Before we start:

If you feel ill, go home

Keep your distance to others

Wash or sanitize your hands

Disinfect table and chair

Respect guidelines and restrictions

30552 Satellite Geodesy – E20

Lecture 6
Introduction to Global Navigation Satellite Systems (GNSS)

by
Daniel Olesen, DTU Space

Content

- Introduction to GNSS
- Introduction to the GPS system
  - Space, control and user segment
  - Satellite signals
  - Error sources
  - Reference system WGS84
  - Code and phase based positioning
  - Basic principle for positioning
- Examples of GNSS applications
- Assignment 4

GNSS

- Global Navigation Satellite Systems

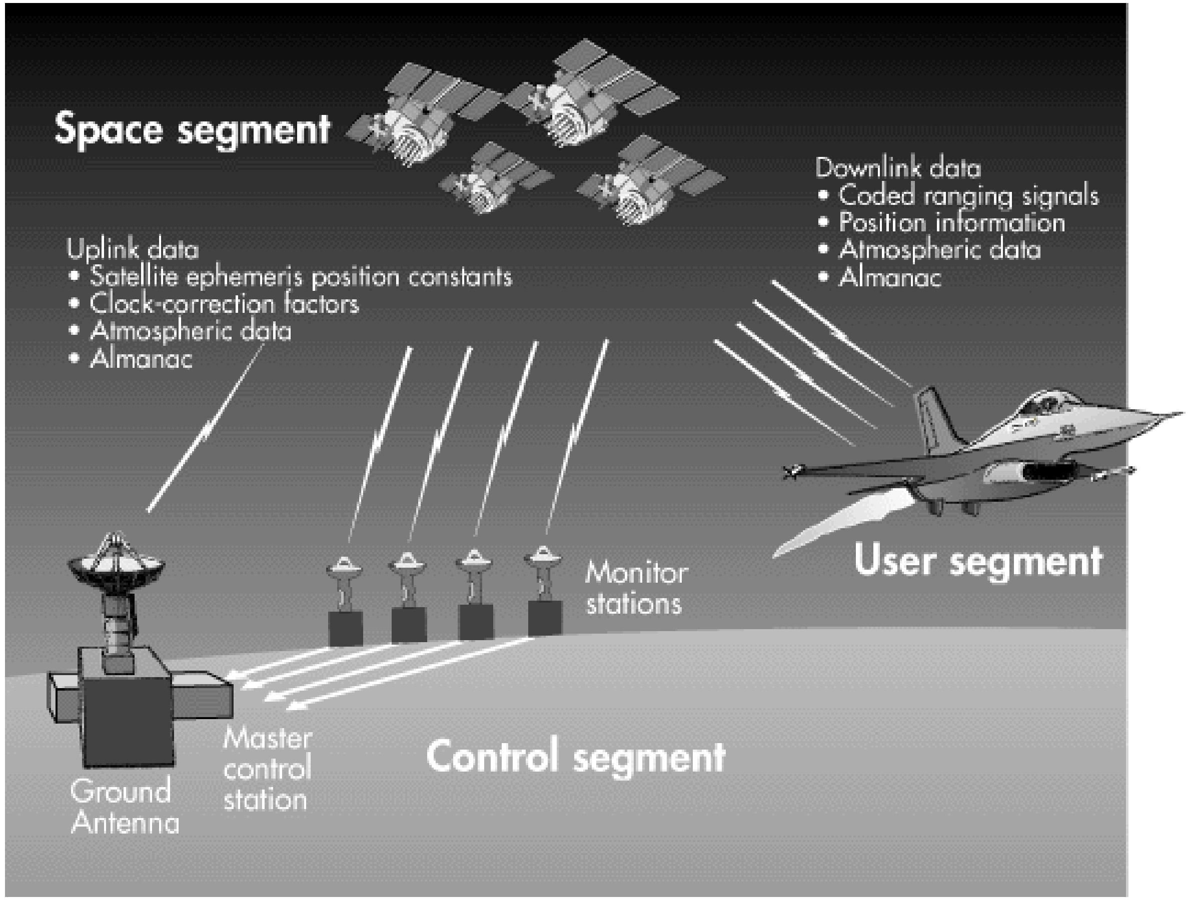
- GNSS includes GPS and similar systems:
  - Galileo (Europe)
  - GLONASS (Russia)
  - Beidou (China)

- As well as augmentation systems
  - EGNOS (Europe)
  - WAAS (USA)
  - QZSS (Japan)
  - Etc.

GPS

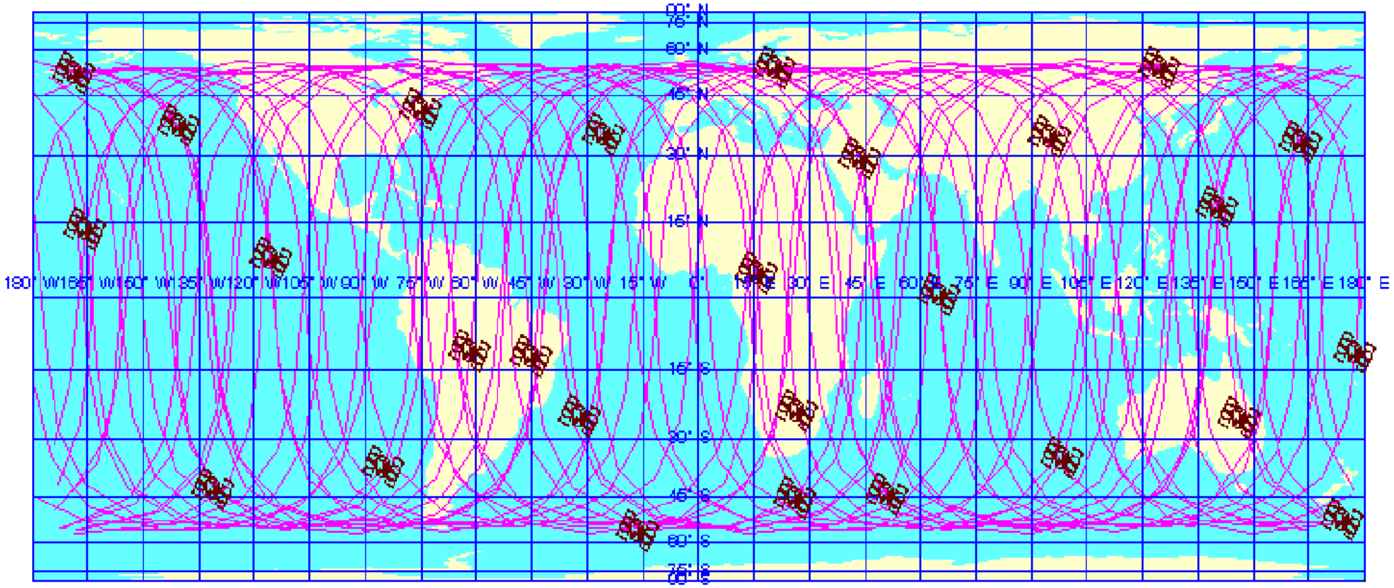
- Global Positioning System (NAVSTAR)

- Consists of 3 segments:
  - –Space segment
  - –Control segment
  - –User segment



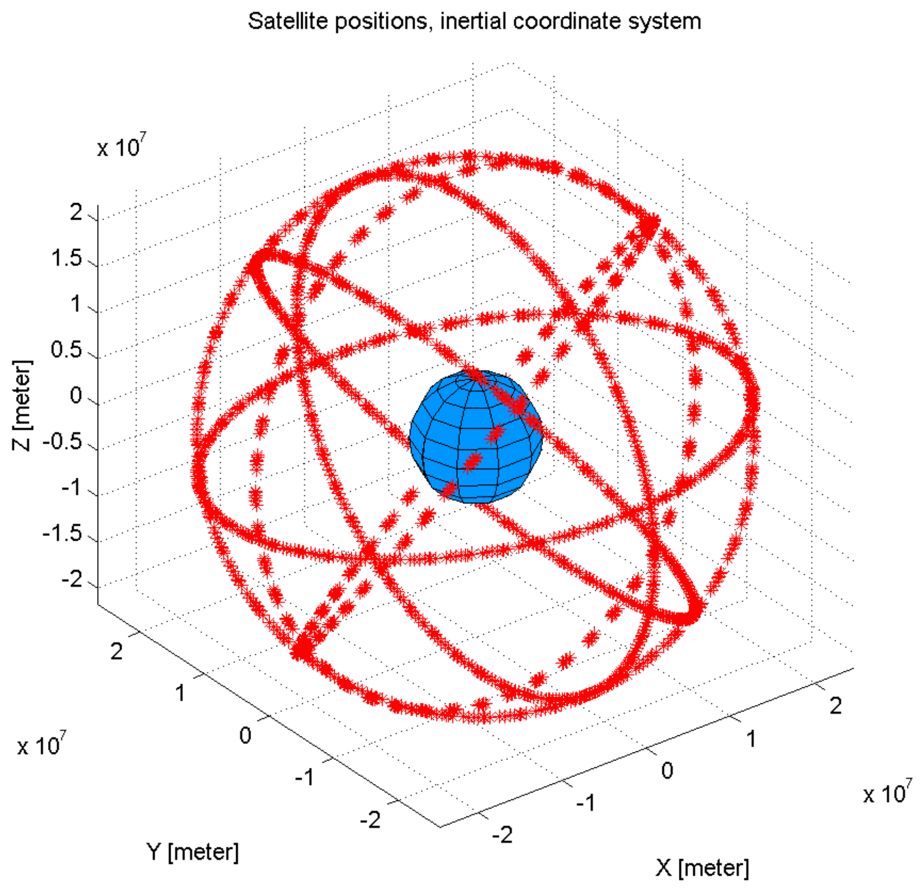
GPS Space Segment

- 24 satellites
- 6 orbital planes with 55 degrees inclination
- 20.200 km above surface of the Earth
- Speed: ~ 4 km/s, period: 11 hours and 58 minutes



Global Positioning System Satellites and Orbits
for 27 Operational Satellites on September 29, 1998
Satellite Positions at 00:00:00 9/29/98 with 24 hours (2 orbits) of Ground Tracks to 00:00:00 9/30/98

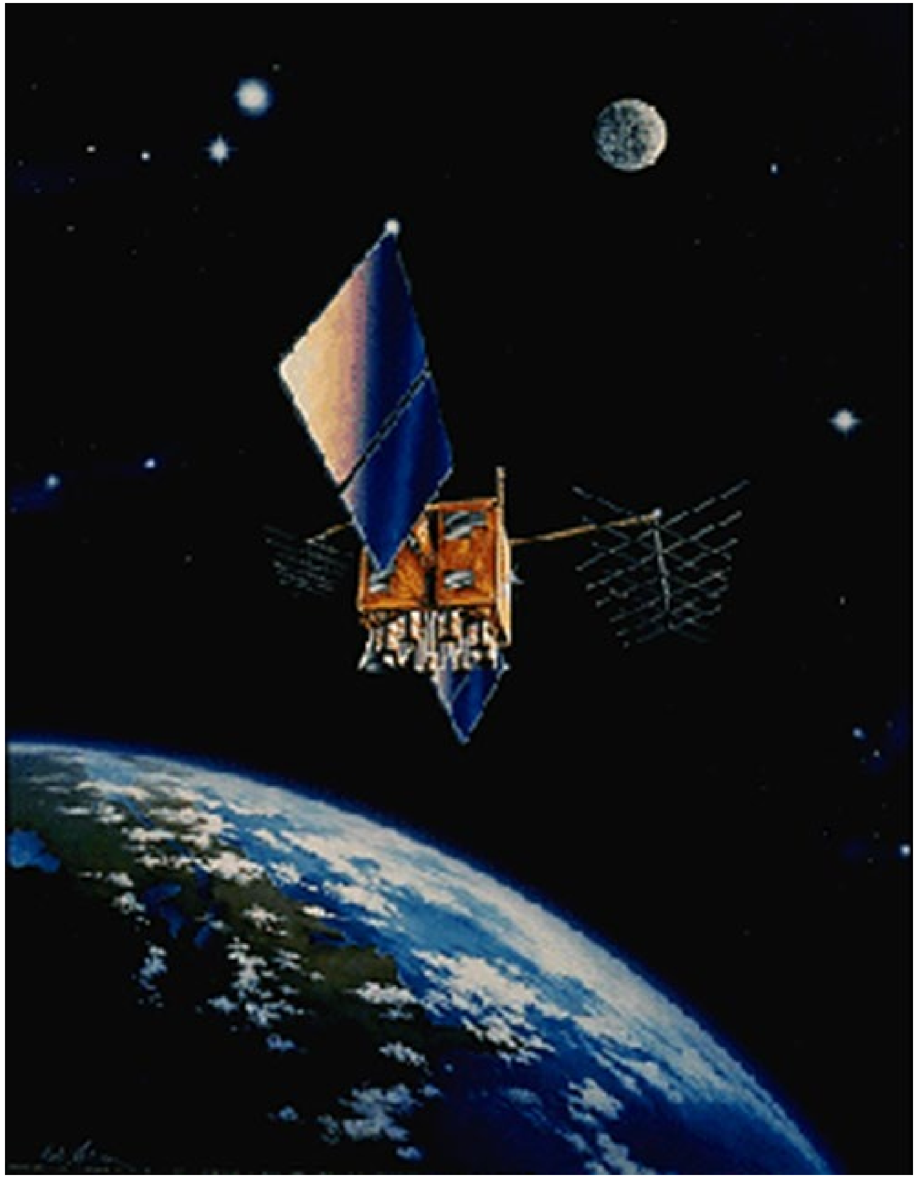
GPS Orbits - simulation



https://www.youtube.com/watch?v=nd9uD4WUnrg : GPS orbits

GPS Satellites

- Rubidium and Cesium atomic clocks for precise timing
- Transmits binary codes and electromagnetic waves
- Solar panels primary source of energy
- Block IIF satellites have expected life time of 12 years



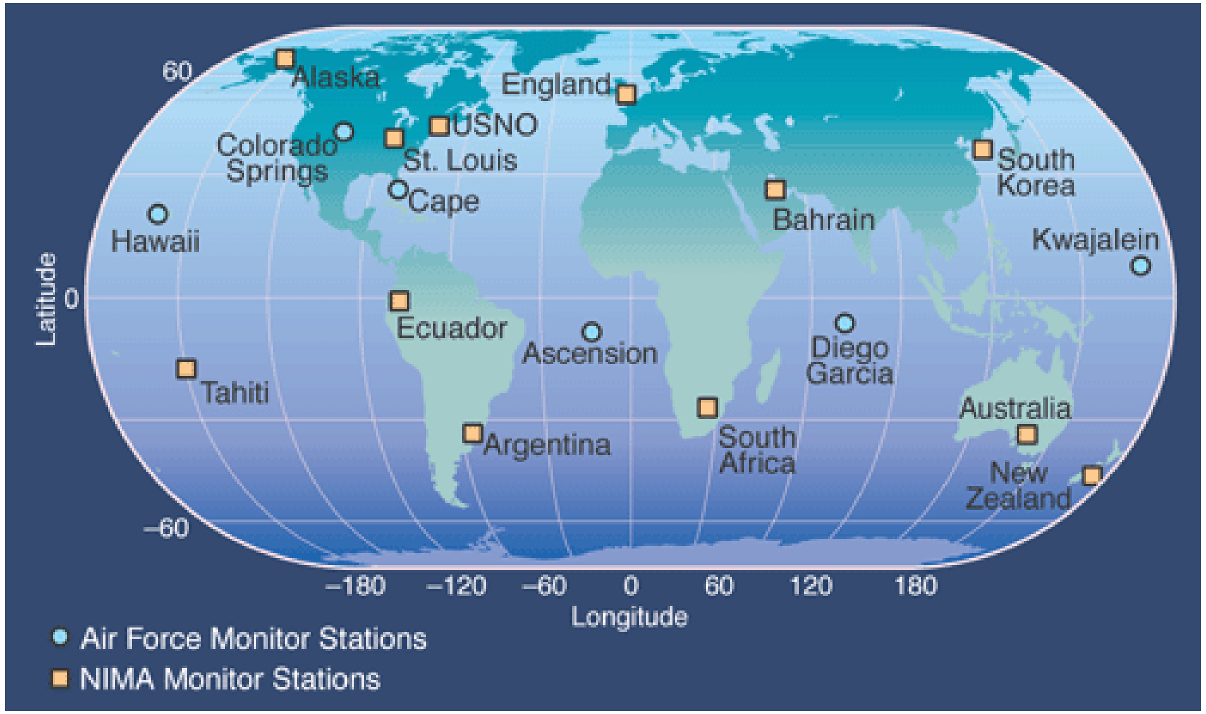
Picture from U.S. Naval Observatory

Control Segment

- The control stations monitor the condition and location of the satellites, collect data, compute orbit parameters and clock corrections

- Upload data to the satellites

- Operated by the US Department of Defence



User Segment

- Military and civil users

- Equipment:
  - GPS antenna
  - GPS receiver
  - Power

- Equipment prices range between 500 and 200.000 DKK

- High correlation between price and obtainable accuracy

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GNSS signals

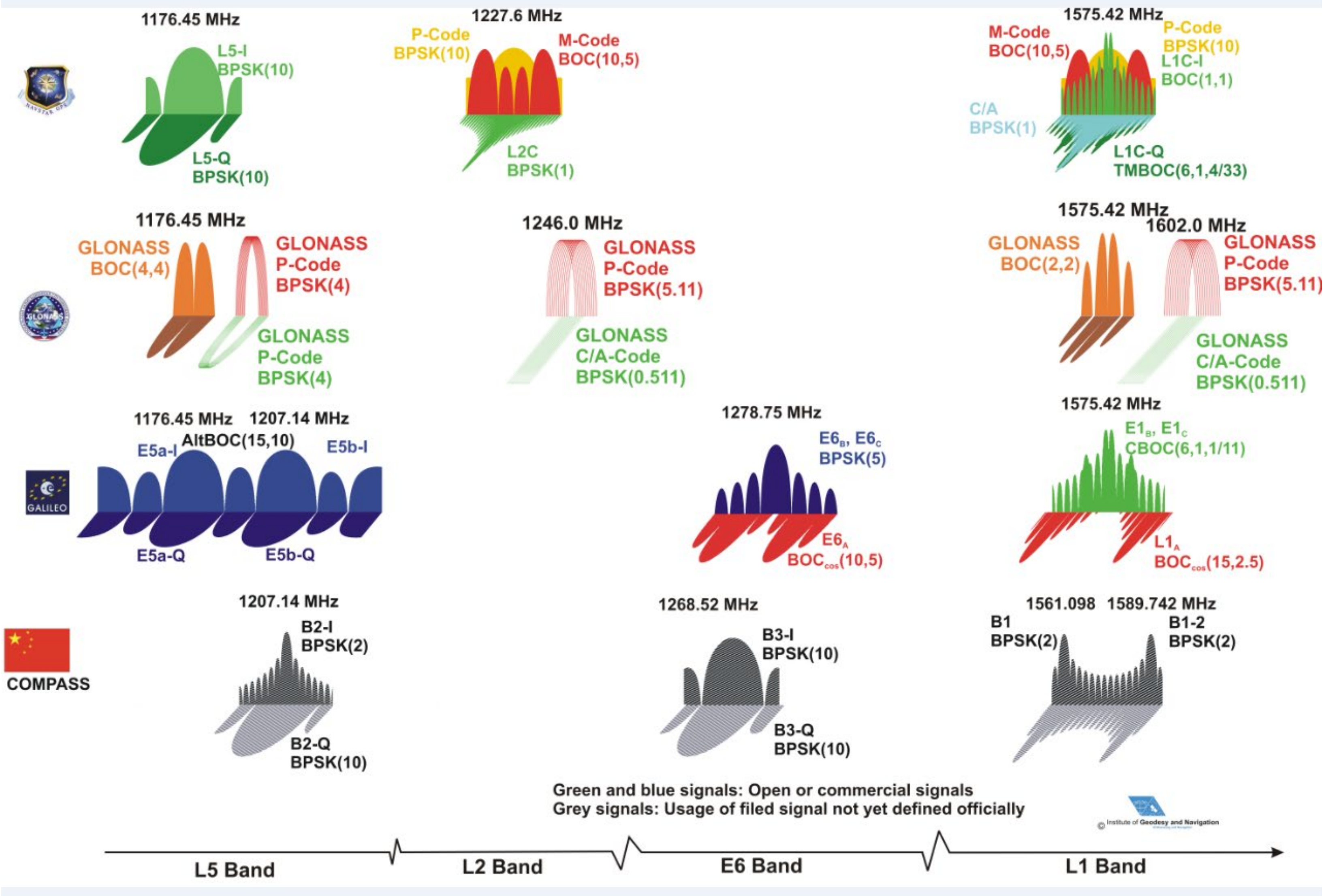
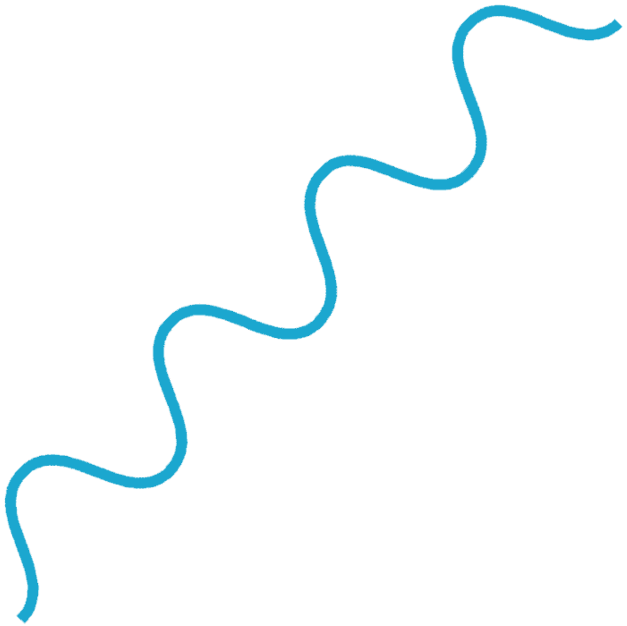


Figure from Navipedia.net

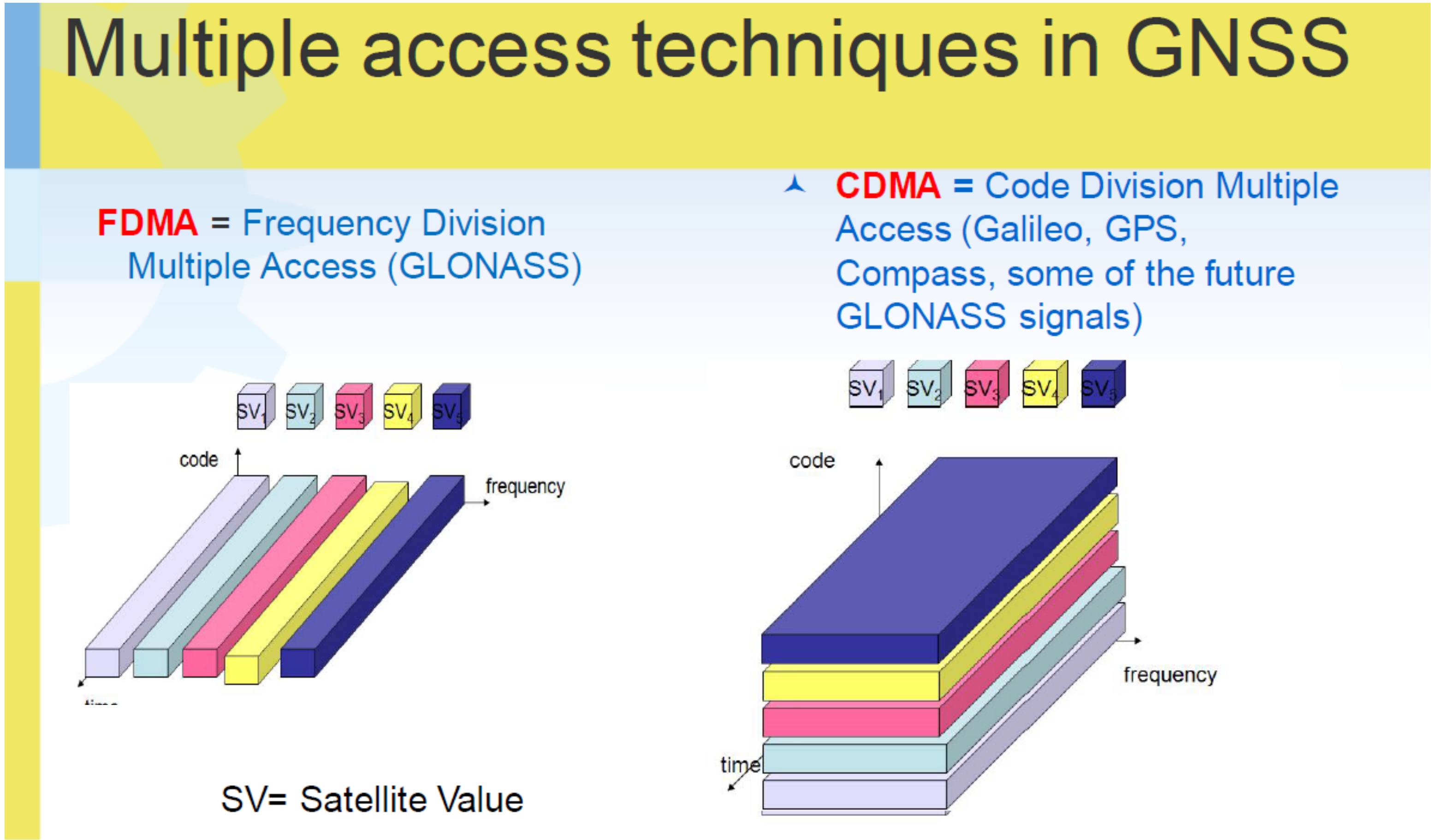
GPS Satellite Signals - Frequencies

- L1: 1575.42 MHz
  - Wave length: 19 cm
- L2: 1227.60 MHz
  - Wave length: 24 cm
- L5: 1176.45 MHz
  - realized with block IIF satellites, the first set operational on Aug. 27, 2010



How can all GPS satellites share the same frequency?

FDMA and CDMA



Code modulation in the satellites

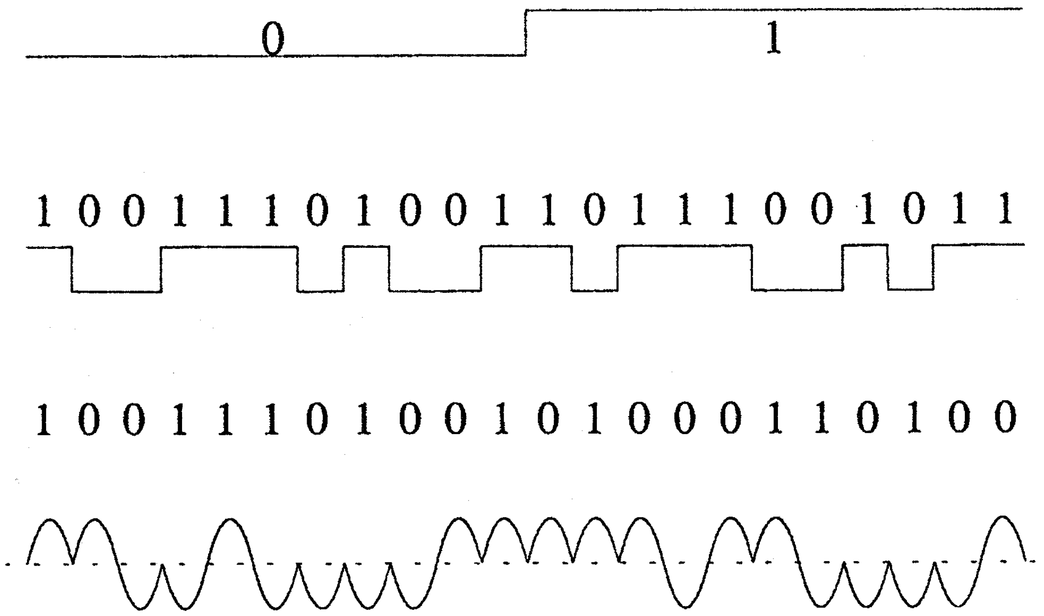
Navigation message

PRN code

Binary addition

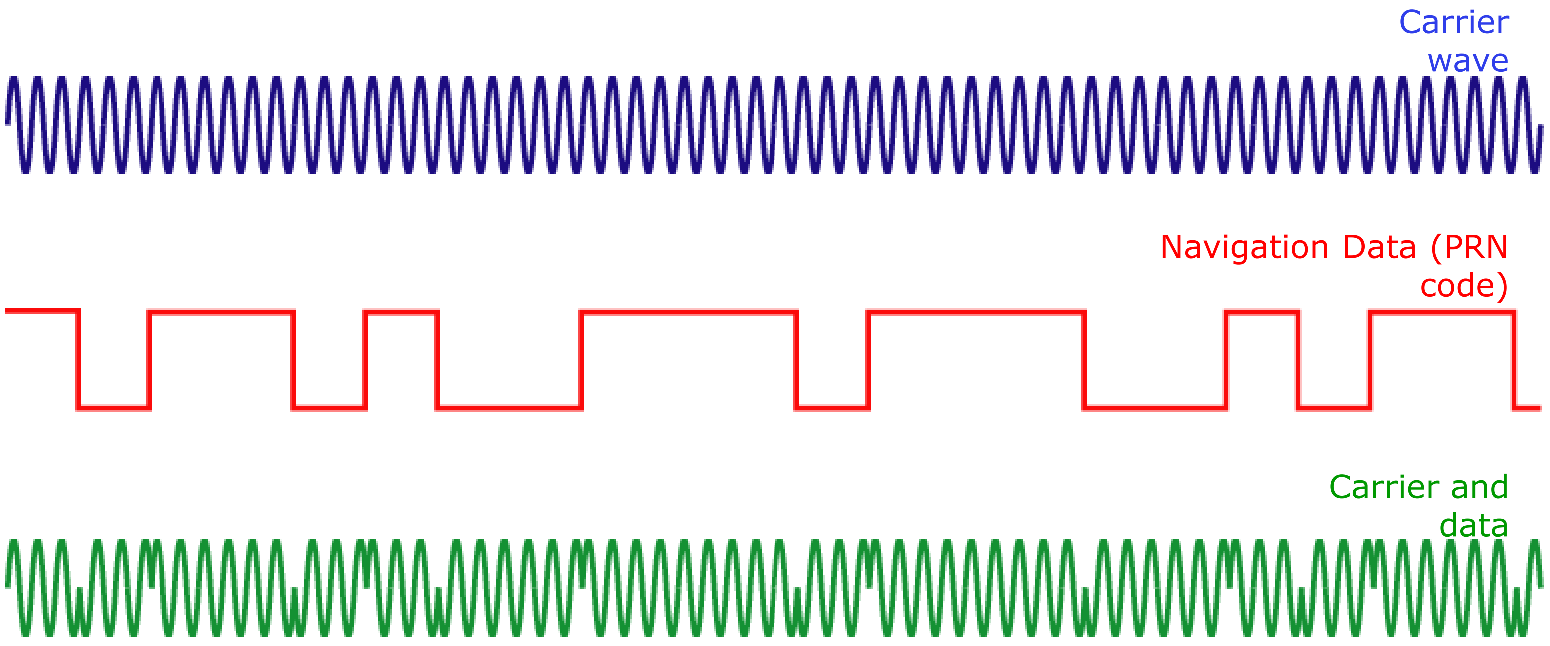
Phase change of

carrier wave



From: ”GPS” by K. Dueholm, M. Laurentzius, A.B.O. Jensen, Nyt Teknisk Forlag, 2005

Signal modulation in satellite



Modulation: Binary Phase-Shift Keying (BPSK)

Error Sources

- Generated errors:
  - AS (Anti Spoofing): Encryption of P code
  - SA (Selective Availability): Errors in navigation message
    Deactivated since May 2000

- Natural error sources:
  - Effects on the signal in the atmosphere
  - Multipath (signal reflection) and electronic noise
  - Inaccuracies in the navigation message

- Receiver noise and clock errors

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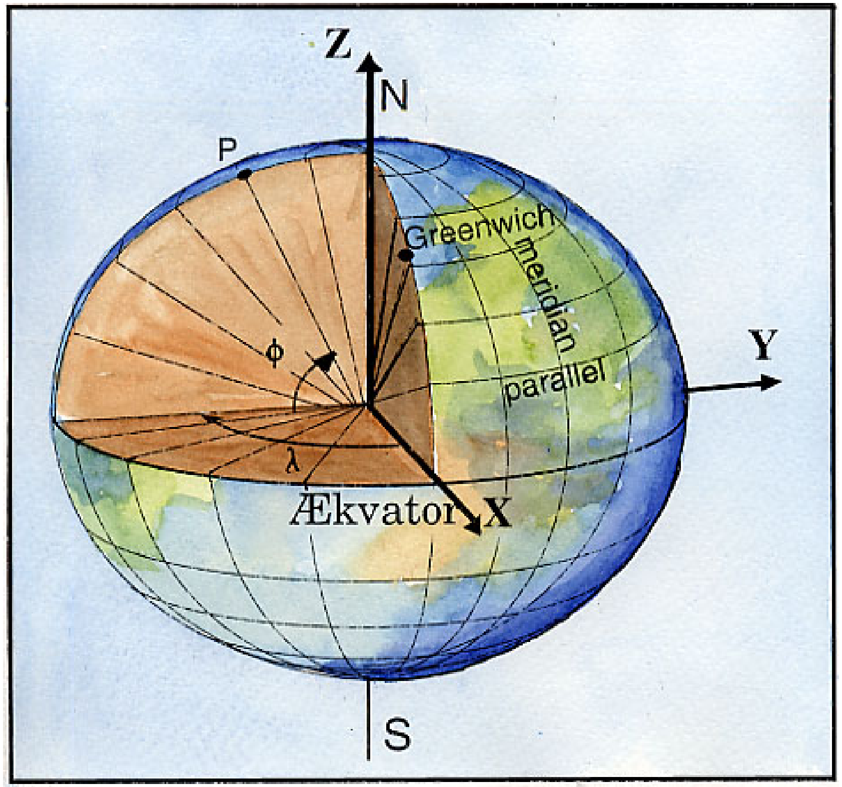
Reference system: WGS84

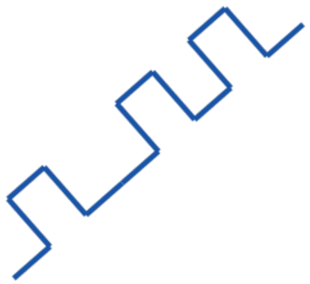
- World Geodetic System

- Coordinates given as:
  - X, Y, Z
  - Latitude, longitude, height



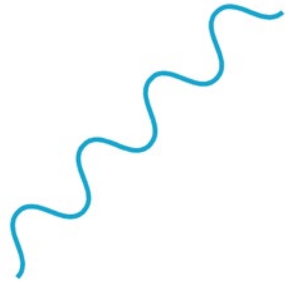
– Figure from Prof. Ole Jacobi

Code based positioning



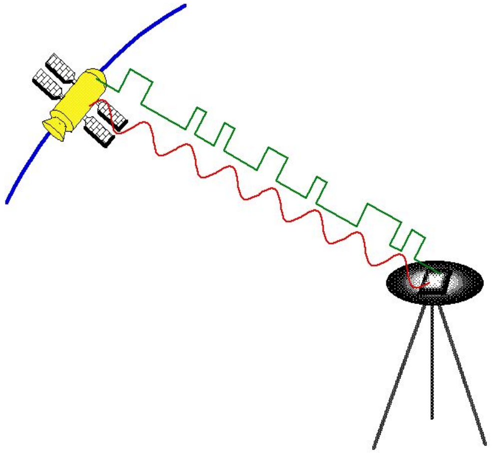
- Transmission time determined with C/A code
- Transmission time converted to pseudorange with accuracy at about 0.3 – 3 meter
- Pseudorange corrected for atmospheric effects using models, and satellite clock error is corrected using corrections from navigation message
- Position determined based on at least 4 pseudo-ranges
- Position accuracy: 3 – 10 meter

Carrier phase based positioning



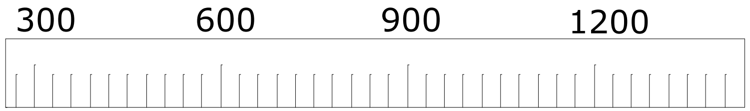
- Preliminary position from code observations
- Distance from satellite to receiver is determined as the full number of cycles + the last fraction of a cycle
- The last fraction of a cycle (the phase) is measured continuously by the receiver with an accuracy better than 1% of the wavelength ~ 2 mm
- Models are used for correction of atmospheric effects and satellite clock error
- The number of full cycles (the ambiguity) is determined using more advanced mathematical methods
- Obtainable position accuracy: < 1 cm
- Much more fragile than code-based observations

GPS observables overview



Analogy with tape

P code



30 m grading

C/A code

300 m grading

Carrier phase

20 cm grading without numbers

Illustration from Milan Horemuz, KTH - Royal Institute of Technology

GPS code-based Positioning – basic principle

- The GPS receiver determines the distance to the satellites based on received satellite signals. The distance is called a pseudorange

- Pseudorange = speed of light * transmission time

- The pseudoranges and the known positions of the satellites are used to determine the position of the GPS receiver (X, Y, Z) and the receiver clock error

- At least 4 pseudoranges are required for a 3D position

GPS code-based Positioning – basic principle

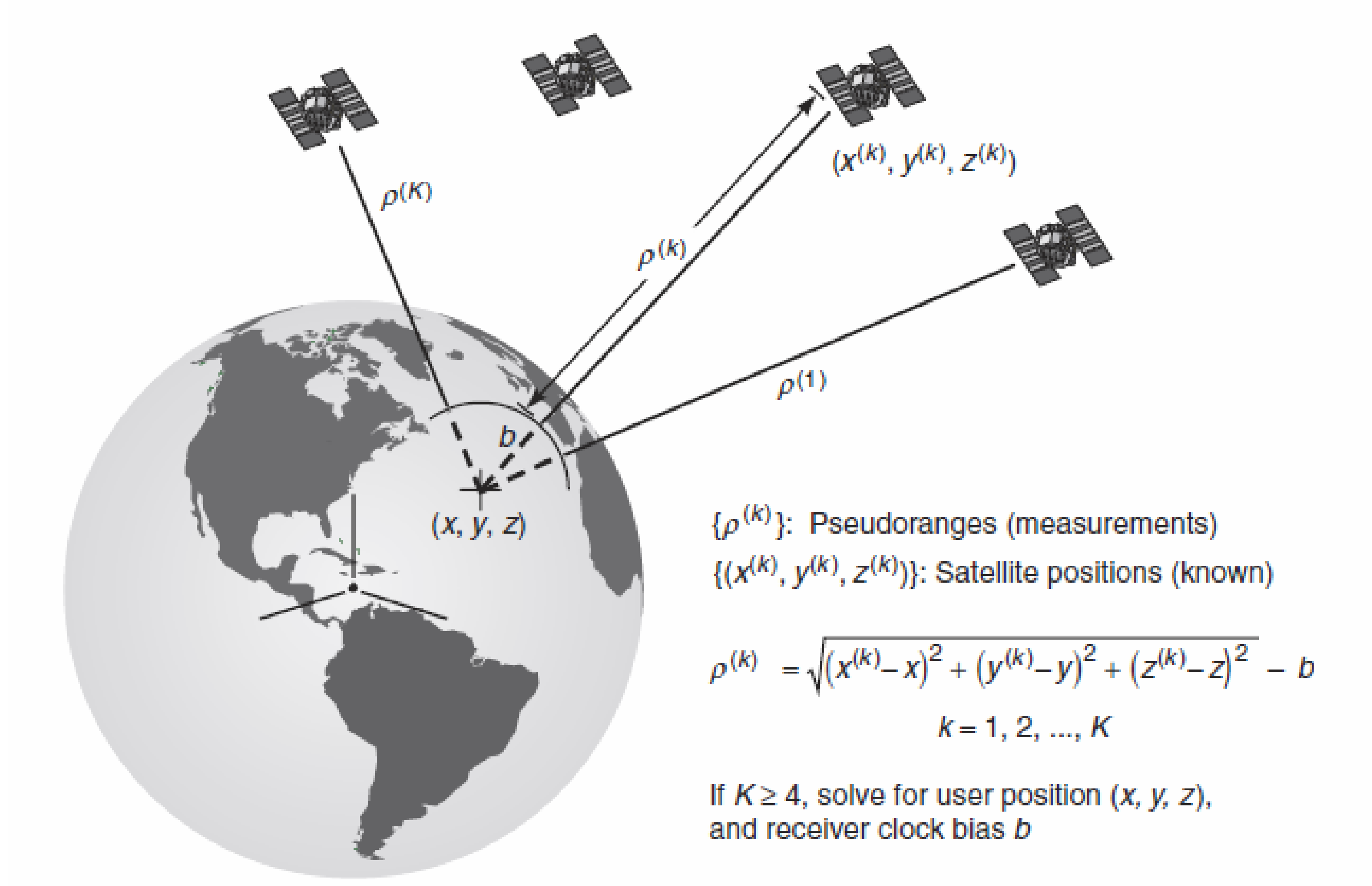


Illustration from "Global Positioning System..." by P. Misra and P. Enge

GPS code-based Positioning – basic principle

- Observation equation:

$$R_r^s = \rho_r^s + c \cdot \Delta\delta_r = \sqrt{(X^s - X_r)^2 + (Y^s - Y_r)^2 + (Z^s - Z_r)^2} + c \cdot \Delta\delta_r$$

- Where:

R_r^s	measured pseudorange between satellite, s and receiver, r
ρ_r^s	geometric distance between satellite and receiver
X_r, Y_r, Z_r	unknown coordinates of the receiver
X^s, Y^s, Z^s	known coordinates of the satellite
c	speed of light
$\Delta\delta_r$	GPS receiver clock error in seconds (unknown)

- We have 4 unknowns in the equation, so it can be solved with a minimum of 4 observations

GPS code-based Positioning – basic principle

- Four equations and four unknown

$$R_r^{I} = \sqrt{(X^{I} - X_r)^2 + (Y^{I} - Y_r)^2 + (Z^{I} - Z_r)^2} + c \cdot \Delta\delta_r$$

$$R_r^{II} = \sqrt{(X^{II} - X_r)^2 + (Y^{II} - Y_r)^2 + (Z^{II} - Z_r)^2} + c \cdot \Delta\delta_r$$

$$R_r^{III} = \sqrt{(X^{III} - X_r)^2 + (Y^{III} - Y_r)^2 + (Z^{III} - Z_r)^2} + c \cdot \Delta\delta_r$$

$$R_r^{IV} = \sqrt{(X^{IV} - X_r)^2 + (Y^{IV} - Y_r)^2 + (Z^{IV} - Z_r)^2} + c \cdot \Delta\delta_r$$

- Normally observations to more than 4 satellites are available and mathematical methods for adjustment and error estimation can provide better position accuracies

- The terms we would like to solve for is, $\theta = [X_r \ \ Y_r \ Z_r \ \Delta\delta_r]$

GPS code-based Positioning – basic principle

- As the equations are non-linear, we first need to linearize them around an operating point $\theta_0 = [X_0 \; Y_0 \; Z_0 \; \Delta\delta_r]$, we then assume that $\theta \approx \theta_0 + \Delta\theta$

$$R_r^s(\theta) \approx R_r^s(\theta_0) + \frac{\partial R_r^s(\theta)}{\partial X_r}\Delta X_r + \frac{\partial R_r^s(\theta)}{\partial Y_r}\Delta Y_r + \frac{\partial R_r^s(\theta)}{\partial Z_r}\Delta Z_r + \frac{\partial R_r^s(\theta)}{\partial \delta_r}\Delta\delta_r$$

Where the partial derivatives can be calculated as:

$$\frac{\partial R_r^s(\theta)}{\partial X_r}\Big|_{\theta_0} = \frac{X^s - X_0}{\rho_0^s}, \qquad \frac{\partial R_r^s(\theta)}{\partial Y_r}\Big|_{\theta_0} = \frac{Y^s - Y_0}{\rho_0^s},$$

$$\frac{\partial R_r^s(\theta)}{\partial Z_r}\Big|_{\theta_0} = \frac{Z^s - Z_0}{\rho_0^s}, \qquad \frac{\partial R_r^s(\theta)}{\partial \delta_r}\Big|_{\theta_0} = 1,$$

Where $\rho_0^s = \sqrt{(X^s - X_0)^2 + (Y^s - Y_0)^2 + (Z^s - Z_0)^2}$

GPS code-based Positioning – basic principle

$$\frac{\partial R_r^s(\theta)}{\partial X_r} = \frac{\partial(\sqrt{(X^s - X_r)^2 + (Y^s - Y_r)^2 + (Z^s - Z_r)^2} + c\delta_r)}{\partial X_r}$$

Apply the chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \Rightarrow \frac{d\,[u^n]}{dx} = nu^{n-1} \cdot u'$$

$$\frac{\partial R_r^s(\theta)}{\partial X_r} = \frac{1}{2}\left((X^s - X_r)^2 + (Y^s - Y_r)^2 + (Z^s - Z_r)^2\right)^{-1/2} \cdot (2X_r - 2X^s) =$$

$$\frac{\frac{1}{2}(2X_r - 2X^s)}{\sqrt{(X^s - X_r)^2 + (Y^s - Y_r)^2 + (Z^s - Z_r)^2}} = \frac{X^s - X_r}{\rho_r^s}$$

GPS code-based Positioning – basic principle

$$\underbrace{\begin{bmatrix} R_r^1 - \rho_0^1 - c\delta_{r,0} \\ R_r^2 - \rho_0^2 - c\delta_{r,0} \\ \vdots \\ R_r^n - \rho_0^n - c\delta_{r,0} \end{bmatrix}}_{z} = \underbrace{\begin{bmatrix} \frac{X^1 - X_0}{\rho_0^1} & \frac{Y^1 - Y_0}{\rho_0^1} & \frac{Z^1 - Z_0}{\rho_0^1} & 1 \\ \frac{X^2 - X_0}{\rho_0^2} & \frac{Y^2 - Y_0}{\rho_0^2} & \frac{Z^2 - Z_0}{\rho_0^2} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ \frac{X^n - X_0}{\rho_0^n} & \frac{Y^n - Y_0}{\rho_0^n} & \frac{Z^n - Z_0}{\rho_0^n} & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \\ \mathrm{c}\Delta\delta_r \end{bmatrix}}_{\Delta x}$$

The above systems of equations needs to be iteratively solved until the increments of Δx is lower than a specified bound. In Matlab the solution to the above system can be found as; $\Delta x = A\backslash\mathrm{z}$

$X_0, Y_0, Z_0, \delta_{r,0}$ is updated by Δx for each iteration

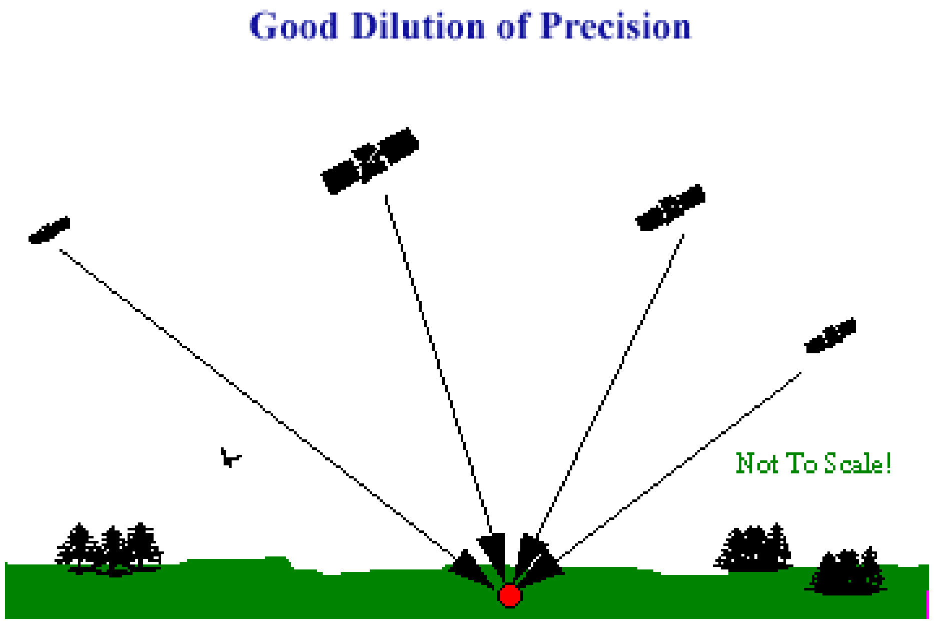
Satellite - receiver geometry (1)

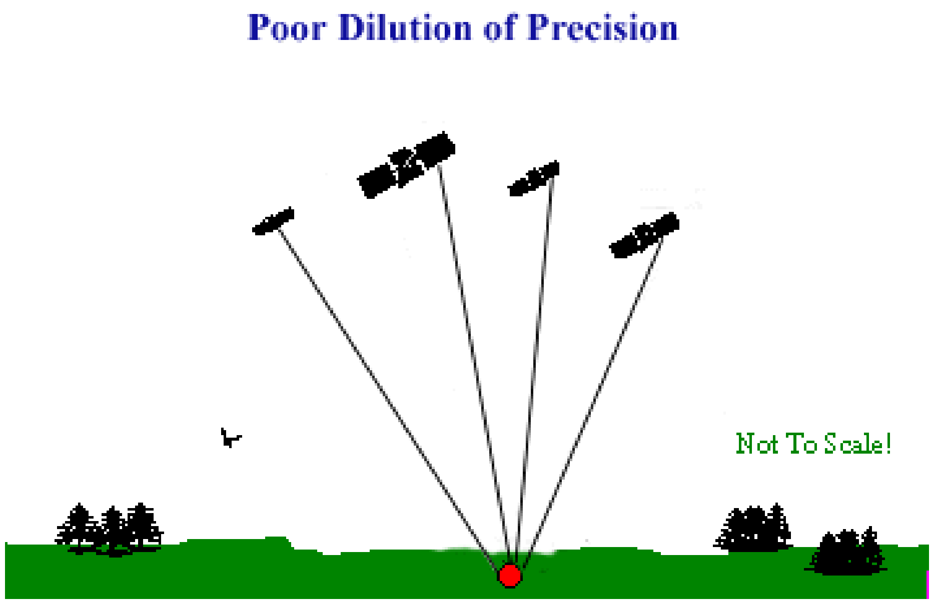
- GPS positioning is basically a geometric problem, therefore also the geometry of receiver and satellites affect position accuracy

- In general:
  – More satellites => more data => better position accuracy
- But also
  – Better satellite-receiver geometry => better position accuracy

- This means that using four satellites for positioning will not always provide the same level of accuracy. The reliability of the position solution is dependent on the location of the satellites with respect to the receiver

Satellite-receiver geometry (2)





Figures from: http://compsci02.snc.edu/cs225/2004/gps/webpage/Accuracy.html

Dilution of precision (DOP)

A value for characterising the satellite-to-receiver geometry, the so-called DOP, is generated by the covariance matrix (***Q_x***) of the observation equations

$$Q_X = (A^T A)^{-1} \cdot \sigma_0^2$$

Where A is composed by the partial derivatives of the linearized observation equation (more in the next lecture):

$$A = \begin{bmatrix} \frac{X^1 - X_0}{\rho_0^1} & \frac{Y^1 - Y_0}{\rho_0^1} & \frac{Z^1 - Z_0}{\rho_0^1} & 1 \\ \frac{X^2 - X_0}{\rho_0^2} & \frac{Y^2 - Y_0}{\rho_0^2} & \frac{Z^2 - Z_0}{\rho_0^2} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ \frac{X^n - X_0}{\rho_0^n} & \frac{Y^n - Y_0}{\rho_0^n} & \frac{Z^n - Z_0}{\rho_0^n} & 1 \end{bmatrix}$$

Dilution of precision (DOP)

$$Q_X = \begin{bmatrix} q_{XX} & q_{XY} & q_{XZ} & q_{Xt} \\ q_{YX} & q_{YY} & q_{YZ} & q_{Yt} \\ q_{ZX} & q_{ZY} & q_{ZZ} & q_{Zt} \\ q_{tX} & q_{tY} & q_{tZ} & q_{tt} \end{bmatrix} \cdot \sigma_0^2$$

$$TDOP = \sqrt{q_{tt}}$$

$$PDOP = \sqrt{q_{XX} + q_{YY} + q_{ZZ}}$$

$$GDOP = \sqrt{q_{XX} + q_{YY} + q_{ZZ} + q_{tt}}$$

- Horizontal and vertical DOPs can be determined by conversion of the DOP matrix to the local coordinate system:

$$HDOP = \sqrt{q_{NN} + q_{EE}} \qquad VDOP = \sqrt{q_{UU}}$$

Dilution of precision (DOP)

- Lower DOP-value => better position accuracy

- Rule of thumb: $$\sigma_{pos} = DOP \cdot \sigma_{pseudo}$$

- where:
  - σ_{pseudo} is standard deviation of pseudorange (URE)
  - σ_{pos} is standard deviation of position

- Generally:
  - PDOP < 4 is good, and PDOP > 6 is bad

  - But “good” and “bad” is also depending on user requirements

Observation weigting

Define an observation weight:

$$w_i = \frac{1}{\sin(elevation_i)^2}$$

Puts more emphasis on pseudo-ranges with higher-elevation angles

Why is that often a good idea? Satellites with high-elevation angles are closer to the receiver, has to travel through a smaller cross-section of the ionosphere & troposphere and is less prone to multipath effects.

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$$w = [w_1 \; w_2 \; \dots \; w_n]^T$$

Is incorporated into the solution as;

$$\Delta x = (diag(w) \cdot A) \backslash (diag(w) \cdot z)$$

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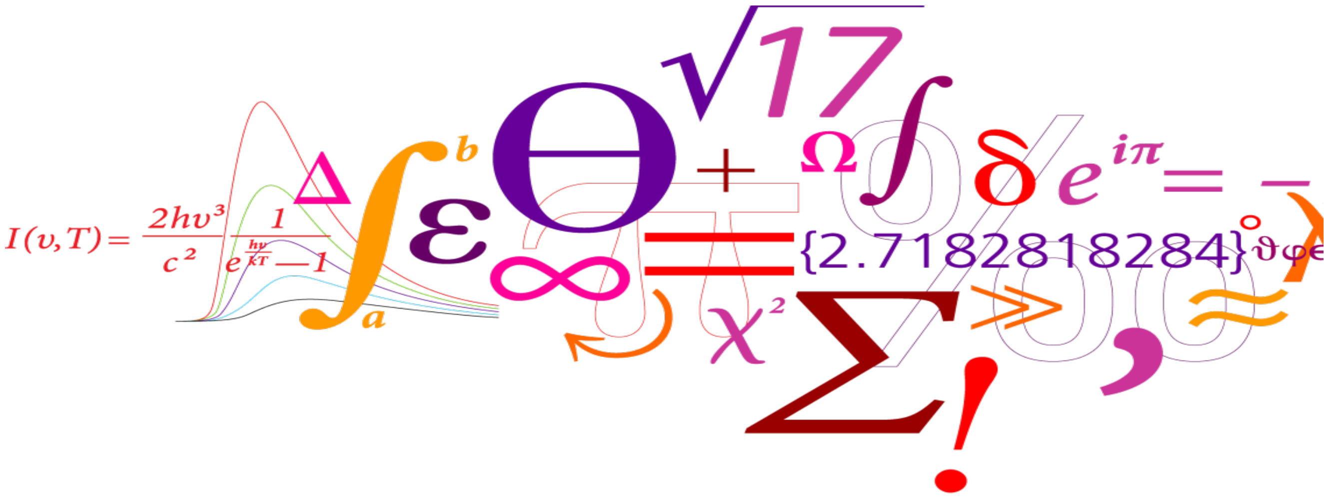
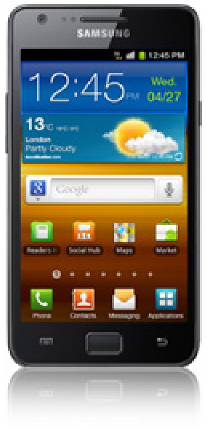
Examples of GNSS applications



Conventional GPS positioning

- GPS in mobile phones
- Might be assisted with other information; this is called A-GPS
- Personal services
- Where is the closest bar, how do I get there, and when is happy hour?



Photos from www.apple.com and www.samsung.dk

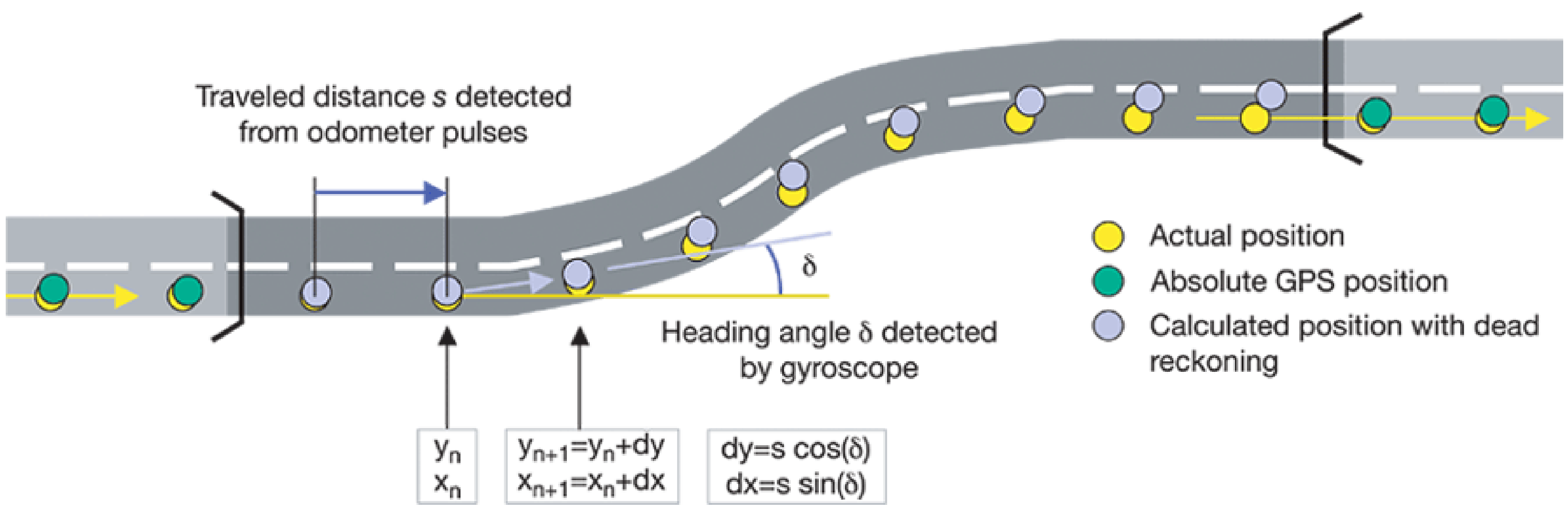
Vehicle navigation and fleet managemet

- Car navigation
  - Units composed of GPS, digital map and perhaps also real time traffic information from external source
  - Used by private and professionals

- Fleet management
  - Car navigation system connected via data link to the order, storage and/or management systems of the company
  - Used by for instance:
    - Taxi companies, ambulance service providers etc.
    - Shipping and transportation companies like UPS, DSV etc.

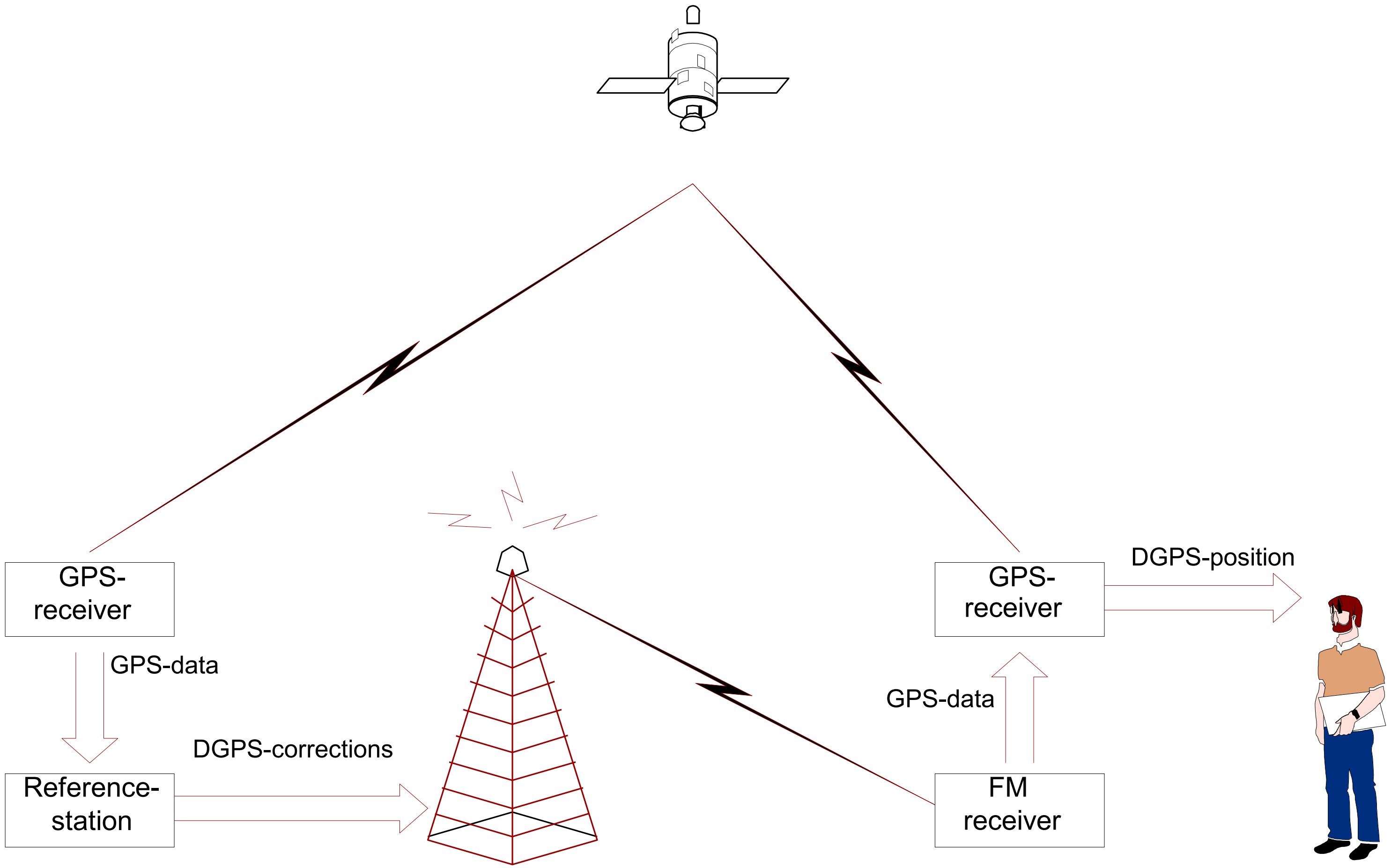
Illustration from firmabilen.dk

Internal aiding, dead reckoning



”Dead reckoning” navigation units acts as a backup solution if GPS satellites are missing for instance in tunnels

DGPS (Differential GPS)



Air navigation

- GPS is being used for more and more phases of flight. Augmentation is needed to enhance safety.

- Accuracy required: from meter en route, to cm at landing

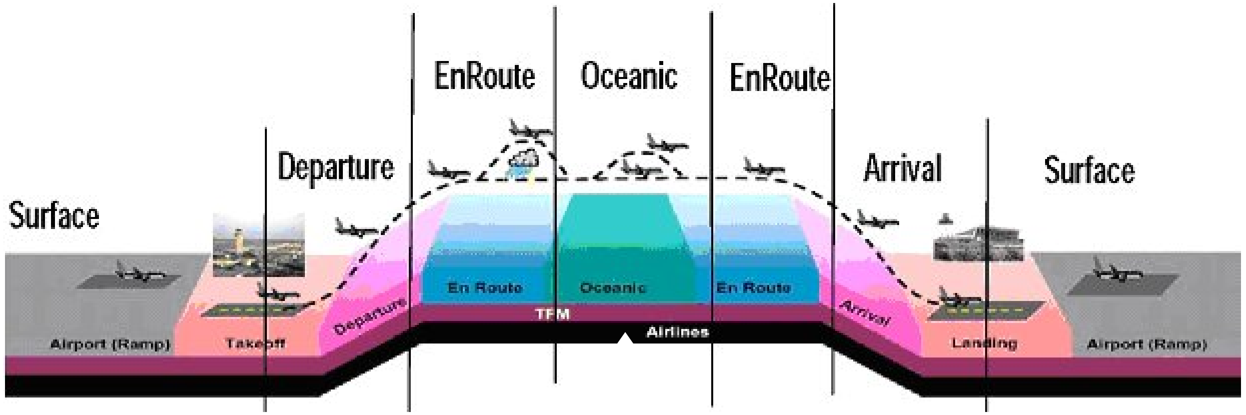


Figure from: atc-network.com

Land surveying

- Requirement: Position accuracy at the 1–10 cm level
- Use of so-called RTK technique (Real Time Kinematic)
- Same technique used for precision farming and for machine guidance in construction work, open mining etc.

- Figure: http://ciscokidz.com/index.htm

Sports and training

- The RTK-technique is also used for performance optimization of high level athletes

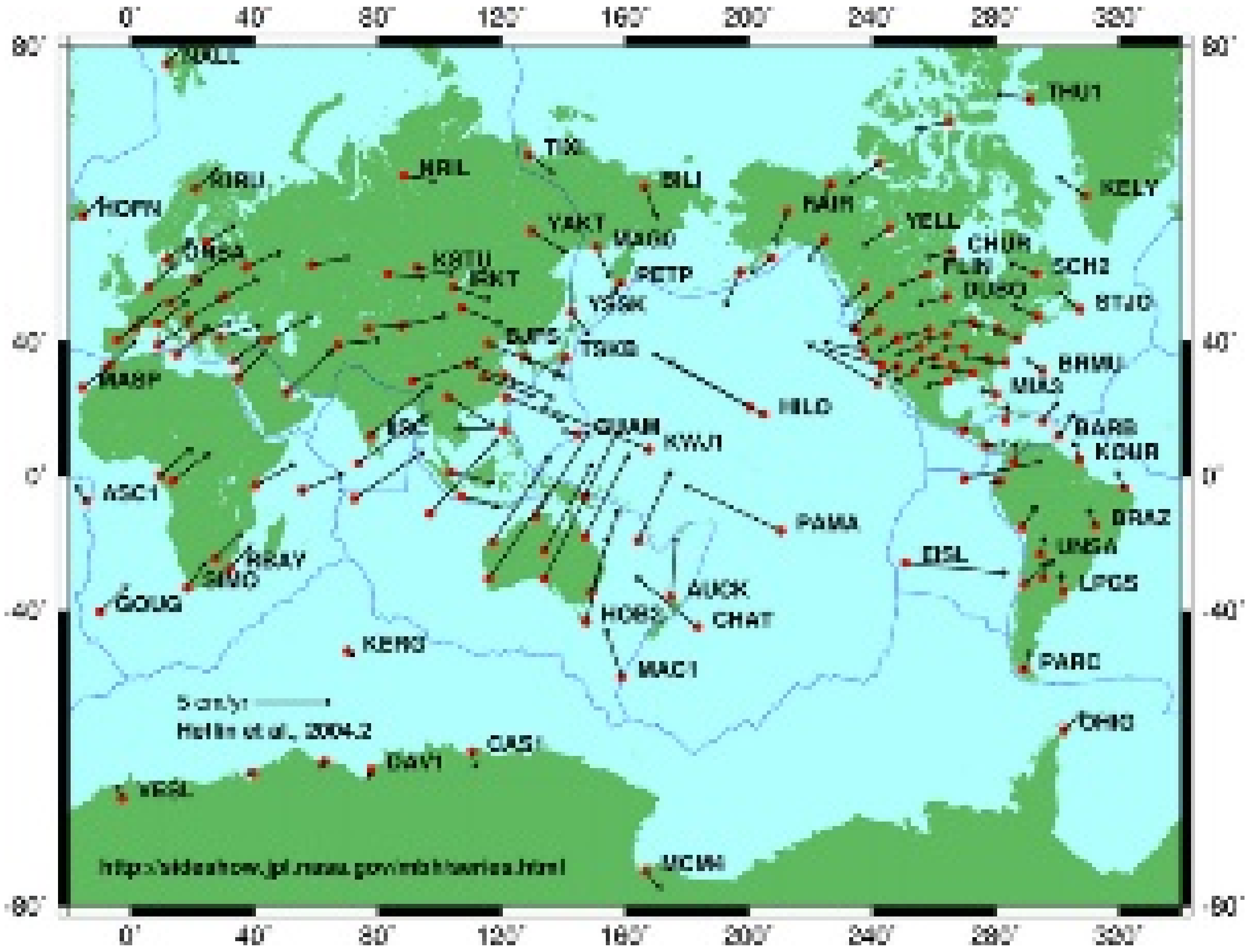
Figure: www.gpsworld.com

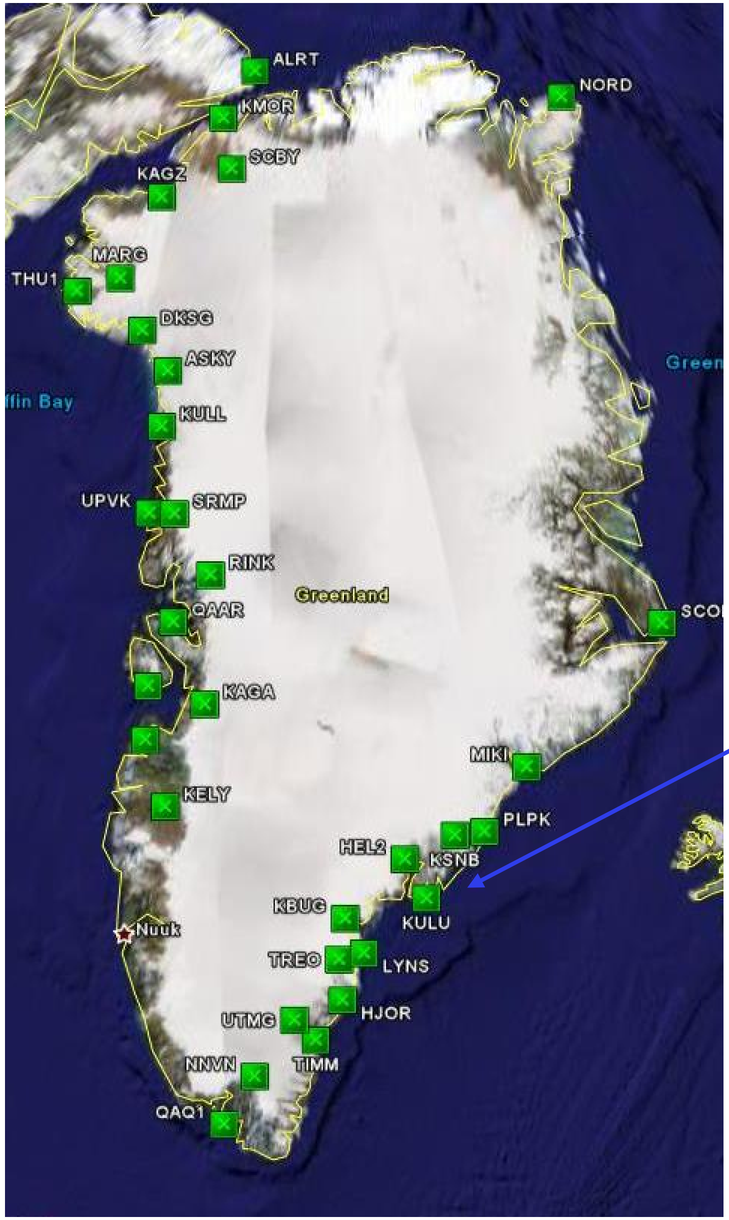
Geodynamics

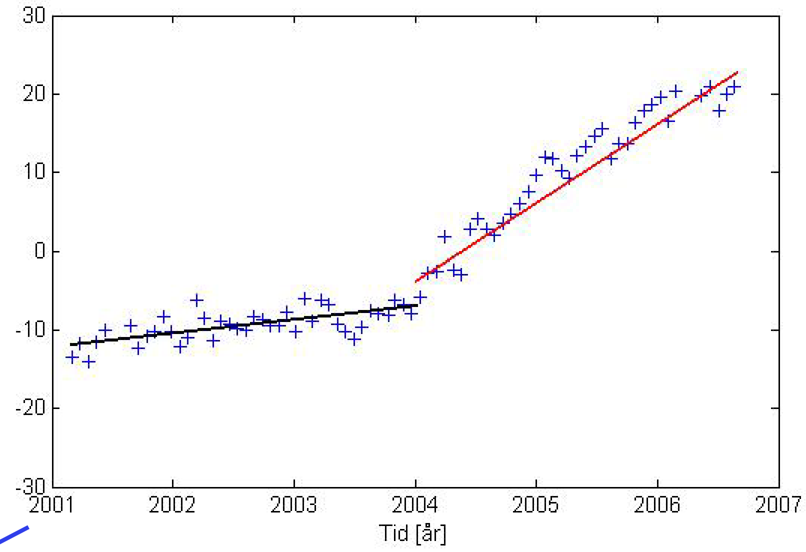
- Movements in the Earths crust
- Permanent GPS reference stations can provide positions with accuracy of a few mm

Figure from: Danish Geodata Agency

Geodynamics – Global Movements







Uplift acceleration due to mass loss (in mm)

GPS Applications - Accuracy

Real time

- Absolute code positioning, 5 - 10 meter
- DGPS, code differential positioning, 1 – 5 m
- RTK, phase differential positioning, 1 - 5 cm

Post processing

- Kinematic, phase differential, 1 – 10 cm
- Static, phase differential, < 1 cm

Military use of GPS

- GPS is a very important tool in the modern high-technology warfare

- GPS is for instance used for military navigation, missile guidance, and communication

Large scale operations

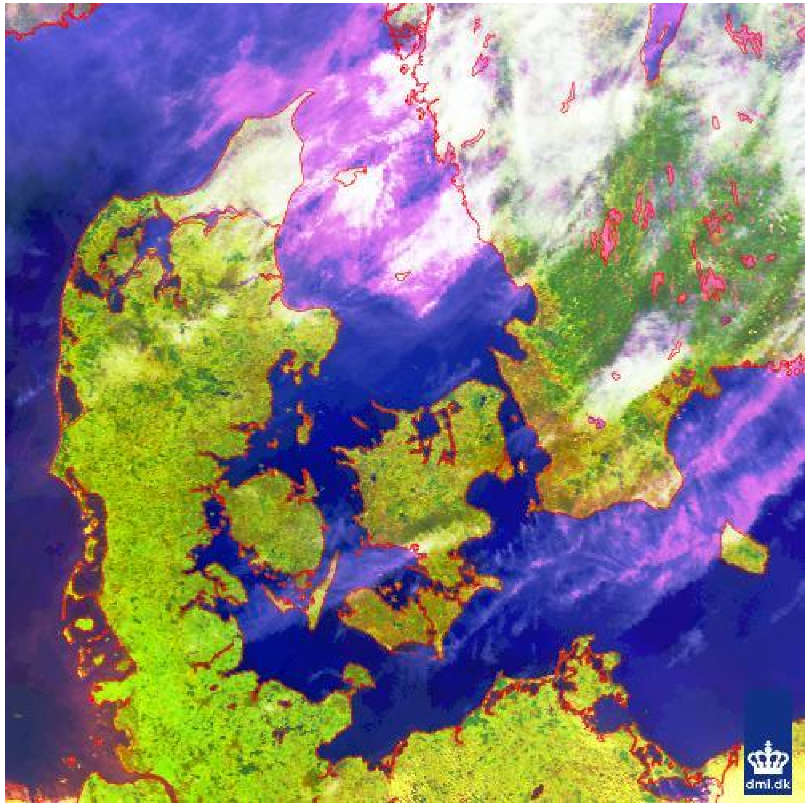
- In coordinated actions and large scale operations different positioning techniques are combined. For instance for:
  - Emergency and disaster management
  - Environmental monitoring
  - War scenarios

- GPS is just one tool along among many other sensors and communication techniques, and they must all work together

- Research goal is to obtain seamless indoor-outdoor high accuracy positioning for this type of operations

Other Applications of GPS

- Surveillance of activities in the ionosphere
- Modeling of water vapor content and distribution in the atmosphere => improved weather predictions
- Timing and time tagging



Assignment 4

Given

- Pseudoranges, ρ, and Satellite Positions

x_0 = [0 0 0 0]; % Initially assume position to be center of earth and receiver clock delay of 0

While(iter < itermax && max(abs(dx)) > threshold)

{

Update estimates of geometric distance:

$$\rho_0^i = \sqrt{(X^i - X_0)^2 + (Y^i - Y_0)^2 + (Z^i - Z_0)^2}$$

Form measurement vector, z

$$z = \rho - \rho_0 - c\delta t_{r,0}$$

Form A matrix according to slide 29,

Solve LS problem (Matlab notation),

$$\Delta x = A \backslash \mathrm{z}$$

Updated previous estimates

$$X_0 = X_0 + \Delta x(1); \quad Y_0 = Y_0 + \Delta x(2); \quad Z_0 = Z_0 + \Delta x(3); \quad \delta t_{r,0} = \delta t_{r,0} + \Delta x(4)$$

end