GNSS Satellite Orbits

Lecture notes for 30550

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Content:

1. Introduction

This lecture note provides a basic introduction to the theory on GNSS satellites orbits. Keplers laws are reviewed, the orbital coordinate system and the expressions for position and velocity within the orbit are described. The conventional inertial reference system is introduced along with the procedure for conversion of satellites positions to the inertial coordinate system. Also factors perturbing the theoretically smooth satellite motion are briefly described, before the introduction of orbital parameters for the GPS satellites. Broadcast and precise GPS orbits are discussed, and finally the orbital parameters of Galileo and GLONASS are described.

The notes are intended as an introduction to the field of satellite geodesy for students and others working with GNSS. As such, the notes are not providing an in depth derivation of the equations for a dynamic motion in the earths gravity field. The reader is referred to the list of references for this.

In the text, vectors are denoted in bold lowercase and matrices are bold uppercase. Any suggestions for improvements of the notes are welcomed by the author.

2. Kepler's laws

All positioning of satellites today is based on the laws of Johannes Kepler who lived in Germany from 1571 to 1630. Keplers work was based on observations carried out by the Danish astronomer Tycho Brahe (1546-1601). Brahe built an observatory on the island of Ven located between Denmark and Sweden, and he performed numerous observations of the motions of the planets. Kepler was his student, and after Brahe died, Kepler devoted most of the remaining part of his life to analyzing the observations collected by Brahe. The legacy of Brahe and Kepler is strong within the communities of satellite navigation and space technology, and their names are often remembered and honored.

Kepler developed a number of theorems and laws describing the motion of the planets in their orbits around the sun. These laws do, in general, also describe the motion of a satellite orbiting around the earth and the laws are therefore repeated below.

Keplers 1. law
The orbit of each planet is an ellipse with the sun in one of the foci.

Effect on satellites:
The orbit of a satellite is an ellipse with the gravitational centre of the earth in one of the foci.

Referring to Figure 1:
F are the two foci of the ellipse
P is perigee, the point on the orbit closest to the earth
A is apogee, the point on the orbit farthest away from the earth
a is the semi major axis of the ellipse
b is the semi minor axis of the ellipse

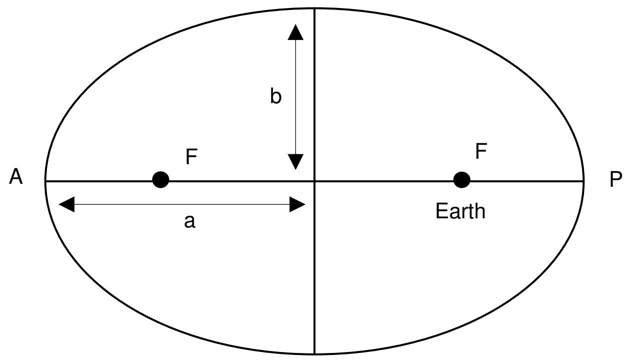

Figure 1. Satellite orbital ellipse.

Keplers 2. law
The planets revolve with constant area velocity, e.g. the radius vector of the planet sweeps out equal areas in equal lengths of time, independent of the location of the planet in the orbit.

Effect on satellites:
Satellites revolve with a constant area velocity within the orbit. The speed of the satellite is not constant, but varies with the location of the satellite in the orbit, so the speed is higher when the satellite is close to the earth (see Figure 2).

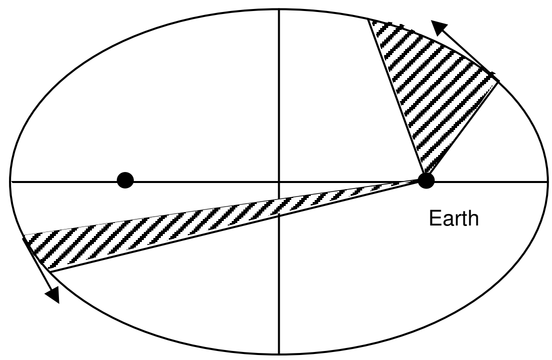

Figure 2. The satellite sweeps out equal areas in the ellipse in equal time intervals while orbiting

Keplers 3. law
The relation between the square of the period, T, and the cube of the semi major axis, a, is constant for all planets:

$$\frac{T^2}{a^3} = const \tag{1}$$

Effect on satellites:
Two satellite orbits with the same size of their semi major axes, will have the same T even if the eccentricities of the orbital ellipses are different (see Figure 3).

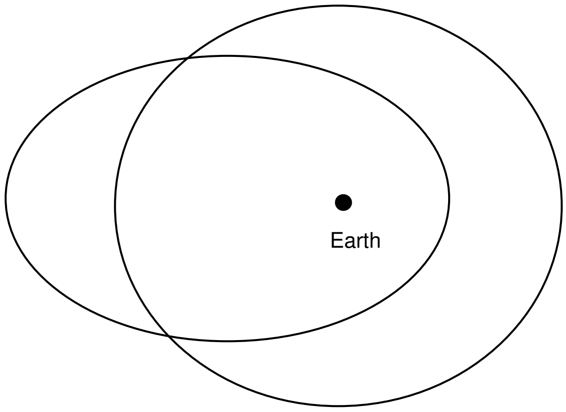

Figure 3. Two orbits with same size of semi major axis and period, but with different eccentricity.

The value of the constant given in Equation (1) was determined several years later by Isac Newton (1624 - 1727) based on his work on gravity.

$$\frac{T^2}{a^3} = const = \frac{4\pi^2}{GM} \Leftrightarrow T = \frac{2\pi}{\sqrt{GM}} a^{3/2} \qquad (2)$$

Where *GM* is the earths gravitational constant of 3986004.418 x 10^8 m^3/s^2 (Misra and Enge, 2001)

Keplers three laws would be true for satellites today if the satellite and the earth were point masses (or homogeneous bodies with a spherical mass distribution), and if no other forces than earths gravity were affecting the satellites.

This is of course not the case, and the expressions of satellite motions are therefore more complicated since we have to account for the variations in the earths gravity field, and several external forces e.g. lunar gravity and solar radiation affecting the satellites. This will be discussed later in the notes.

3. Orbital coordinates system

In order to describe the motion of a satellite within its orbit, we define an orbital coordinate system, called *q*.

The axis of the coordinate system are defined so origo is located in the mass center of the earth, the first axis, *q1*, is directed towards perigee, the second axis, *q2*, is located in the orbital plane, perpendicular to the first axis in the direction of the satellite motion, and the third axis, *q3*, is perpendicular to both first and second axis to form a right hand system. In Figure 4 the *q3* axis is thus pointing out of the plot towards the reader.

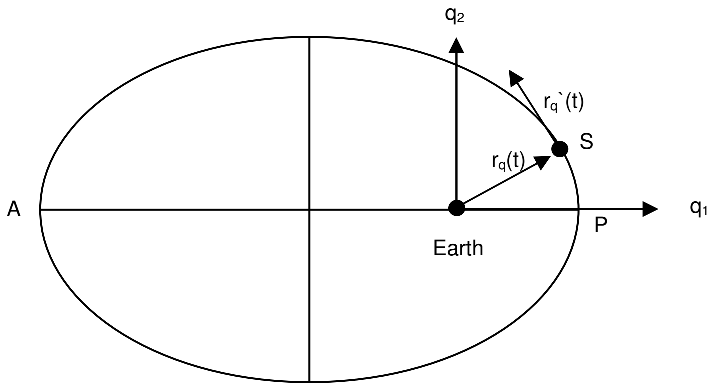

Figure 4. Elements of the orbital coordinate system, q.

Further, in order to describe the location of the satellite within the orbital coordinate system we need to define a number of parameters for the orbital ellipse (Figures 4 and 5):
S, satellite
a, semi major axis of the ellipse
e, eccentricity of the ellipse
E(t), eccentric anomaly of satellite position
v(t), true anomaly of satellite position
$\mathbf{r_q}$(t), position vector of satellite in orbital coordinate system
$\mathbf{r'_q}$(t), velocity vector of satellite in orbital coordinate system

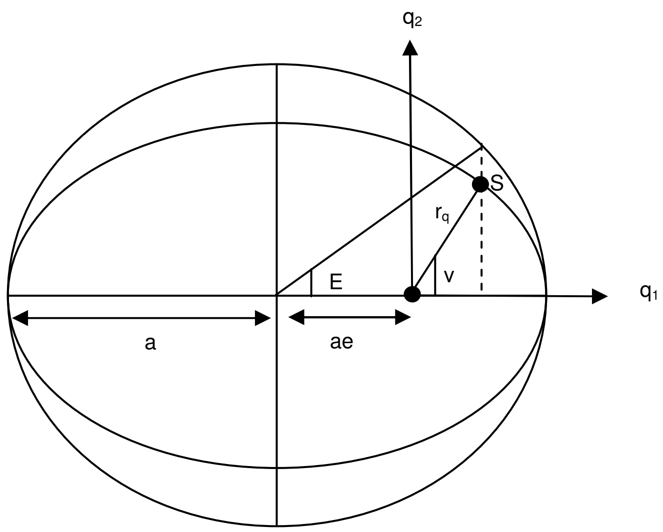

Figure 5. Parameters for describing the location of a satellite in the orbital coordinate system, q. Figure inspired by Kaula (1969).

The position of the satellite for a given epoch in time is given as:

$$\mathbf{r} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \frac{a\left(1-e^2\right)}{1+e\cos(v)} \begin{bmatrix} \cos(v) \\ \sin(v) \\ 0 \end{bmatrix} \tag{3}$$

The *q3* coordinate is zero, since the coordinate system is defined so the q3-axis is perpendicular to the orbital plane. The satellite motion is, according to the laws of Kepler, a 2D motion within the q coordinate system.

The expression in Equation (3) can also be given as below, where the satellite motion is described using the eccentric anomaly as the angular variable.

$$\mathrm{r} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} a\cos(E) - ae \\ a\sqrt{1-e^2}\sin(E) \\ 0 \end{bmatrix} \tag{4}$$

The eccentric anomaly, *E* and the true anomaly, ν are two different angles, both indicating the satellite position in the orbit as a function of time. Depending on the use of the expressions, and the variables given, one expression is usually preferable to the other.

The relation between eccentric anomaly, *E* and the true anomaly, ν can be given by:

$$n = \sqrt{GM}\; a^{-3/2} \tag{5}$$

$$M(t) = n \cdot (t - t0) \tag{6}$$

$$E(t) = M(t) + e\,\sin(E(t)) \tag{7}$$

$$\tan(v(t)) = \frac{\sqrt{1-e^2}\sin(E(t))}{\cos(E(t)) - e} \tag{8}$$

Where *n* is the mean motion of the satellite, and *M* is called the mean anomaly.

Please note that the mean anomaly, M in Equation (6) and (7) is not the same as the mass of the earth, M in Equation (5). It has been decided to use this notation, however, because it complies with the standard in most literature.

The derivative of the position vector in the orbital plane is given as:

$$\mathbf{r}` = \frac{na}{\sqrt{1-e^2}}\begin{bmatrix} -\sin(v) \\ e+\cos(v) \\ 0 \end{bmatrix} = \frac{na}{1-e\cos(E)}\begin{bmatrix} -\sin(E) \\ \sqrt{1-e^2}\cos(E) \\ 0 \end{bmatrix} \tag{9}$$

as functions of the true and eccentric anomalies, respectively.

The expressions given in Equation (3) and (4) are solutions to the basic equation of motion in a force field, Equation (10), which is a second order non-linear differential equation.

$$\mathbf{r}'' = -\frac{GM}{r^3}\mathbf{r} \tag{10}$$

For a derivation of the Equations (3) to (9) the reader is referred to Kaula (1969) or Seeber (2003), where also the basic expressions for motion in the earths gravity field are derived and discussed.

4. Conventional inertial reference system (CIS)

We have now defined a coordinate system for describing the motion of a satellite within its orbit, but in order to use the satellites for positioning on the surface of the earth, we need a better relation between the orbital coordinate system and the coordinate systems we use for referencing of the positions on the surface of the earth, such as for instance the WGS84.

The Conventional Inertial System (CIS) is necessary as an intermediate step in this conversion. The CIS is used for positioning and orientation of the earth in space. The coordinate system is defined by orienting the axes towards distant quasars.

The Conventional Inertial System (CIS) is defined with origo coinciding with the center of mass of the earth. The third axis, *Z*, is defined to be coinciding with the rotational axis of earth rotation, the first axis, *X*, is located in the equatorial plane towards the vernal equinox, and finally the second axis, *Y*, is located in the equatorial plane to complete a right handed cartesian coordinate system.

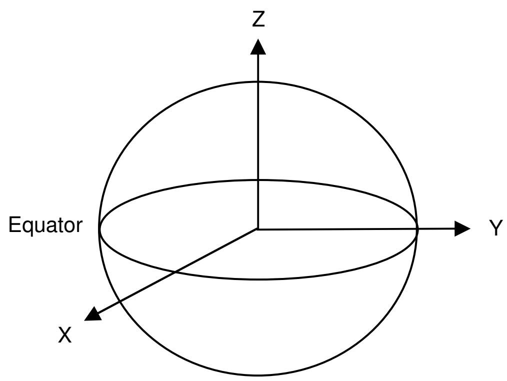

Figure 6. Coordinate axes of the inertial reference system.

The *vernal equinox* is the point in space where the equatorial plane of the earth intersects with the ecliptic (the plane of the earth and the sun) in the spring time. I.e. the direction to the sun as seen from the earth when the sun is moving from the southern to the northern hemisphere. The point is also called the *spring equinox*.

Important is that the CIS does not rotate with the earth, what makes it convenient for positioning of satellites.

Since the mass distribution of the earth is not homogenous, the rotational axis of the earth is time variant, and the motion of the axis is composed of two periodic movements called *precession* and *nutation*. Precession is caused by gravitational attraction of the sun, the moon and other celestial objects, and it causes the spin axis to move in a slow circular motion like a top. Nutation is a smaller movement with a shorter period superimposed on the precession (Bock, 1996).

The axis of the CIS are thus not constant in time, and when converting positions from the inertial reference system to an earth fixed system as for instance the WGS84, this motion must be taken into consideration. More information on precession, nutation and conversions between the inertial and terrestrial reference systems is given by for instance Seeber (2003).

5. Conversion of satellites positions between orbital system and CIS

The CIS and the orbital coordinate system both have the center of mass of the earth as origo. This means that conversion of coordinates from one system to the other does not include translations, but only rotations of the coordinate axes with respect to each other.

The three rotation angles are given in the inertial reference system, they are shown in Figure 7 and are denoted as:

Ω - right ascension of the ascending node. The angle between the first axis of the CIS, and the vector in the CIS pointing from origo to the point in the Equatorial plane where the orbital plane intersects with the Equatorial plane. This point is denoted the ascending node, and the right ascension of the ascending node identifies the point where the satellite moves from the southern hemisphere of the earth to the northern hemisphere.

i - is called the inclination, and is the inclination angle of the orbital plane with respect to the Equatorial plane.

ω - is the argument of perigee. The angle between the position vector of the ascending node and the position vector of the satellite at the current epoch in time.

Coordinates of the satellite position as given in the orbit coordinate system can now be converted to coordinates in the inertial reference system by rotating about the first and the third axis of the CIS, using the three rotation angles; Ω, i, and ω, and corresponding rotation matrices.

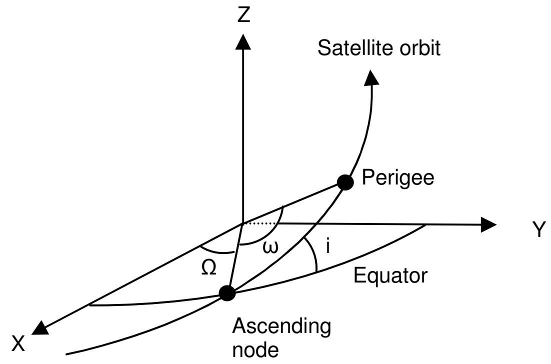

Figure 7. Rotation angles between orbital and inertial coordinate systems.
Figure inspired by Kaula (1969).

As before we refer to the orbital coordinate system as q, and the inertial coordinate system is referred to as x. Conversion of a position vector, $\mathbf{r_q}$ to a position vector in the inertial coordinates system, $\mathbf{r_x}$ is carried out using the following expression:

$$\mathbf{r}_x = \mathbf{R}_{xq}\,\mathbf{r}_q \tag{11}$$

where

$$\mathbf{R}_{xq} = \mathbf{R}_3(-\Omega)\mathbf{R}_1(-i)\mathbf{R}_3(-\omega) \tag{12}$$

The three dimensional rotation matrices, $\mathbf{R}$, with the given rotation, α, are constructed as follows:

$$\mathbf{R}_1(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix} \qquad \mathbf{R}_2(\alpha) = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix}$$

$$\mathbf{R}_3(\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\mathbf{R}_2$ is not used in this context, but included for the sake of completeness.

Converting coordinates in the opposite direction i.e. from x to q is carried out rotating in the opposite sequence and direction:

$$\mathbf{r}_q = \mathbf{R}_{qx}\,\mathbf{r}_x \tag{13}$$

$$\mathbf{R}_{qx} = \mathbf{R}_3(\omega)\mathbf{R}_1(i)\mathbf{R}_3(\Omega) \tag{14}$$

Kepler elements

Summing up, the parameters we need for describing the satellite orbit and its relation to the inertial reference system are the following six variables, which are normally referred to as the *Kepler Elements*

Satellite orbit size and shape:
a – semi major axis
e - eccentricity

Location of orbit in the inertial reference system:
i - inclination
Ω – right ascension of the ascending node
ω – argument of perigee

Further, to describe the location of a satellite in its orbit, we need:
ν – true anomaly
or
E - eccentric anomaly

6. Perturbed satellite motion

The satellite motion is affected by external forces dragging and pushing the satellite from the theoretically smooth orbit as described in the previous. The most important perturbing effect is, however, caused by variations in earths gravity field. The earth is not a point mass and the mass is not homogeneously distributed inside the earth. The deviation of the gravity field from a central sphere, and the variations in the earth gravity field as a function of the distribution of masses inside the earth are well modeled today, mainly because of many years of studies of satellite orbit perturbations, but also because of a very dense network of gravity reference stations on the surface of the earth, where gravity is measured precisely at regular intervals. The models of the earths gravity field are therefore also used to model the effect of the satellite orbits.

The non-spherical and non-central gravity field causes a rotation of the orbital plane within the inertial coordinate system. The gravity field basically tries to drag the satellite orbit into the equatorial plane. The effect on the Kepler elements, describing the size, shape and location of the satellite orbit, is rather large (see Table 2), and must be considered when dealing with real satellite positions. The effect is larger for satellites located in orbits close to the surface of the earth, the so-called LEO satellites (low earth orbiters).

Other forces affecting the satellite motion are gravitational effects of the sun and the moon, solar radiation pressure, albedo (reflection of solar light from the surface of the earth back into space), effects of earth and ocean tides, radiation from space, atmospheric drag etc.

The perturbing forces and their effects on satellite orbits are described in detail by Kaula (1969) and Seeber (2003), where also the equations of motion considering perturbing effects are derived. Misra and Enge (2001) cover the basics of perturbed satellite motions in a lighter description. Some examples of the effects on GPS satellites orbits are given in Table 1 and 2.

Table 1. Implications of perturbations on GPS satellite orbit. From Seeber (2003)

Perturbation	Effect on satellite acceleration m / s^2
Deviation of earth gravity field from a sphere	$5 \cdot 10^{-5}$
Variations in earth gravity field	$3 \cdot 10^{-7}$
Solar and lunar gravitation	$5 \cdot 10^{-6}$
Earth and ocean tides	$1 \cdot 10^{-9}$ each
Solar radiation pressure	$1 \cdot 10^{-7}$
Albedo	$1 \cdot 10^{-9}$

Table 2. Deviation of true satellite orbit from Keplerian reference orbit of a GPS satellite after 4 hours. From Seeber (2003)

Kepler element	Deviation of earth gravity field from a sphere	Variations in earth gravity field	Solar and lunar gravitation	Solar radiation pressure
a	2600 m	20 m	220 m	5 m
e	1600 m	5 m	140 m	5 m
i	800 m	5 m	80 m	2 m
Ω	4800 m	3 m	80 m	5 m
ω + M	1200 m	4 m	500 m	10 m

The combined perturbing forces are difficult to model. This means that even though we are able to predict the position of a satellite, given the Kepler elements and models of the perturbations, there is a limit as to how well this can be done.

7. GPS satellite orbits

The Kepler elements for the nominal 24 GPS satellite constellation originally designed is given in Table 3.

Tabel 3. Nominal GPS satellite constellation (Misra and Enge, 2001).

Slot ID	Semi-major axis [km]	Eccentricity [-]	Inclination [deg]	RAAN [deg]	Arg. of Perigee [deg]	Mean Anomaly [deg]
A3	26559.8	0.00	55.0	272.85	0.0	11.68
A4	26559.8	0.00	55.0	272.85	0.0	41.81
A2	26559.8	0.00	55.0	272.85	0.0	161.79
A1	26559.8	0.00	55.0	272.85	0.0	268.13
B1	26559.8	0.00	55.0	332.85	0.0	80.96
B2	26559.8	0.00	55.0	332.85	0.0	173.34
B4	26559.8	0.00	55.0	332.85	0.0	204.38
B3	26559.8	0.00	55.0	332.85	0.0	309.98
C1	26559.8	0.00	55.0	32.85	0.0	111.88
C4	26559.8	0.00	55.0	32.85	0.0	241.57
C3	26559.8	0.00	55.0	32.85	0.0	339.67
C2	26559.8	0.00	55.0	32.85	0.0	11.80
D1	26559.8	0.00	55.0	92.85	0.0	135.27
D4	26559.8	0.00	55.0	92.85	0.0	167.36
D2	26559.8	0.00	55.0	92.85	0.0	265.45
D3	26559.8	0.00	55.0	92.85	0.0	35.16
E1	26559.8	0.00	55.0	152.85	0.0	197.05
E2	26559.8	0.00	55.0	152.85	0.0	302.60
E4	26559.8	0.00	55.0	152.85	0.0	333.69
E3	26559.8	0.00	55.0	152.85	0.0	66.07
F1	26559.8	0.00	55.0	212.85	0.0	238.89
F2	26559.8	0.00	55.0	212.85	0.0	345.23
F3	26559.8	0.00	55.0	212.85	0.0	105.21
F4	26559.8	0.00	55.0	212.85	0.0	135.35

The eccentricity of the orbits is zero, implying circular orbits. The argument of perigee is also zero and that is possible with circular orbits, since perigee is not a distinct point on the orbit as it is with ellipsoidally shaped orbits. With the argument of perigee being zero the direction of the q_1 axis with respect to the inertial coordinate system is thus defined to be located in the Equatorial plane pointing towards the point of the ascending node (ref. Figure 7).

Also note that the value of the right ascension of the ascending node (RAAN) is identical for each four satellites, indicating that these four satellites are revolving in the same orbit. The satellites are thus distributed into six orbital planes (Figure 8).

The mean anomaly indicates the distribution of satellites within the orbit. The satellites are not distributed evenly within the orbit, as seen in Figure 9. This design of the satellite constellation ensures a better global satellite coverage, compared to if the satellites were evenly distributed in the orbit.

The US DoD is aiming at keeping the nominal satellite constellation as described above, but because of the perturbing forces, and because some satellites wear out and new satellites are launched ahead of schedule, the actual constellation is slightly different. Plot of the current satellite constellation are maintained and updated on a regular basis by for instance the University of New Brunswick in Canada, and an example of such a plot is given in Figure 9. Satellites marked with red in the plot are unhealthy and thus not useable for positioning, and satellites marked in orange are set as spare satellites.

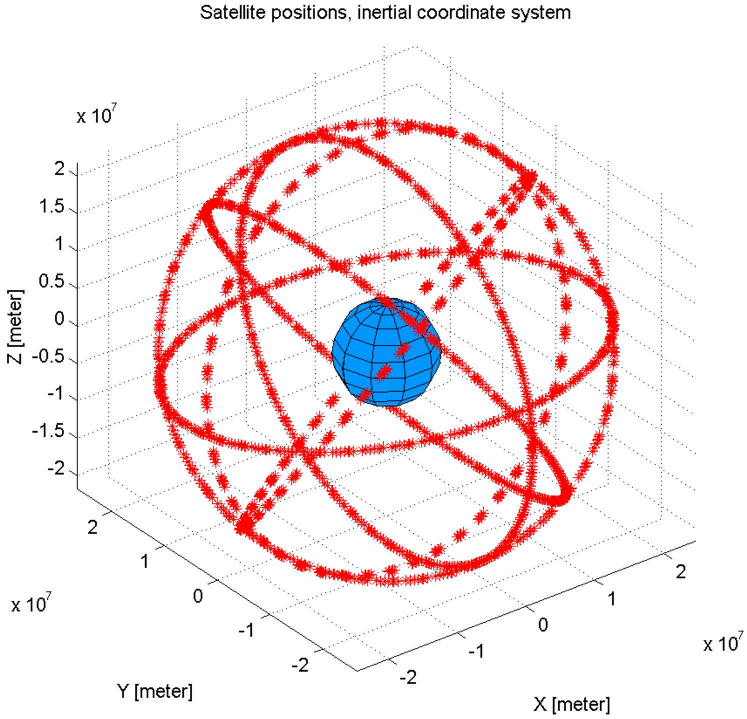

Figure 8. GPS satellite positions.
Plot based on 24 hour simulation of nominal constellation from Table 3.

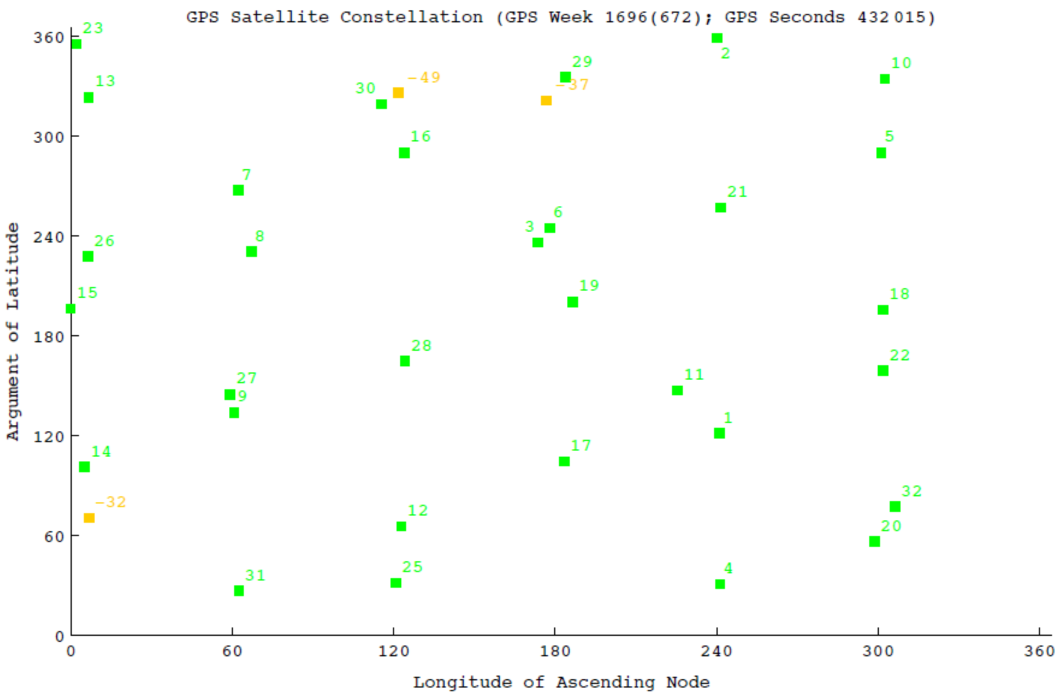

Figure 9. Actual GPS satellite constellation of July 16, 2012.
Plot from prof. R. Langley, University of New Brunswick, Canada

The chosen design of the GPS satellite orbits, causes the satellite availability to be depending on the location of the observer on the surface of the earth. Figure 10 shows the apparent satellite tracks during 24 hours in so-called sky plots, for four different locations on the earth.

With a sky plot the virtual observer is located in the center of the plot, and the plot shows the location, or tracks, of the satellites as they would be seen in the sky above the observer - if they were visible. The outermost ring of the plot corresponds to the horizon around the observer, and full circles indicating elevation angles of 30 º and 60º are also plotted. The dotted circle illustrates an elevation mask of 15º, which is often used for high accuracy GPS applications, where satellites located below this level are eliminated in the data processing to reduce the noise level.

The best GPS receiver position accuracy is in general obtained with an even distribution of satellites in the sky above and around the receiver. With the GPS satellite constellation the best satellite availability is obtained close to the Equator, in Figure 10 this is illustrated by Qatar. No satellites are, however, visible directly north of the observer, generating a void of observations from the north, which in practice will cause the accuracy of the north-south oriented coordinate component (e.g. latitude or UTM Northing) to be slightly worse than the east-west oriented component (e.g. longitude or UTM Easting) of the position. In practice this difference is, however, marginal.

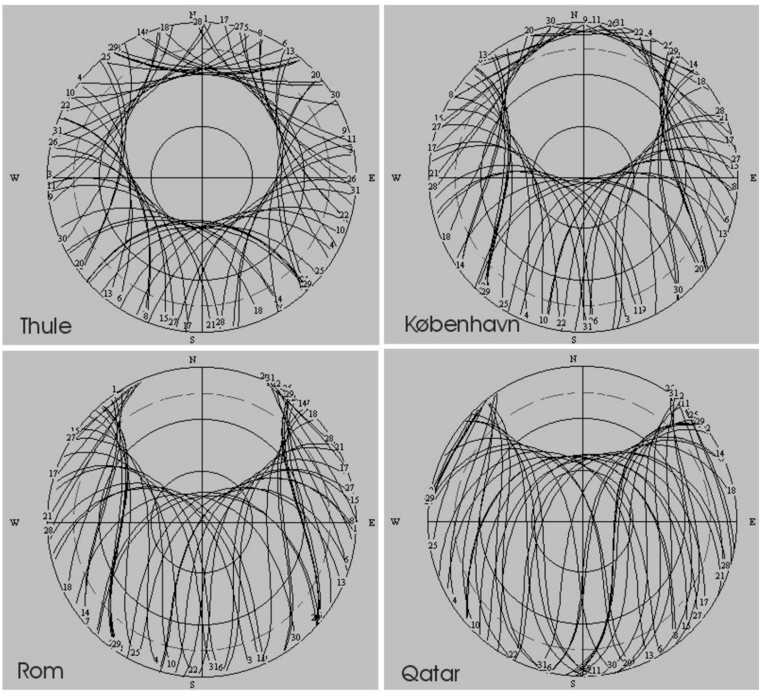

Figure 10. GPS sky plots generated for March 1. 2002 (Dueholm et. al, 2005).

For a location close to the poles, as for instance at Thule located in the northern most part of Greenland, signals will be received from satellites located directly north of the observer. However, these satellites are located close to the horizon, and the signals are thus affected by a higher noise level, and not as attractive as signals received from satellites with higher elevation angle affected by a lower signal to noise ratio and this a better signal quality.

Close to the Equator there are several satellites located in zenith (directly above the observer, at a 90º elevation angle), and close to the pole, no satellites will be observed in the zenith direction. This is affecting the accuracy of the height component of the GPS-derived position, which is generally better at the Equator than at the poles.

GPS broadcast ephemeris

For GPS positioning, the GPS receiver needs information on the satellite positions at the point in time where the satellite signals were transmitted from the satellites. This information is called *broadcast ephemeris* and is transmitted from the satellites with the *navigation message*.

The broadcast ephemeris are composed of the following variables (Misra and Enge, 2001):

- The ephemeris reference time, t_0
- The Kepler elements given as:
  - $\sqrt{a}$, e, i at t_0
  - Ω at beginning of the GPS week
  - ω and M at t_0
- A correction factor to the mean motion, Δn
- The rate of change of i and Ω
- Coefficients for correction models of ω, a and i

The broadcast ephemeris are estimated by the GPS control stations. The control stations continuously collect data from the satellites, and based on observations of previous satellites positions, future positions are predicted by the Kepler elements and their derivatives, to compose the broadcast ephemeris.

The ephemeris parameters are uploaded once daily, and by transmitting the Kepler elements with derivatives, in stead of satellite positions with derivatives, estimation of the satellite positions at the given epoch in time can be carried out more precisely considering the perturbations.

When receiving the broadcast ephemeris the GPS receiver estimates the satellite positions as X, Y, Z coordinates in the WGS84 reference system, and the actual estimation of the position of the GPS receiver can be carried out.

The formal procedure for estimation of satellite positions in the WGS84 based on the broadcast ephemeris is given in the GPS Interface Control Document, the official user manual of GPS, which is available on the web site of the US Coast Guard:

http://www.navcen.uscg.gov/pubs/gps/icd200/default.htm

9. Precise orbits

The broadcast ephemeris are predicted, and the location of a satellite can be estimated with an accuracy of about 3 meter based on the broadcast ephemeris. More precise satellite positions are estimated by a number of organizations world wide and are made available on the internet.

The *International GNSS Service* (IGS) provide precise orbits estimated from several days of satellite data collected in more than 200 permanent GPS reference stations distributed globally. Each perturbation effect is modeled independently, using advanced estimation techniques, and the Kepler elements are estimated by combination of the models leading to the final positions of the satellites given in the data files.

The IGS generate a number of different orbit products:

- *Predicted orbits* with a standard deviation of about 1 m are available in real time
- *Rapid orbits* with a standard deviation of about 10 cm are available after 1 day.
- *Precise orbits* with a standard deviation of 6 cm are available after 11 days.

The orbit products are available from http://igscb.jpl.nasa.gov in the sp3 format which is described in the next section.

Precise orbits are used for a limited number of GNSS applications with high accuracy requirements. Typically when the station separation between reference and rover receiver in carrier-phase based differential mode is long (more than 50 km), and/or when positions are post processed (not real time) so the extra time needed to wait for the precise orbits is of lower priority than the better position accuracy obtainable with precise orbits. But also precise orbits are very beneficial for research and development purposes.

The sp3 format

The sp3 format is a text data format used for precise orbit information. The most important parts of the format is discussed below, a full description of the format is given on: http://www.navcen.uscg.gov/ftp/GPS/PRECISE/FORMAT.TXT

Example of an sp3 data file

```
#cP2006  5 29  0  0  0.00000000      96 ORBIT IGb00 HLM  IGS
## 1377  86400.00000000   900.00000000 53884 0.0000000000000
+   29   G01G02G03G04G05G06G07G08G09G10G11G13G14G15G16G17G18
+        G19G20G21G22G23G24G25G26G27G28G29G30  0  0  0  0  0
+          0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
+          0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
+          0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
++         3  3  3  3  3  3  3  3  3  3  3  3  3  3  3  3  3
++         4  3  3  4  3  4  3  3  3  4  3  3  0  0  0  0  0
++         0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
++         0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
++         0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
%c G  cc GPS ccc cccc cccc cccc cccc ccccc ccccc ccccc ccccc
%c cc cc ccc ccc cccc cccc cccc cccc ccccc ccccc ccccc ccccc
%f  1.2500000  1.025000000  0.00000000000  0.000000000000000
%f  0.0000000  0.000000000  0.00000000000  0.000000000000000
%i    0    0    0    0      0      0      0      0         0
%i    0    0    0    0      0      0      0      0         0
/* FINAL ORBIT COMBINATION FROM WEIGHTED AVERAGE OF:
/* cod emr esa gfz jpl mit ngs sio
/* REFERENCED TO IGS TIME (IGST) AND TO WEIGHTED MEAN POLE:
/* CLK ANT Z-OFFSET (M): II/IIA 1.023; IIR 0.000
*  2006  5 29  0  0  0.00000000
PG01  14003.983504 -20531.964425   9495.875403     55.833061  9 10  6 152
PG02  -4857.749099  14526.560859  21435.621040      0.595589 12 11 10 188
PG03   7573.446990 -22332.880868 -12077.575640    102.321813 11 10 11 144
PG04 -17065.511518   8524.720000  18582.647681    241.170978 12 11  9 172
PG05  13397.115513  21415.123417   8278.836680     56.878513  8  8 10 129
PG06  18691.208577   1177.506743  19057.727051    543.825785 10 12 10 150
```

```
PG07  20035.215079 -17199.791044   3809.418273    539.473229  9  9 11 116
PG08 -23035.896854   1859.631763 -13587.696226    -67.199315  9  8  8 186
PG09  16286.007420  15179.953835 -15243.135657     22.099821 12 11 12 172
PG10    287.313626  25303.883467   7425.263642     84.134932  9  9  9 168
```
etc.

The content of the header is:
\# Date and time for the first position in the file. Also various information on reference frame and types of orbits.
\## Date and time of first position given by GPS week number and seconds of week, followed by the time interval (in seconds) of the positions in the file. The last numbers are the modified julian day, integer and fraction.
\+ The number of satellites and the corresponding satellite numbers. G indicate a GPS satellite; several alternatives are possible e.g. R for GLONASS or E for Galileo.
++ Accuracy of the satellite orbits. The numbers are exponents and should be interpreted in the following way: If the accuracy exponent is 4, the accuracy is 2^4 mm. The sequence of accuracy exponents corresponds to the satellite numbers in rows 3 and 4.
%c, %f, %i Indicate that the following numbers are either chars (c), floats (f) or integers (i). Most of the spaces in these lines are not used at present, but reserved for further purposes. However it should be mentioned, that in line number 13 , the fourth slot indicate the time system used for the satellite positions, which in this example is GPS time. An alternative could be UTC time.
/* Are comments. In this case the comment lines contain information on processing centers that contributed with data and preliminary solutions to the final precise orbits. *cod* is for instance an abbreviation for Center for Orbit Determination at the Astronomical Institute, the University of Berne in Switzerland.

Following the header, satellite positions are given with the epoch interval indicated in line two, which in this case is 900.0 seconds. When the first character of a line is * positions for a new epoch in time are following.

* Year, month, day, hour, minute, seconds for the lines to follow. Valid until next line starting with *

Following is information on one satellite per line:
P is for position and clock records, the alternatives are *V* for velocity and clock correction rate-of-change, *EP* for position and clock correlation, or *EV* for velocity and clock rate-of-change.

When the files contain satellite position and clock records, each line starting with a P contains the satellite number, the satellite position, X, Y, Z in km, and a precise estimate of the satellite clock error (in micro seconds). The last four numbers in each line are the standard deviation in mm for the X, Y and Z coordinates, and the standard deviation in pico seconds for the clock error.

Several flags can be set in the file, and the reader is referred to the full format description for details. There are for instance flags to indicate satellite maneuvers or discontinuities in the clock corrections.

The SP3 format has been revised a couple of times, each time, however, with backward compatibility. The version described above is called sp3-c. The first version was sp3-a.

GPS orbit interpolation

Interpolation in the sp3 files with precise satellite position information should be carried out using a Lagrange interpolation routine. Lagrange interpolation is based on polynomial functions fitted to the data set. Studies referenced by Hofmann-Wellenhof et al. (2001) show that a 17. order interpolator can provide mm-accuracy in the interpolated satellite positions.

Lagrange interpolation is briefly described by Hofmann-Wellenhof et al. (2001), and more thoroughly by for instance Eldén et al. (2004).

10. Galileo and GLONASS satellite orbits

The nominal Kepler elements of the Galileo satellites are given in Table 4. The full Galileo satellite constellation will consist of 30 satellites. The last three satellites are not given in this list, since they will be placed in the orbits at locations where the need for extra satellites is largest.

Table 4. Nominal Galileo satellite constellation. Provided by Galileo Project Office, ESA.

SV#	PRN#	Semi-major axis [km]	Eccentricity [-]	Inclination [deg]	RAAN [deg]	Arg. of Perigee [deg]	Mean Anomaly [deg]
1	111	29600.318	0.00	56.00	0.00	0.00	0.00
2	112	29600.318	0.00	56.00	0.00	0.00	40.00
3	113	29600.318	0.00	56.00	0.00	0.00	80.00
4	114	29600.318	0.00	56.00	0.00	0.00	120.00
5	115	29600.318	0.00	56.00	0.00	0.00	160.00
6	116	29600.318	0.00	56.00	0.00	0.00	200.00
7	117	29600.318	0.00	56.00	0.00	0.00	240.00
8	118	29600.318	0.00	56.00	0.00	0.00	280.00
9	119	29600.318	0.00	56.00	0.00	0.00	320.00
10	121	29600.318	0.00	56.00	120.00	0.00	13.33
11	122	29600.318	0.00	56.00	120.00	0.00	53.33
12	123	29600.318	0.00	56.00	120.00	0.00	93.33
13	124	29600.318	0.00	56.00	120.00	0.00	133.33
14	125	29600.318	0.00	56.00	120.00	0.00	173.33
15	126	29600.318	0.00	56.00	120.00	0.00	213.33
16	127	29600.318	0.00	56.00	120.00	0.00	253.33
17	128	29600.318	0.00	56.00	120.00	0.00	293.33
18	129	29600.318	0.00	56.00	120.00	0.00	333.33
19	131	29600.318	0.00	56.00	240.00	0.00	26.66
20	132	29600.318	0.00	56.00	240.00	0.00	66.66
21	133	29600.318	0.00	56.00	240.00	0.00	106.66
22	134	29600.318	0.00	56.00	240.00	0.00	146.66
23	135	29600.318	0.00	56.00	240.00	0.00	186.66
24	136	29600.318	0.00	56.00	240.00	0.00	226.66
25	137	29600.318	0.00	56.00	240.00	0.00	266.66
26	138	29600.318	0.00	56.00	240.00	0.00	306.66
27	139	29600.318	0.00	56.00	240.00	0.00	346.66

Note that the Galileo satellites will be distributed into three orbital planes as opposed to the six planes used with GPS. A plot showing the Galileo satellite orbits is shown in Figure 11.

The GLONASS satellites are distributed into three orbital planes with semi-major axis of 25440 km and an inclination angle of 64.8 degrees. The satellites are distributed evenly within the orbits, i.e. a spacing of 45 degrees with the nominal constellation of 24 satellites. The three orbital planes are distributed evenly around the Equator, which gives a spacing of 120 degrees between the ascending nodes (Forssell, 1997). Also the GLONASS orbits are nearly circular with an eccentricity close to zero.

Figure 12. shows a plot of the GLONASS satellite constellation. Satellites marked with red, indicate unhealthy satellites at the time of plotting.

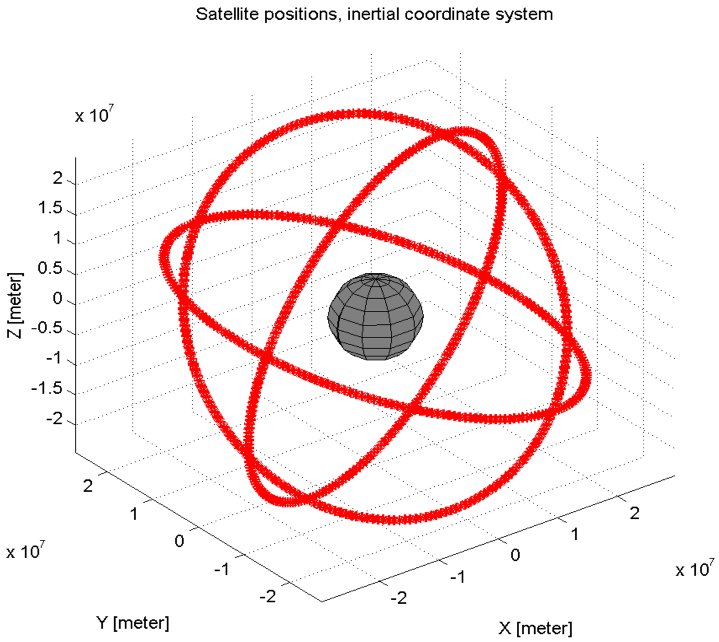

Figure 11. Galileo satellite positions.
Plot based on 24 hour simulation of nominal constellation from Table 4.

Also note that the GLONASS orbits have a higher inclination angle than the GPS and Galileo orbits, thus providing a better satellite availability at higher latitudes. GLONASS is therefore met with a particular interest in areas of high latitude, for instance in Norway where reliable satellite positioning for marine navigation in narrow fiords is important.

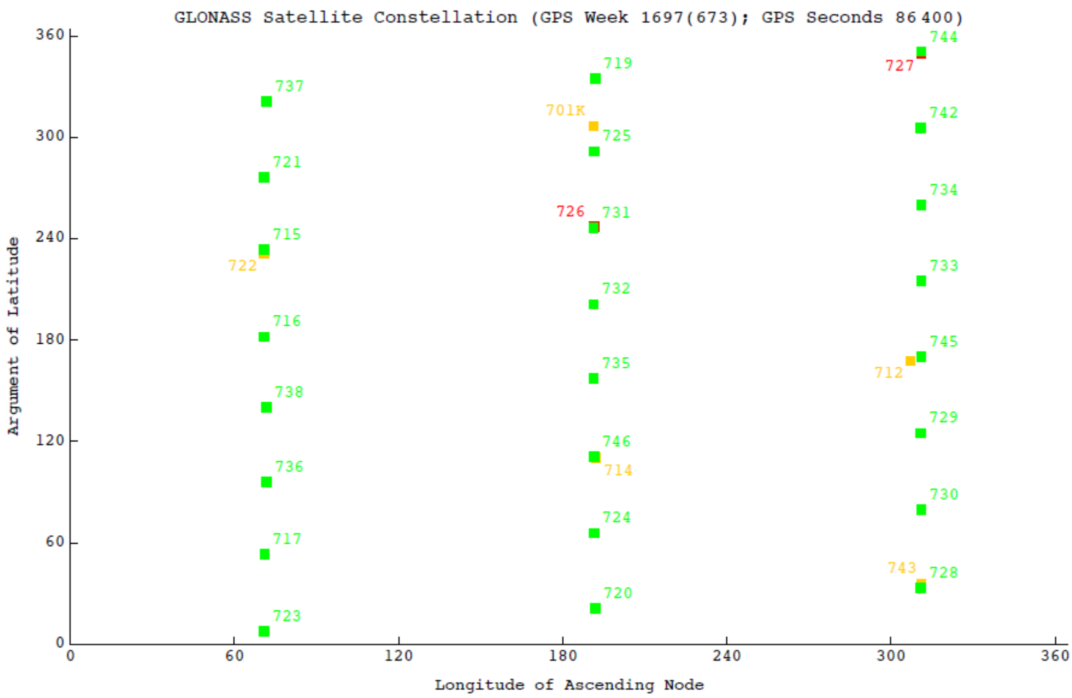

Figure 12. GLONASS satellite constellation of July 17, 2012.
Plot from prof. R. Langley, University of New Brunswick, Canada

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