Before we start:

If you feel ill, go home
Keep your distance to others
Wash or sanitize your hands
Disinfect table and chair
Respect guidelines and restrictions

30552 Satellite Geodesy – E20

Lecture 4
Observation concepts used in satellite geodesy

by
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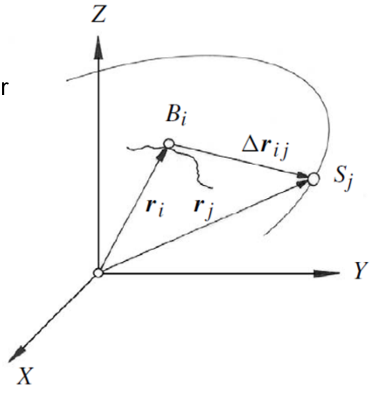
Outline

- Satellite geodesy as a parameter estimation problem
  - Introduction to least squares adjustment
- Observables and concepts
  - Directions
  - Ranges
  - Range differences (Doppler observations)
  - Range rates
  - Interferometric measurements
  - Synthetic Aperture Radar (SAR)
- Assignment 3

Satellite geodesy as a parameter estimation problem

- The fundamental problem:
  - We have a satellite, S_j
  - We want information about something related to the Earth or on the Earth, B_i
  - We can have many satellites, denoted by $_j$, and many observation points on the Earth, denoted by $_i$
- The observations are given in the vector between the observation point and the satellite, $\Delta\mathbf{r}_{ij}$
- The parameters we want to estimate are given in the position vectors $\mathbf{r}_i$ and/or in $\mathbf{r}_j$
- In some cases both (precise) location of the observation point, B and (precise) location of the satellite, S are unknown

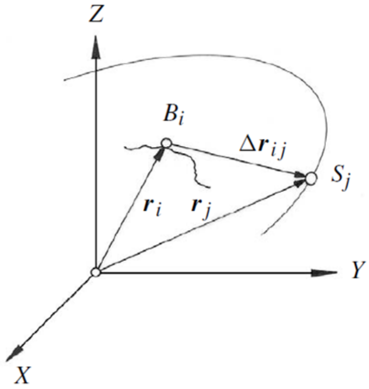


Satellite geodesy as a parameter estimation problem

- General observation model:

$$\mathbf{L} + \mathbf{v} = \mathbf{\Phi}(\mathbf{X})$$

- Where:
  - **L** is vector with observations e.g. ranges
  - **v** is vector of residuals i.e. unmodelled components of the estimation process e.g. observation errors or noise
  - **X** is vector of unknown parameters e.g. coordinates
  - **Φ** is nonlinear vectorial function



Satellite geodesy as a parameter estimation problem

- The model can be linearised when approximate values, $\mathbf{X}_0$ are introduced for the unknown:

$$\mathbf{L}_0 = \mathbf{\Phi}(\mathbf{X}_0)$$

- Then we have vector of residual observations: $\mathbf{l} = \mathbf{L} - \mathbf{L}_0$
- And a vector of residual parameters: $\mathbf{x} = \mathbf{X} - \mathbf{X}_0$
- The linear form of the observation model **L** + **v** = **Φ(X)** is therefore:

$$\mathbf{l} + \mathbf{v} = \mathbf{A}\mathbf{x}$$

- Where A is a design matrix with partial derivatives of the observations with respect to the parameters, determined around the approximate point of expansion, $\mathbf{X}_0$

Satellite geodesy as a parameter estimation problem

- Examples of parameters (or unknown) in satellite geodesy:
  - Parameters describing the motion of the observation station including e.g.
    - Geodynamic parameters, Earth rotation, polar motion
  - Parameters describing the satellite motion e.g.
    - Parameters describing the perturbing forces on the satellite orbits
  - Parameters influencing the observations e.g.
    - Atmospheric effects on the satellite signals

- It is not possible to provide a complete or “correct” solution to these problems. We therefore talk about an *estimation* problem.

- Depending on the problem we want to solve, some parameters are more important than others
  - Less important parameters are considered “known” with a given un-certainty

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Satellite geodesy as a parameter estimation problem

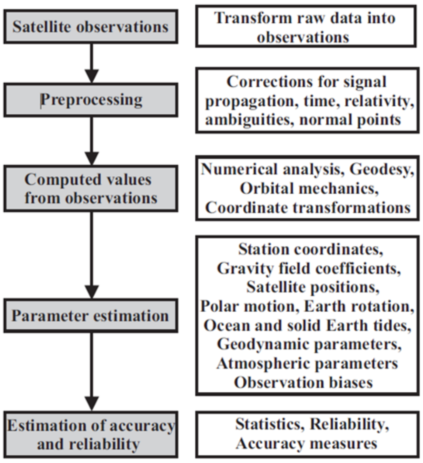

Figure from: Seeber, G. *Satellite Geodesy*. 2nd edition, 2003. Walter de Gruyter

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Principle of least squares

- Consider a number of observations or measurements, l:

$$l_1, l_2 \ldots l_n$$

- Think about the observations as a sum of the ”true” value, L and a residual (an error or some noise), v:

$$\begin{aligned} l_1 &= L + v_1 \\ l_2 &= L + v_2 \\ &\vdots \\ l_n &= L + v_n \end{aligned}$$

Principle of least squares

- Now find the value for L which minimises the sum of the square of the residuals:

$$\begin{aligned} v_1^2 &= (l_1 - L)^2 \\ v_2^2 &= (l_2 - L)^2 \\ &\vdots \\ v_n^2 &= (l_n - L)^2 \end{aligned}$$

- Find L so:

$$\min \left[\sum_{i=1}^{n} v_i^2 \right]$$

Least squares adjustment

- Using matrix notation the problem can also be described as:

$$\mathbf{l} = \mathbf{A}\mathbf{x}$$

- Where:
  - **l** is a vector with the observations
  - **x** is a vector with the unknowns (e.g. the L on the previous slide)
  - **A** is a design matrix which maps the unknown onto the observations

- Adding the residuals, **v**, the expression becomes:

$$\mathbf{l} = \mathbf{A}\mathbf{x} + \mathbf{v}$$

- Note: A weight matrix **P** should also be included but is omitted here for simplicity

Least squares adjustment

- Given the observation equation:

$$\mathbf{l} = \mathbf{A}\mathbf{x} + \mathbf{v}$$

- We want to find the best solution according to the least squares principle, namely the solution which minimises the residual (or the influence of the noise and errors in our observations):

$$\min\left[\mathbf{v}^T\mathbf{v}\right]$$

- Rewriting:

$$\min\left[(\mathbf{l} - \mathbf{A}\mathbf{x})^T(\mathbf{l} - \mathbf{A}\mathbf{x})\right]$$

Least squares adjustment

- The minimum of a function can be found by setting the first derivative to zero:

$$\frac{d\left(\mathbf{v}^T\mathbf{v}\right)}{d\hat{\mathbf{x}}} = \mathbf{A}^T\mathbf{A}\hat{\mathbf{x}} - \mathbf{A}^T\mathbf{l} = 0$$

- The derivation is carried out with respect to a new vector $\hat{\mathbf{x}}$ which is an estimate of **x**
- The estimate of **x** providing the minimum sum of the squared residuals is then given as:

$$\hat{\mathbf{x}} = \left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T\mathbf{l}$$

Least squares adjustment

- When $\hat{\mathbf{x}}$ has been estimated, the residuals can be estimated as well:

$$\mathbf{v} = \mathbf{l} - \mathbf{A}\hat{\mathbf{x}}$$

- The residuals in the **v** vector provides information on the quality of the solution; smaller residuals indicates a better solution, or in other words, a better internal consistency between the observations

- Finally, the cofactor matrix of the parameters, $\mathbf{Q}_{\hat{x}}$ can be useful in some cases e.g. with GPS:

$$\mathbf{Q}_{\hat{x}} = \left(\mathbf{A}^T \mathbf{A}\right)^{-1}$$

Weighting of observations

- In these slides, all observations are given an equal weight, they are all considered equally important

- Normally, different weights are introduced in the estimation process, to give good observations a higher weight and poor observations a lower weight

- Weights can be set based on:
  - Satellite elevation angle relative to the horizon at the observation point
  - Signal to noise ratio (S/N) of received signal from satellite
  - Quality of the data logging equipment
  - Un-certainty estimate from pre-processing
  - Etc.

Weighting of observations

- The weights are introduced in a weight matrix, P, so:

$$\hat{\mathbf{x}} = \left(\mathbf{A}^T\mathbf{P}\mathbf{A}\right)^{-1}\mathbf{A}^T\mathbf{P}\mathbf{l}$$

- Where P is a diagonal matrix:

$$\mathbf{P} = \begin{bmatrix} P_{11} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & P_{nn} \end{bmatrix}$$

- and the element $P_{ii} = p_i$ which is the weight of observation i

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Covariance matrix

- Considering also the weight matrix, the cofactor matrix becomes:

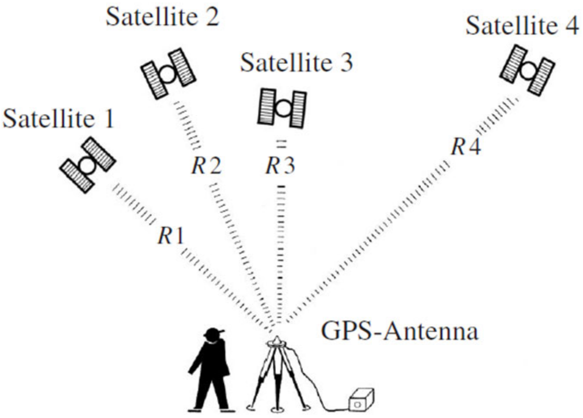
$$\mathbf{Q}_{\hat{x}} = (\mathbf{A}^T\mathbf{P}\mathbf{A})^{-1}$$

- The a posteriori covariance matrix: $\boldsymbol{\Sigma}_{\hat{X}} = \hat{\sigma}_0{}^2\mathbf{Q}_{\hat{x}}$

- where $\hat{\sigma}_0{}^2 = \frac{\hat{\boldsymbol{v}}^T\boldsymbol{P}\hat{\boldsymbol{v}}}{r-u}$ is the a posteriori standard error of unit weight
  - r is the number of observations
  - u is the number of parameters or unknown

Example

- Application of least squares adjustment to find an estimate of position using conventional absolute code-based GPS positioning
- On the board…



Least squares adjustment

- Least squares adjustment is not the only estimation method used in satellite geodesy, but it is used a lot
- The challenge is to establish the observation equations for the relevant problem. In many cases these can be found in the literature e.g. in books or scientific journal papers
- An introduction to least squares adjustment is for example:
  - A. A. Nielsen: “Least Squares Adjustment: Linear and Nonlinear Weighted Regression Analysis”, DTU Compute: https://www2.imm.dtu.dk/pubdb/edoc/imm2804.pdf

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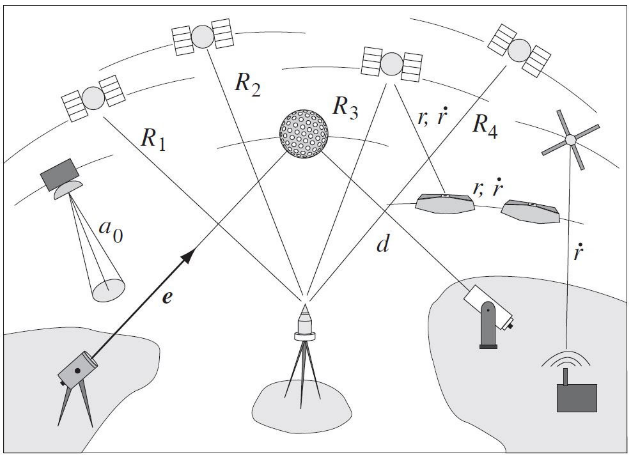
- Observables and concepts
  - Directions
  - Ranges
  - Range differences (Doppler observations)
  - Satellite altimetry
  - Range rates
  - Interferometric measurements
  - Other techniques
- Assignment 3

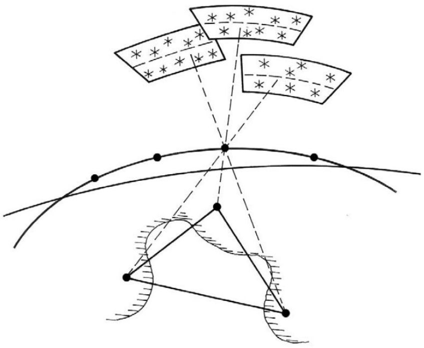
Observation methods in satellite geodesy

- Brief introduction to the observation methods
- More details on most of the methods provided during the following lectures of this course
- Figure from: Seeber, G. *Satellite Geodesy*. 2nd edition, 2003. Walter de Gruyter
- Same source used for illustrations in the following slides, unless otherwise noted



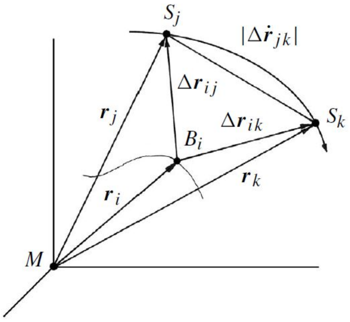
Directions as observables in satellite geodesy

- Directions, or angles, in satellite geodesy are often measured or determined between:
  - Different astronomical objects (stars) and satellites
    - E.g. for orbit determination of a satellite
  - A space object and different ground stations
    - E.g. for geodetic reference networks and reference frame realisation
- Relatively low accuracy compared to other methods



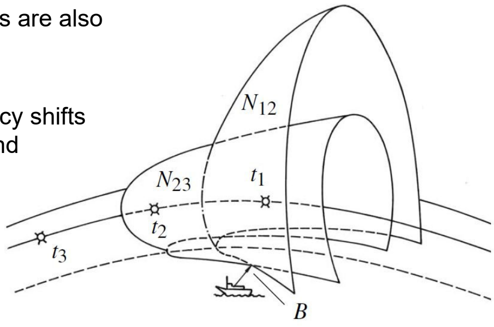
Ranges as observables in satellite geodesy

- Determination of ranges, or distances, is normally done using the transmission time of an electromagnetic signal between transmitter and receiver
  - One-way as e.g. for GPS where transmitter is on the satellite and receiver is on the ground
  - Two-way, out and return, e.g. for radar and laser where transmitter and receiver are on the same platform, and the satellite moves between transmission and reception (time j to time k in the figure)
- Transmission time is converted to distance considering the signal propagation velocity (approximated by the speed of light) and corrections of time and clock offsets and/or drifts
  - Atmospheric corrections most often also apply (more in Lecture 7)



Range difference observables in satellite geodesy

- Performing observations based on range differences are also referred to as the Doppler method
- Range differences are determined from the frequency shifts caused by the change of range between receiver and transmitter
  - In figure, one transmitter is used. It is moving with the satellite from t_1 to t_3



- The Doppler method is used e.g.
  - with GPS to determine velocity of objects on the ground
  - with the DORIS system to determine positions of satellites
  - with satellite-to-satellite tracking (SST) where changes in the distance between satellites is of relevance e.g. with the GRACE and GRACE-FO missions

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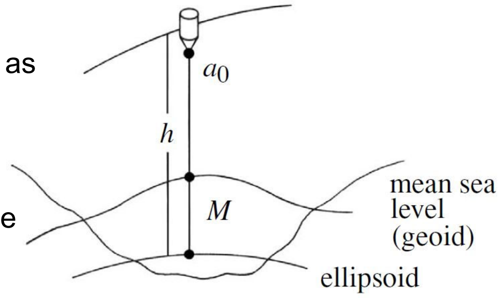
Satellite altimetry in satellite geodesy

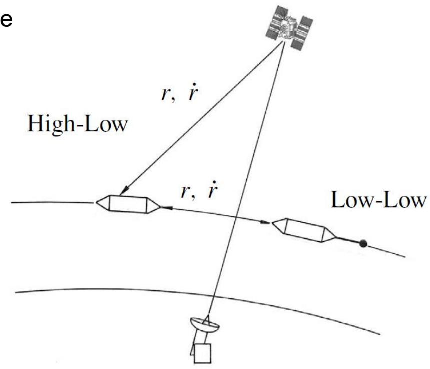
- Satellite altimetry is a special case of range observations between satellite and a reflection surface on the Earth
  - In particular used with the ocean surface as target, or reflection surface, but ice can also be used as reflector



- Used for modelling of sea level, sea surface topography, ice coverage etc.
- Very important observation method in satellite geodesy. Treated in more detail later in the course (Lecture 8+)

Range rates as observables in satellite geodesy

- Observations of both ranges and range rates (relative velocity) between two satellites is frequently used for satellite based gravimetry today
- Two different principles:
  - High-low observations, where range rates are determined between satellites in MEO or GEO orbit and satellites in LEO orbit
    - E.g. with the CHAMP satellite in LEO orbit and GPS satellites in MEO orbit
  - Low-low observations where range rates are determined between satellites in LEO orbits
    - E.g. with the GRACE and GRACE-FO missions
- More in Lecture 12



Range rates used with satellite gravimetry

- Further, with satellite-to-satellite tracking (SST):
  - The low-low technique can be supported by GPS as well, but the main observable is the observation between the two LEO-satellites
    - E.g. GRACE
  - The high-low technique can be supported by other observations like gravity gradient observations performed with a gradiometer
    - E.g. GOCE

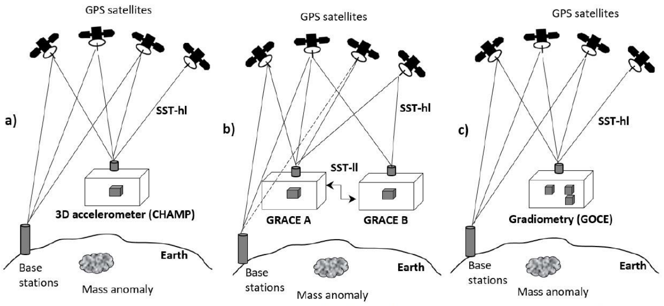
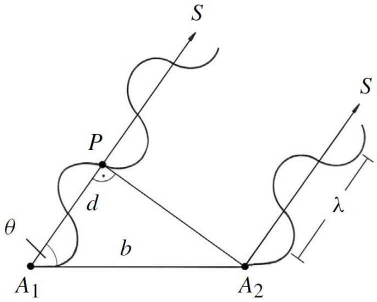


Figure 2.1. Illustrates general concepts of a) high-low satellite-to satellite tracking (SST-hl), b) high-low/low-low satellite-to-satellite tracking (SST-hl/SST-ll) and c) high-low satellite-to-satellite tracking/satellite gravity gradient mode (SST-hl/SGG).

N.A. A. Gido, "*Monitoring lithospheric motions by satellite geodesy*", Ph.D. thesis, KTH, 2020

Interferometric observables in satellite geodesy

- Here two (or more) antennas must be observing signals from the same satellite (or radio frequency source). The distance between the antennas must be much smaller than the distance to the satellite
- The observable is a phase difference which is converted to distance (d) with knowledge of the wavelength of the signal
- Used for instance with:
  - GPS for high accuracy differential positioning where A_1 and A_2 are located 20 – 100 km apart
  - Very Long Baseline Interferometry (VLBI) where S are national radio sources in space (e.g. quasars) and A_1 and A_2 are located on different continents (e.g. Europe and North America)



SAR - Synthetic Aperture Radar (1)

- SAR was originally not considered a method for satellite geodesy. But it is used increasingly for estimation of e.g. height differences of the surface of the Earth
- Radio Detection and Ranging (RADAR) provides observables of signal travel time (i.e. range) and signal strength
- RADAR does not work with satellites because receiving antenna (the aperture) must be very large. But with SAR the aperture is created synthetically by collecting the reflected signals as the satellite travels along the satellite track

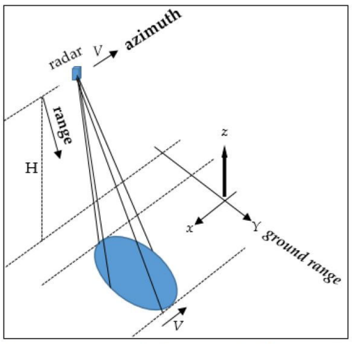


Figure 4.1. Acquisition geometry of SAR system. H and V are the flying height and velocity. The side looking system illuminates a certain area on the ground (blue part).

N.A. A. Gido, "*Monitoring lithospheric motions by satellite geodesy*", Ph.D. thesis, KTH, 2020

SAR - Synthetic Aperture Radar (2)

- SAR Interferometry (InSAR or SAR-in) refers to observations of the same area at the ground from two different satellite locations
  - InSAR is a good tool for mapping of heights and e.g. for elevation models of the surface of the Earth
- Differential SAR Interferometry (DInSAR) refers to observations of the same area at the ground from different satellite passes separated in time
  - DInSAR is a good tool for mapping changes in height e.g. after an earthquake or to monitor land slides or land subsidence
- More in Lecture 11

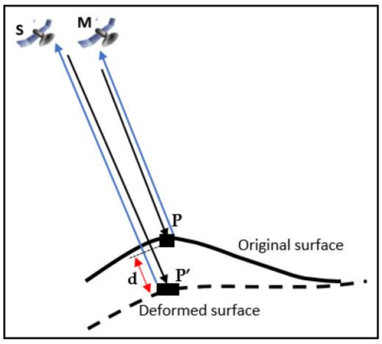


Figure 4.2. Principle of DInSAR deformation measurement. M denotes for image acquired at time (t1) for ground target P (before deformation), S denotes for image acquired at time (t2) for target P′ (target P after deformation). d is the change in the measured range due to surface deformation.

N.A. A. Gido, "*Monitoring lithospheric motions by satellite geodesy*", Ph.D. thesis, KTH, 2020

Examples of satellite/missions used in satellite geodesy

- Following the structure of the book, the next chapter is a presentation of some satellites and missions applied in satellite geodesy (chapter 4.3)
- In this course, you as students will do this, by presenting one satellite mission each next week as part of Assignment 3

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  - ✓Range rates
  - ✓Interferometric measurements
  - ✓Synthetic Aperture Radar (SAR)
- Assignment 3

Assignment 3

Characterise and describe the satellite/mission/constellation considering the following:

- Purpose
  - Briefly describe the overall purpose of the satellite/mission/constellation
  - Mention who has developed or paid (often this is ESA, NASA, DLR etc.)
- Satellite orbits
  - Briefly describe the type of orbit, most characteristic Kepler elements and other factors you find are relevant regarding the orbits
- Satellite design and payload
  - Briefly describe the satellite and the most important payload (i.e. instruments on the satellite)
- Data and results
  - Briefly describe the most important type of data generated by the satellite/mission/ constellation
  - Briefly describe some of the results obtained using the data

Assignment 3

Presentation:

- The description should be provided for the rest of the class with a 10-minute presentation on Monday September 28th.
- When preparing the presentation consider a logical structure of the presentation, consider how much information to include (not too little, not too much), and consider the **10 minutes** allocated for the presentation.

Assignment 3

Hand in:

- After the presentation you must hand in the slides used for the presentation in the pdf-format.
- The slides can be prepared in any software you want, but the hand-in must be in pdf.
- You can also use the black-board during the presentation. If you do so, your notes for what to write on the board should also be handed in
- NB: Be prepared to do the presentation online if Covid pre-causions tell us to do so

Evaluation:

- The presentation will be marked considering the following:
  - Content of the presentation i.e. correctness of the information provided and whether all topics in the list above are covered
  - Structure and clarity of the presentation i.e. whether the presentation is made in a way so it is possible to understand the information provided
  - Timing of the presentation, i.e. whether the presentation is close to 10 minutes

Before we leave:

Disinfect table and chair
Maintain your distance to others
Wash or sanitize your hands
Respect guidelines and restrictions outside