downward continuation amplifies the errors by the reciprocals of the attenuation factors.
  - Thus, there is a critical wavelength ($\approx h$) such that wavelengths much longer than the critical can be resolved, much shorter cannot.

GOCE (medium wavelength)



Measure gravity/accelerations -> no moving parts

Extreme stability->fabulous engeneering achievement



Atomic diameter is ➨ 1 Ångström = 1E-10 m

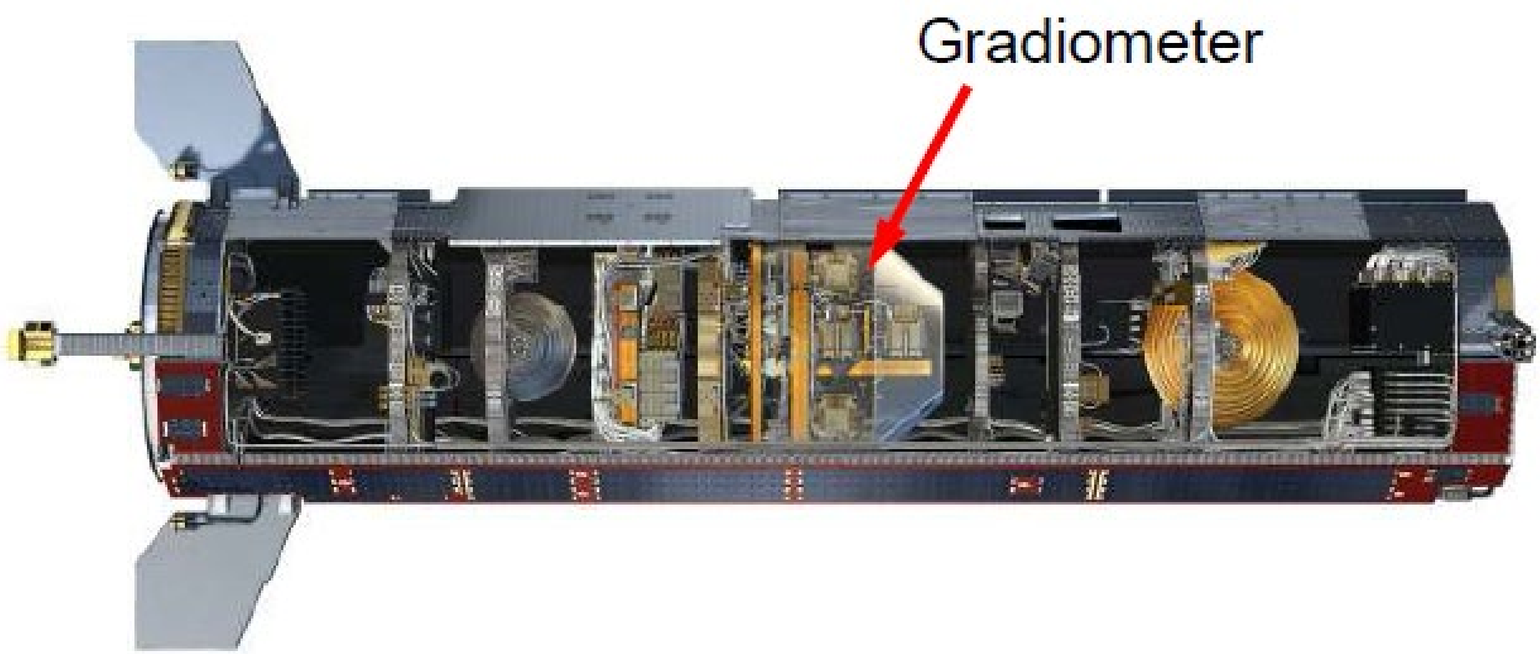
A picometre = 1E-12 m ➨ 1% of an atom !!!

GOCE

Sun-synchronous, 61 day repeat orbit at 256 km altitude

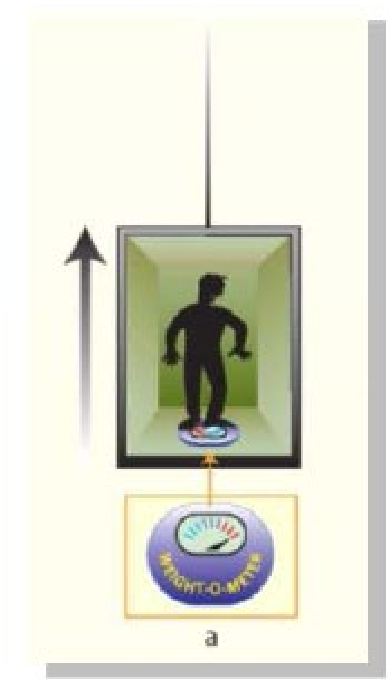
- Orbit as low as possible to maximize signal strength
- Significant aerodynamic forces and torques act on satellite

Gradiometer needs 'quiet environment'



Flying low –>Atmosphere disturbes the Free Fall

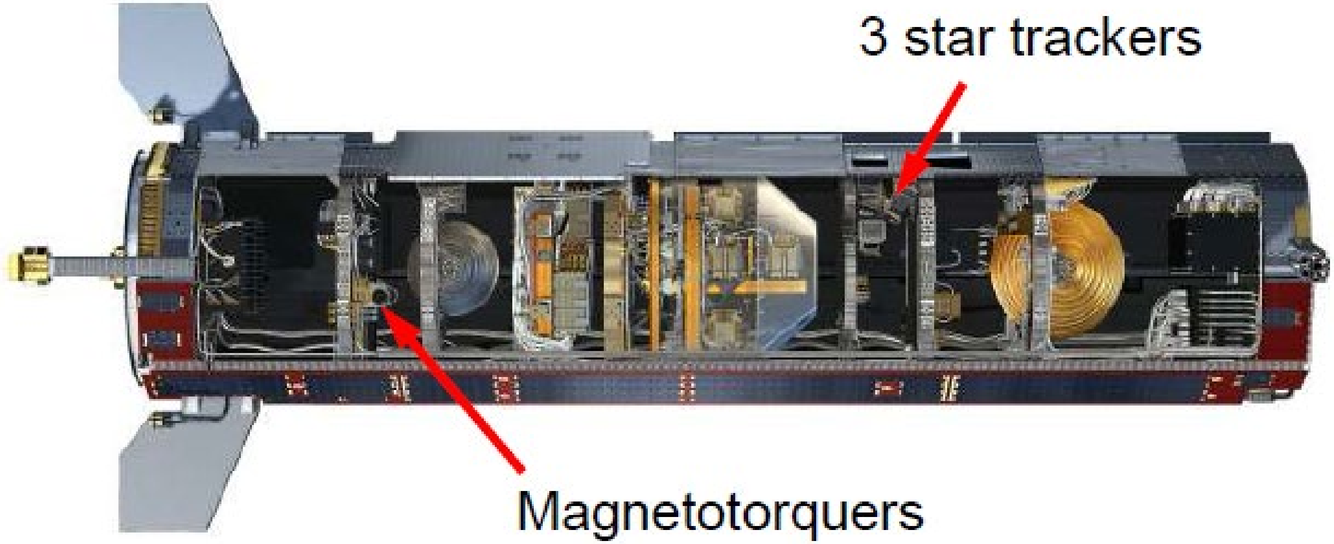
Counteracting the atmospheric drag.....



Navigating GOCE - DTU startrack/camera

Attitude

- Attitude control by magnetotorquers
  - Sun- and nadir-pointing
  - No control about yaw axis at magnetic poles and roll axis at magnetic equator
- Attitude measured by three star trackers
  - Capture also angular rates and angular accelerations



GOCE accelerometer measurements

$$a = -[V] \cdot r + (\omega \times (\omega \times r)) - (\dot{\omega} \times r) + (2\omega \times \dot{r}) + \ddot{r} + d - s \ldots$$

gradient | centrifugal acceleration | angular acceleration | Coriolis | cage | drag | self gravity | magnetic coupling

$$a_i = -(V - \Omega^2 - \dot{\Omega}) r_i + d$$

Forming common and differential accelerations:

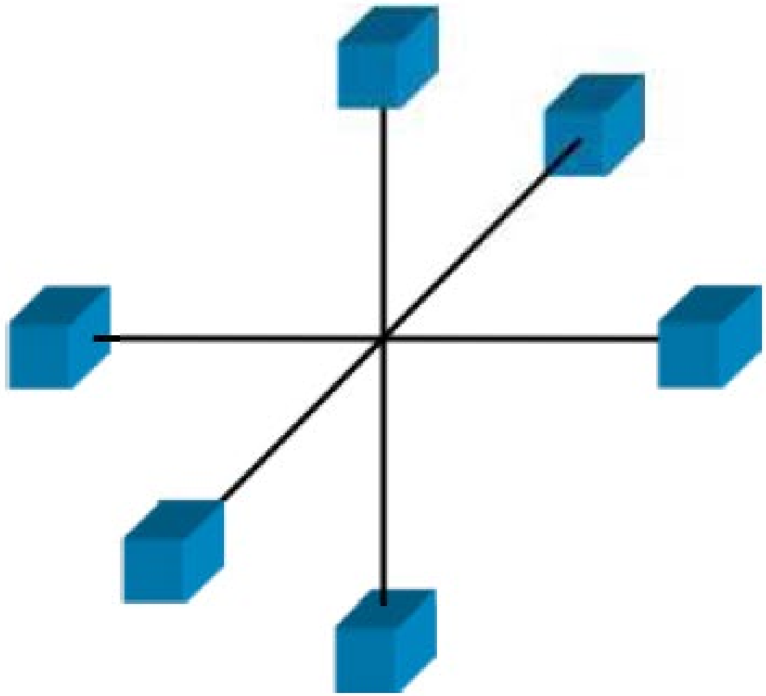
- Common relates to drag and winds
- Differential relates to gravity gradients rotating around the Earth

-> use information from star trackers

Gravity gradiometry

Gradiometer = 6 Accelerometers, each accelerometer measures

$$a_i = -(V - \Omega^2 - \dot{\Omega}) r_i + d$$

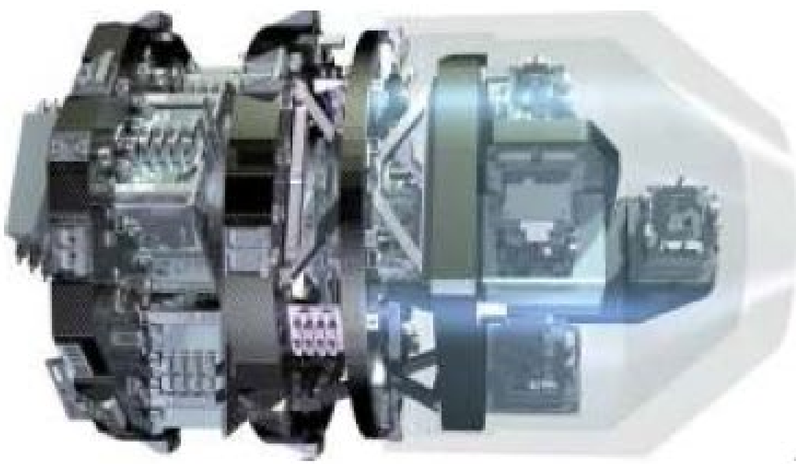


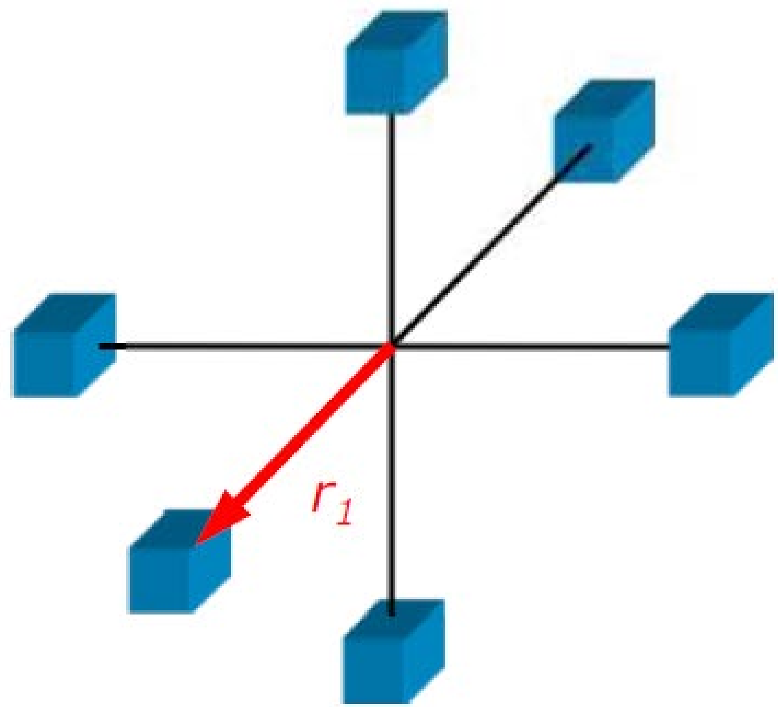
Gravity gradiometry

Gradiometer = 6 Accelerometers, each accelerometer measures

$$a_i = -(V - \Omega^2 - \dot{\Omega})r_i + d$$

position of accelerometer



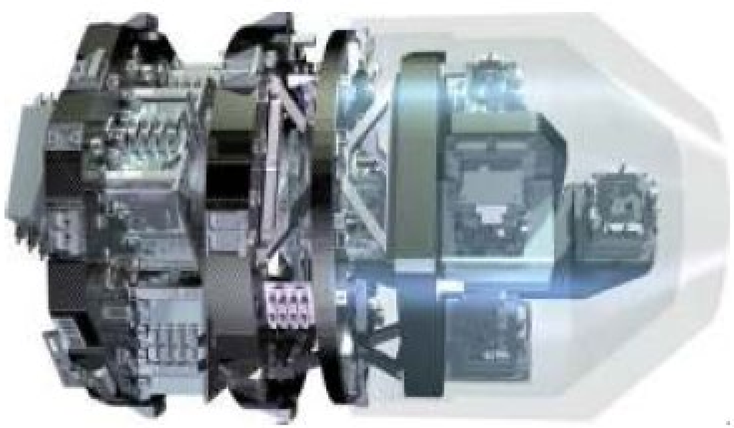


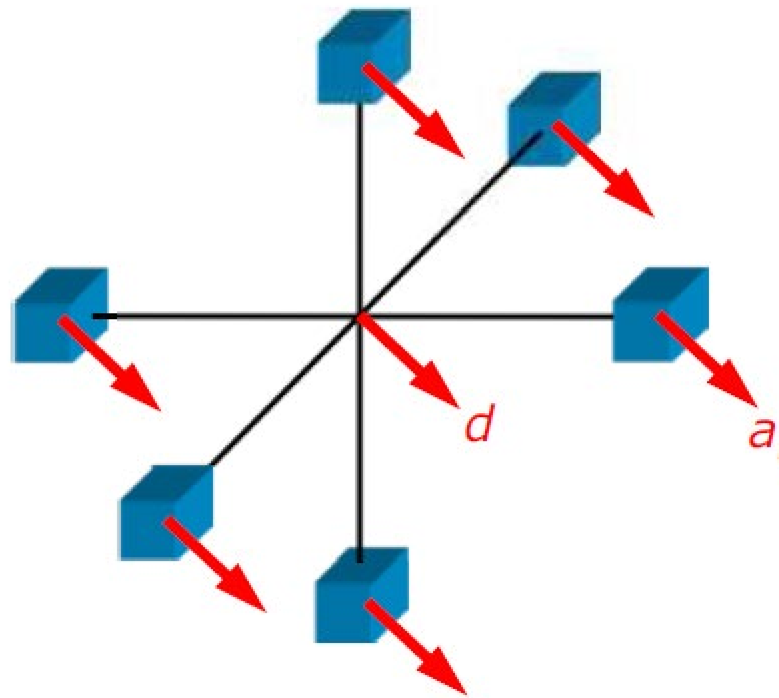
Gravity gradiometry

Gradiometer = 6 Accelerometers, each accelerometer measures

$$a_i = -(V - \Omega^2 - \dot{\Omega})r_i + \boxed{d}$$

linear acceleration of satellite



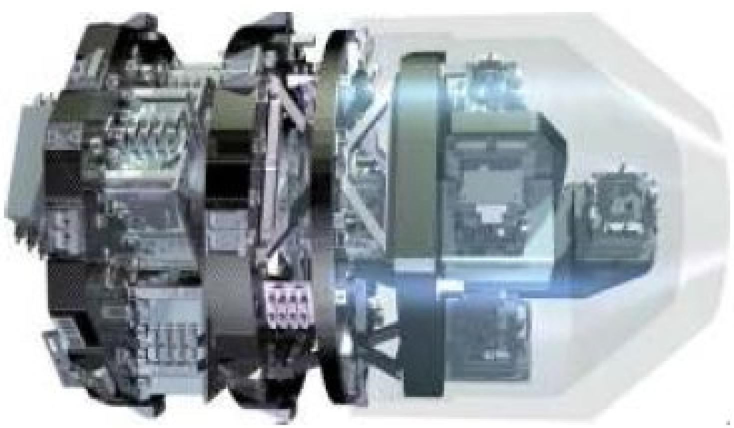


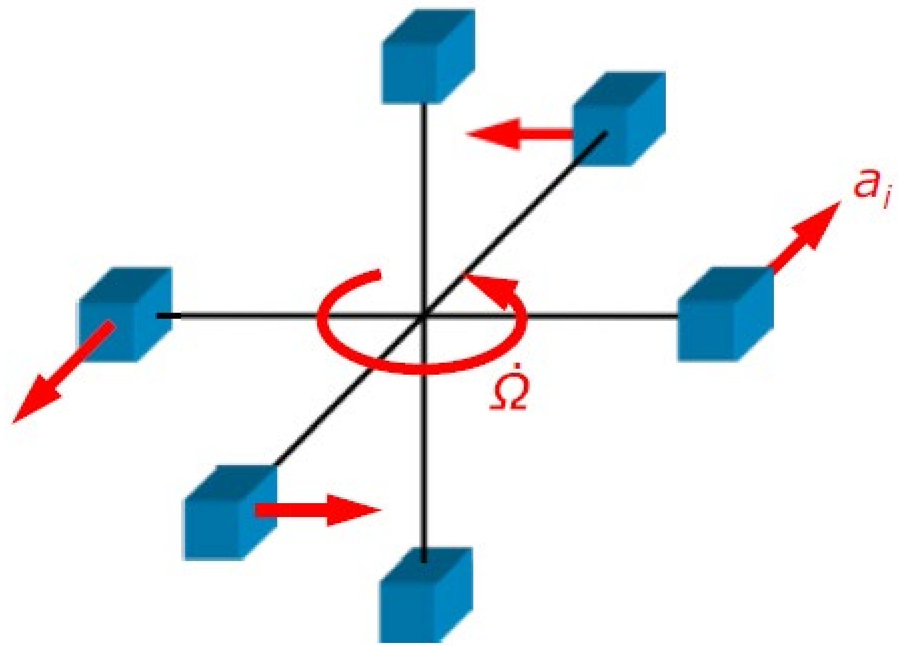
Gravity gradiometry

Gradiometer = 6 Accelerometers, each accelerometer measures

$$a_i = -(V - \Omega^2 - \dot{\Omega}) r_i + d$$

angular acceleration of satellite



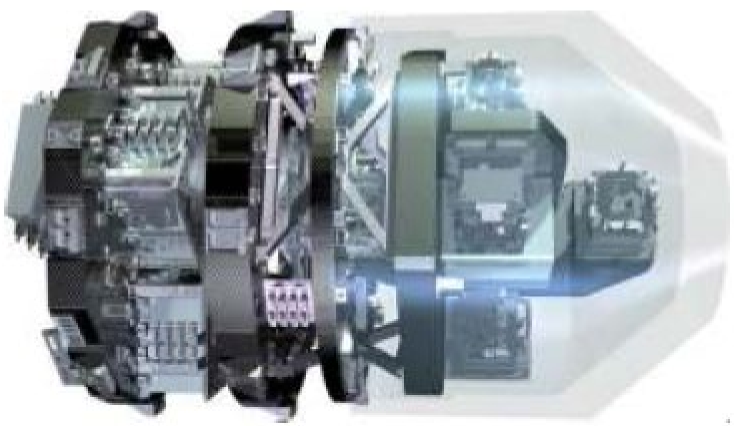


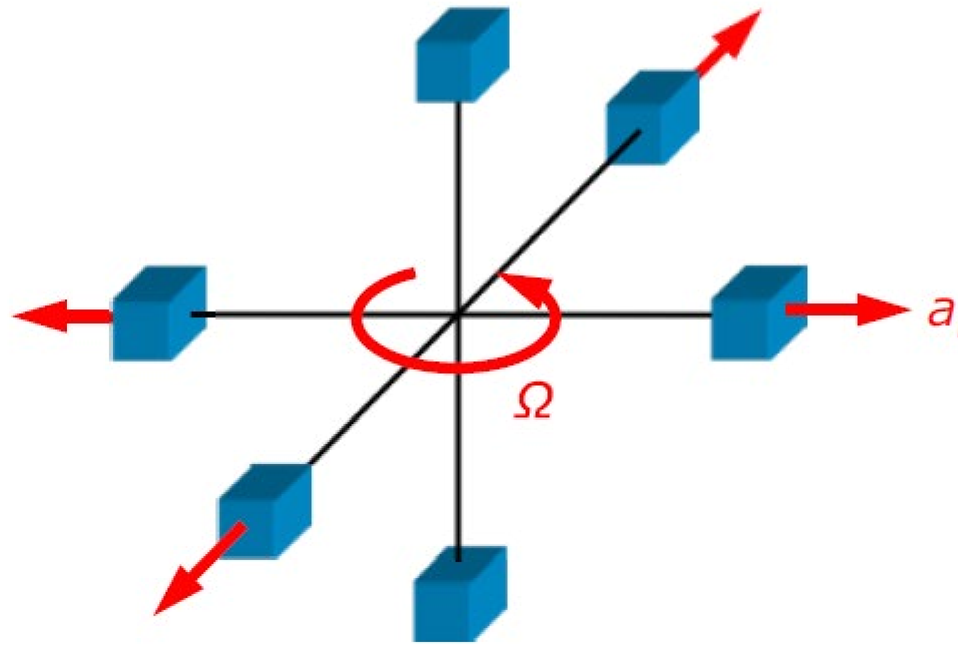
Gravity gradiometry

Gradiometer = 6 Accelerometers, each accelerometer measures

$$a_i = -(V - \boxed{\Omega^2} - \dot{\Omega})r_i + d$$

centrifugal acceleration caused by satellite rotation



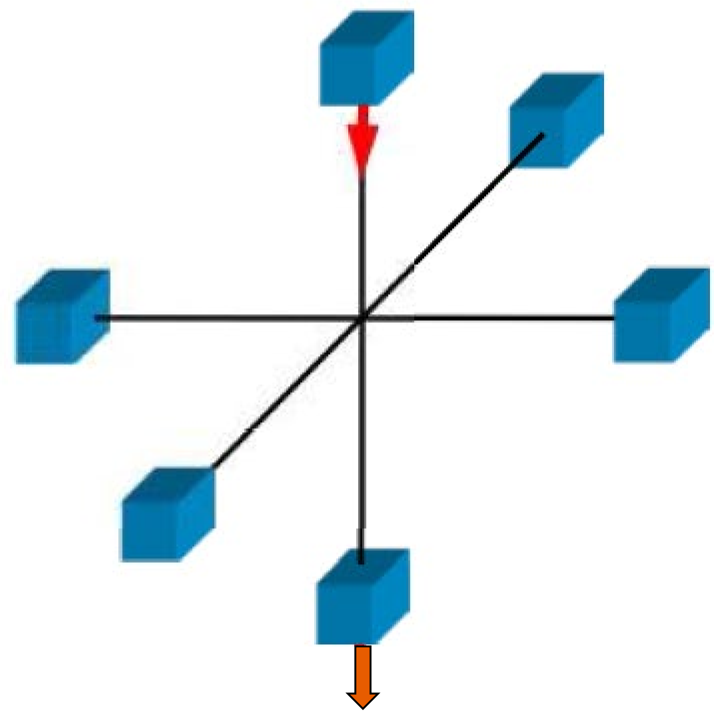


Gravity gradiometry

Gradiometer = 6 Accelerometers, each accelerometer measures

$$a_i = -(V - \Omega^2 - \dot{\Omega})r_i + d$$

gravity gradient between accelerometers



Form common sums and differences....

1. **Common mode rejection** (for accelerometer pair on one gradiometer arm)

$$a_{cij} = \frac{1}{2}(a_i + a_j) \approx d$$ *common mode acceleration*

$$a_{dij} = \frac{1}{2}(a_i - a_j) = -\frac{1}{2}(V - \Omega^2 - \dot{\Omega})(r_i - r_j)$$ *differential mode acceleration*

2. **Separation of symmetric and anti-symmetric parts of**

$$A_d = \begin{bmatrix} a_{d14} & a_{d25} & a_{d36} \end{bmatrix} \qquad L = \begin{bmatrix} r_1 - r_4 & r_2 - r_5 & r_3 - r_6 \end{bmatrix}$$

$$A_d L^{-1} - L^{-T} A_d^T = \dot{\Omega}$$ *angular acceleration*

$$A_d L^{-1} + L^{-T} A_d^T = -V + \Omega^2$$ *gravity gradient + centrifugal acceleration*

Getting the gravity potential

3. **Separation of centrifugal accelerations**

$$V = \boxed{\Omega^2} - (A_d L^{-1} + L^{-T} A_d^T)$$

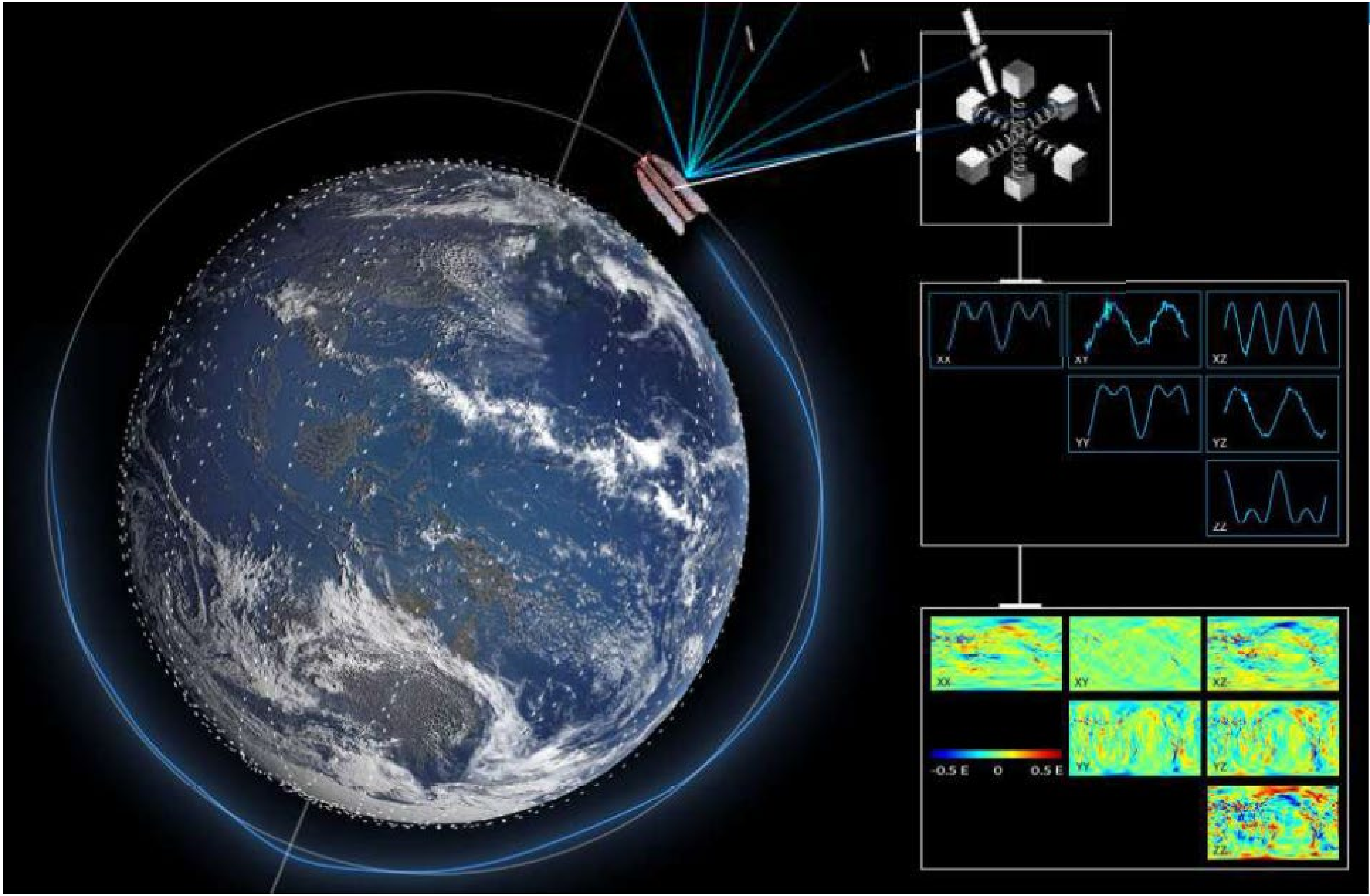
Star trackers provide angular rate as first derivative in time of inertial attitude

Gradiometer provide the angular acceleration

$$-\dot{R}R^T = \Omega_{STR} \Rightarrow \boxed{\text{angular rate reconstruction}} \Leftarrow \int \dot{\Omega}_{GRAD} dt = \int A_d L^{-1} - L^{-T} A_d^T dt$$

$$\Downarrow$$

$$\Omega$$



XX
XY
XZ
YY
YZ
ZZ
XX
XY
XZ
YY
YZ
ZZ
-0.5 E
0
0.5 E

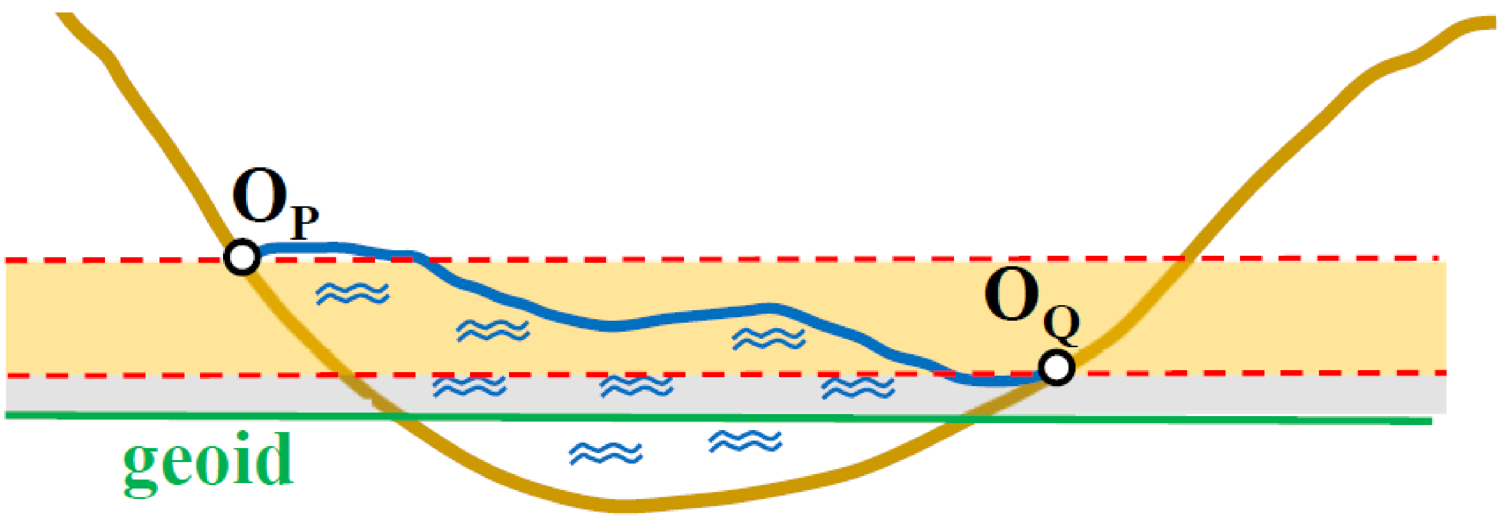
Geoid from satellite data (GOCE)

$$N_P^{GOCE} = \frac{GM}{R\gamma} \sum_{l=2}^{L_{\max}} \sum_{m=0}^{l} \bar{P}_{lm}(\cos\theta_P)\left(\Delta\bar{C}_{lm}\cos m\lambda_P + \Delta\bar{S}_{lm}\sin m\lambda_P\right)$$

limited resolution $L_{\max} \approx 200$

omission error ≈ 30 cm

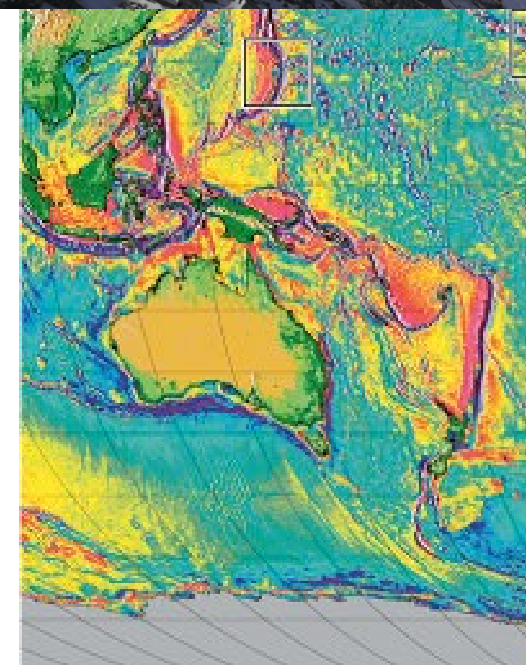
GOCE commission error ≈ 2-3 cm

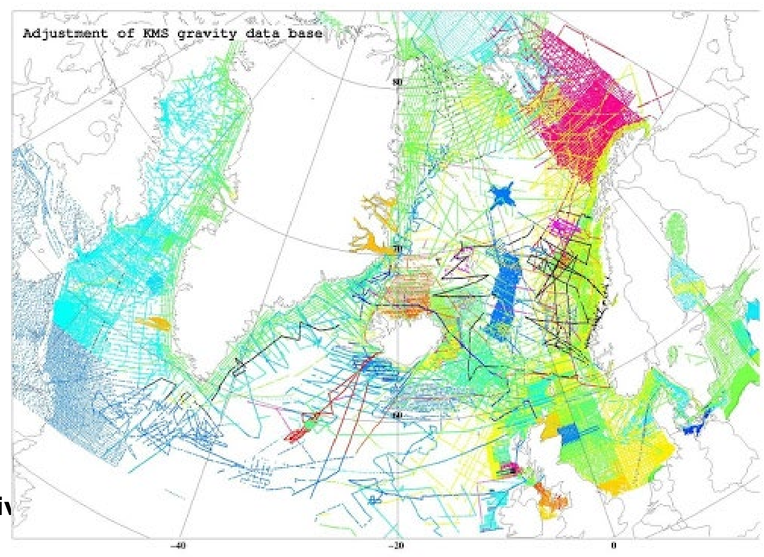


Earth Geopotential Models. Combine local+global data

EGM08 is d/o = 2159 Wavlength =15 km

- **Satellite (200 km)**
- Satellite altimetry (15 km resolution).
- Airborne (10 km resolution.
- Marine and land borne observations (1-10km).





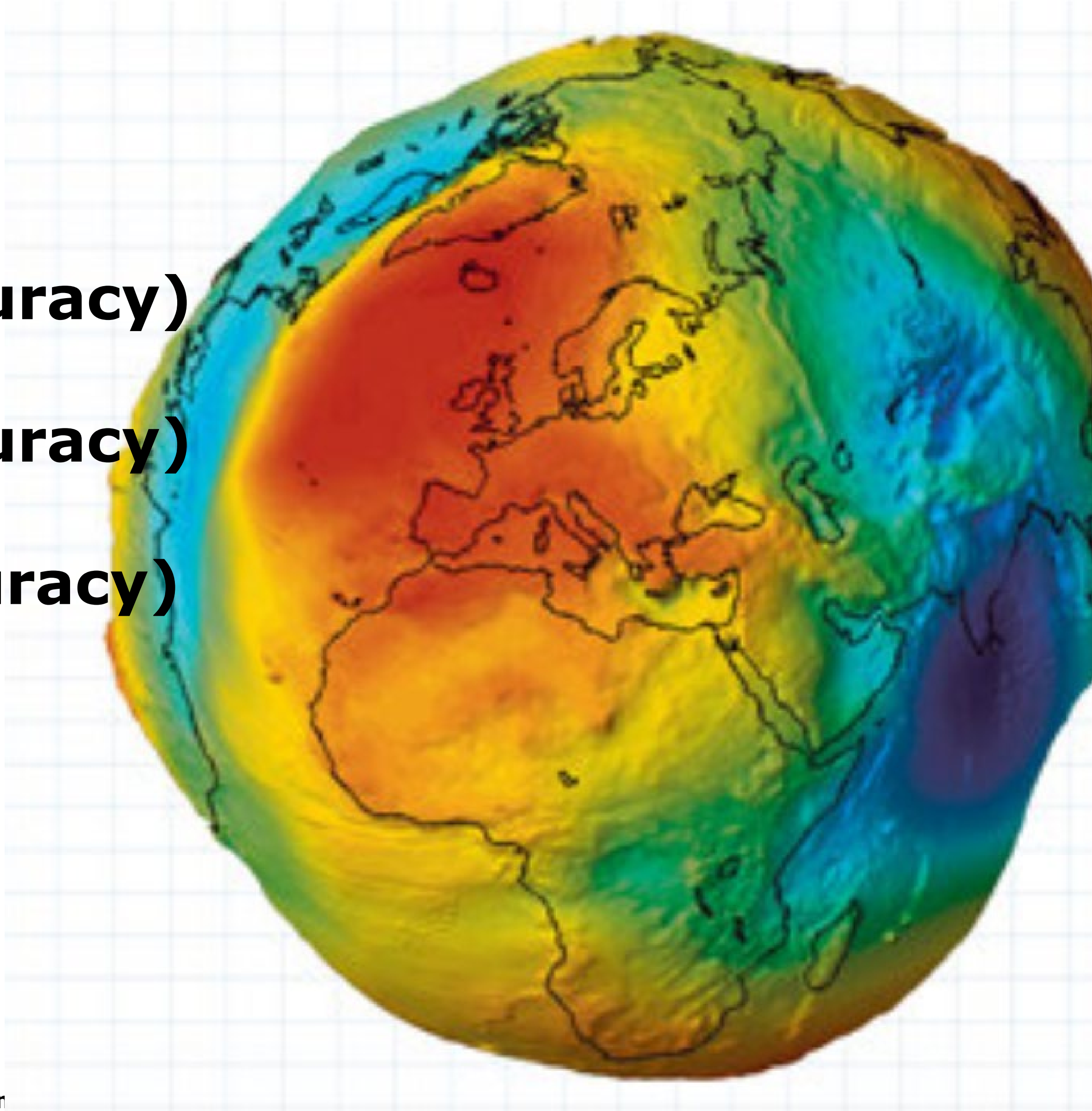
Earth Geopotential Model

Earth
Potato

EGM96 (50 cm accuracy)

EGM08 (15 cm accuracy)

EGM20 (?? cm accuracy)



Thanks – Questions ????