Before we start:

If you feel ill, go home
Keep your distance to others
Wash or sanitize your hands
Disinfect table and chair
Respect guidelines and restrictions

30552 Satellite Geodesy – E20

Lecture 3
Orbit perturbations and orbit determination

by
Anna B. O. Jensen, DTU Space

Outline

- M and E from second lecture
- GPS time from first lecture

- Orbit perturbations
- Perturbations caused by Earth' gravity field
- Perturbations caused by other forces
- Orbit determination
- Example: Precise orbits for GPS
- Types of satellite orbits: LEO, MEO, GEO, IGSO, HEO
- Assignment 3

Angles to describe satellite motion – from lecture 2

- When:
  t is current time, and t0 is time at perigee crossing
- Mean motion: $n = \sqrt{\mu}\, a^{-3/2}$ unit: radians/sec
- Mean anomaly: M(t) = n·(t – t0)
- Eccentric anomaly: E(t) = M(t) + e·sin(E(t))
- True anomaly: $v(t) = \arctan\left[\frac{\sqrt{1-e^2}\sin(E)}{\cos(E)-e}\right]$

Iterative solution of E from M

- When computing satellite coordinates, often Kepler's equation for the eccentric anomaly must be solved. The following iterative solution can be applied for computing E at time k given that M at time k is known:

$$E_0 = M_k$$

$$E_i = M_k + e\sin(E_{i-1}),$$ where i = 1,2,3 … is the iteration counter

The iteration can be stopped if:

$|E_i - E_{i-1}| < \varepsilon,$ where ε is a small number.

- One suggestion is to stop the iterations at $\varepsilon = 1e\text{-}13$. Usually four or five iterations are enough to reach this threshold

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GPS time (1)

- For GPS-positioning it is crucial with a homogeneous *and* continuous time scale. Therefore a new time scale was defined to be used for GPS, called *GPS time*

- GPS time is defined as UTC but without the leap seconds
  - GPS time = UTC time at January 6th, 1980, @ 00:00 hours
  - GPS time in 2020 equals UTC + 18 seconds + a few nanoseconds

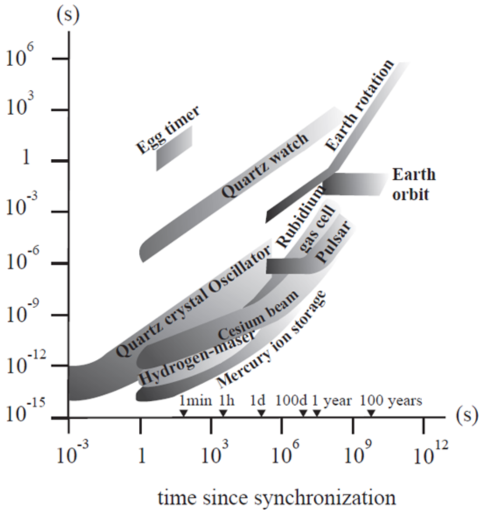
- GPS time is maintained by the US Naval Observatory http://www.navcen.uscg.gov

GPS time (2)

- A time epoch given in GPS time is referenced as:
  - Week number, counted since January 6th, 1980
  - Number of seconds within the week (counter is reset at midnight between Saturday and Sunday, i.e. from 604800 to 0 seconds)
- Current GPS week is: 2123
- For some applications, GPS time is also given as:
  - Week number, day of week, seconds of day

Stability of clocks and frequency standards

- A precise and well defined time reference, only makes sense if we have clocks sufficiently good to "keep the time"
- For example for GPS:
  An error of 1 microsecond in the GPS receiver clock induces an error of 300 meter in the range estimate to the satellite
- Estimation and modelling of both satellite and receiver clock errors is therefore important for geodetic applications
- Figure: G. Seeber, "Satellite Geodesy", 2003



GPS satellite clock error (or instability)

- GPS satellites are equipped with two (or three) rubidium and two cesium atomic clocks that are monitored by the GPS control stations

- The current size of the satellite clock errors are between 2 and 750 microseconds

- Satellite clock errors are modelled by polynomials of second order. Coefficients are transmitted to GPS users via the navigation message from the satellites

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GPS satellite clock error model (1)

- Satellite clock correction model valid at time t:

$$\delta t = a_{f0} + a_{f1}(t - t_0) + a_{f2}(t - t_0)^2 + \Delta t_r$$

- Where:
  - t_0 is the reference time epoch
  - a_{f0} is coefficient for clock offset (seconds)
  - a_{f1} is coefficient for fractional offset (sec/sec)
  - a_{f2} is coefficient for clock drift (sec/sec^2)
  - Δt_r is related to the relativistic effect

- When the model of satellite clock correction is applied to all satellites, their clocks are synchronized to within 5-10 nanoseconds

Practical considerations

- For many applications it is important to remember the difference between especially UTC and GPS time. For instance:
  - When logging GPS data to a PC which runs on UTC time
  - When integrating GPS with other sensors such as cameras, laser scanners or INS equipment that might be referenced to UTC

- Example:
  - An airplane, with a speed of 500 km/h, will move 2,500 meters in 18 seconds
  - When not correcting for the time offset between UTC and GPS time, an equivalent error in the position is introduced

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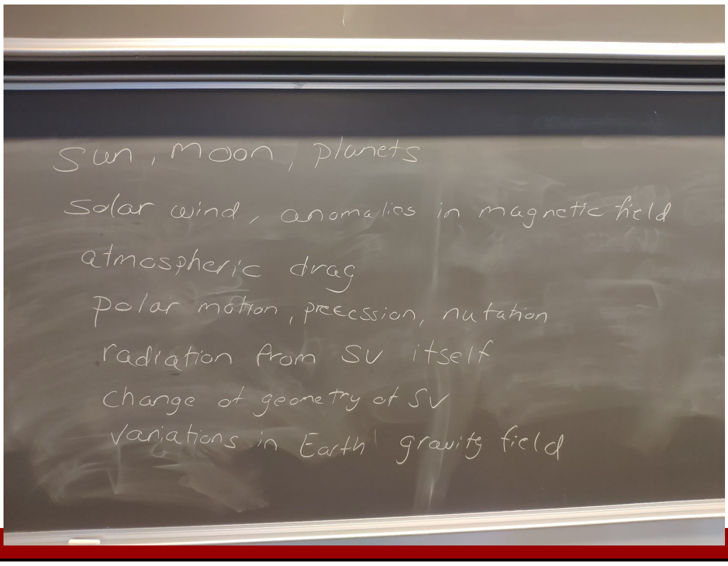
Perturbations of satellite orbits

- Kepler's laws and the theory of the elliptical motion provides a mathematical model for description of the motion of a satellite in its orbits
- This would be true for all satellites if the Earth was a point mass, and if no other forces than Earth's gravity were affecting the satellites
- But there are other forces affecting the satellite motion, and the Earth's gravity field can not be modelled as a point mass
- Remember discussion from last week …

Factors causing perturbations - from last lecture



Sun, moon, planets
solar wind, anomalies in magnetic field
atmospheric drag
polar motion, preecssion, nutation
radiation from SV itself
change of geometry of SV
variations in Earth' gravity field

Perturbed satellite motion (1)

- The largest perturbing forces are caused by:
  - Non-spherical shape of Earth's gravity field
  - Gravitation of Sun and Moon – with variations
  - Variations in Earth's gravity field
  - Solar radiation pressure
  - Ocean and solid Earth tides

- These forces cause perturbations of the satellite orbit and they are, in general, difficult to model
  - ⇒ Satellite positions can be predicted, but with a limited accuracy
  - ⇒ Post processing provides better accuracy

Perturbed satellite motion (2)

Perturbation	Effect on satellite acceleration m / s^2
Deviation of earth gravity field from a sphere	$5 \cdot 10^{-5}$
Variations in earth gravity field	$3 \cdot 10^{-7}$
Solar and lunar gravitation	$5 \cdot 10^{-6}$
Earth and ocean tides	$1 \cdot 10^{-9}$ each
Solar radiation pressure	$1 \cdot 10^{-7}$
Albedo	$1 \cdot 10^{-9}$

From: Seeber, G. (2003). *Satellite Geodesy*. 2nd edition. Walter de Gruyter

Perturbed satellite motion (3)

- Effect on a GPS satellite after 4 hours

Kepler Element	Deviation of Earths gravity field from a sphere	Variations in Earth gravity field	Gravitation of sun and moon	Solar radiation pressure
a	2600 m	20 m	220 m	5 m
e	1600 m	5 m	140 m	5 m
i	800 m	5 m	80 m	2 m
Ω	4800 m	3 m	80 m	5 m
ω + M	1200 m	4 m	500 m	10 m

From: Seeber, G. (2003). *Satellite Geodesy*. 2nd edition. Walter de Gruyter

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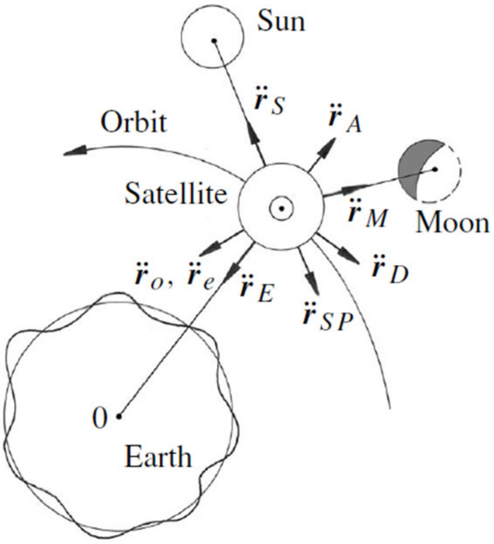
Perturbed satellite motion (4)



From: Seeber, G. *Satellite Geodesy*. 2nd edition, 2003. Walter de Gruyter

Relation between perturbing forces and Kepler elements

Two approaches:

1. Lagrange's perturbation equations
   - Based on relationship between disturbing potential of the perturbing force and Kepler element
   - Relationship established through partial derivatives which are solved through numerical or analytical integration
2. Gaussian form of perturbation equation
   - Formulated at the location of the satellite
   - Based on resolution of perturbing force into three components; 1. perpendicular to orbital plan, 2. perpendicular to radius vector in orbital plane and 3. in direction of radius vector
   - Relationship established through partial derivatives which are solved as above

In the GS-book chapter 3.2 expressions for these relations are provided

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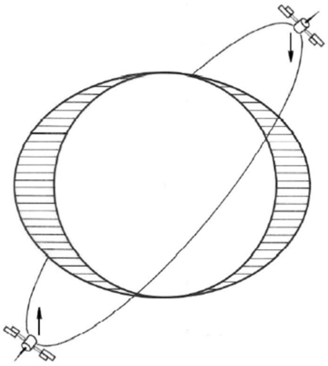
Modelling of Earth's gravity field

- When modelling Earth gravity field there are three parts to cover:
  - Gravity potential caused by a spherically shaped of Earth
  - Gravity potential caused by ellipsoidal shape of the Earth, i.e. the deviation from a sphere
  - Gravity potential caused by variations in the gravity field
    - Localised variations in mass density caused by e.g. heavy granite or light oil in the surface of the Earth

- More on this in the course Physical Geodesy (course no. 30560)



Modelling Earth's gravity field for satellite perturbations

- In Satellite Geodesy, the starting point is the assumption of a spherical homogenous gravity field of the Earth. So we model the gravity field as a *spherical model* with deviations from the sphere caused by the ellipsoidal shape and the local anomalies

- In a non-central force field the potential is given as:

$$V = \frac{GM}{r} + R$$

- Where:
  - GM is gravitational constant and Earth' mass
  - r is distance from centre of mass to evaluation point
  - R is the disturbing potential

- The first part GM/r describes the potential of a homogeneous sphere (also named the Keplerian term)

Spherical harmonics (1)

- For description of the disturbing potential, R, of Earth's gravity field a series expansion with spherical harmonics is beneficial
- See equation 3.110 in the GS-book
- The core is Legendre polynomials, P. In the plot, P_n for degree 1 to 7 is shown (m=0)

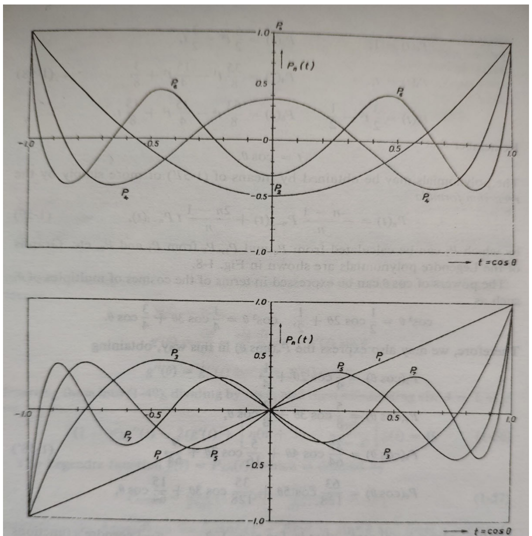


FIGURE 1-8
Legendre's polynomials as functions of $t = \cos\theta$. Top, n even; bottom, n odd.

Figure: Heiskanen and Moritz, *Physical Geodesy*, reprint 1979, TU Graz

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Spherical harmonics (2)

- Harmonics are named based on the order, m, and their geometrical representation of the sphere:
  - Zonal harmonics, m = 0
    - Sphere divided into zones
  - Tesseral harmonics, m ≠ 0
    - Sphere divided into squares or tiles
  - Sectorial harmonics, m = n
    - Sphere divided into sectors

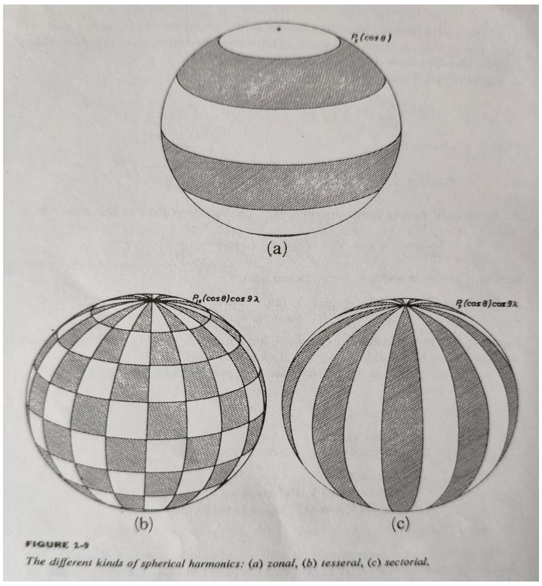
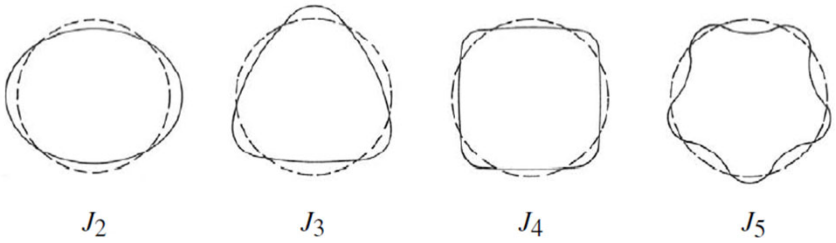


FIGURE 1-9
The different kinds of spherical harmonics: (a) zonal, (b) tesseral, (c) sectorial.

Figure: Heiskanen and Moritz, *Physical Geodesy*, reprint 1979, TU Graz

Spherical harmonics (3)

- In the spherical harmonic series expansion, the harmonic coefficients C_{nm} and S_{nm} are integrals of the mass and describe the mass distribution of the Earth
- Some of the coefficients can be interpreted physically. The variable J is introduced: $J_n = -C_n$
  - $J_2 = -C_2$ corresponds to the Earth flattening
  - J_3 provides a triangular shape, J_4 a quadratic form etc.



From: Seeber, G. *Satellite Geodesy*. 2nd edition, 2003. Walter de Gruyter, chapter 12.2

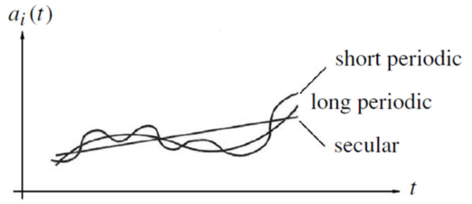
Relation between spherical harmonic coefficients and the Kepler elements

- After some exercises discussed in the GS-book pages 90-93 we can develop expressions for the relation between Kepler elements and the spherical harmonic coefficients used to describe the disturbing potential of the gravity field of the Earth:

$$\begin{aligned}
\frac{d\Omega_{nmpq}}{dt} &= \frac{GMa_e^n F'_{nmp} G_{npq} S_{nmpq}}{\overline{n} a^{n+3}\sqrt{1-e^2}\sin i}, \\
\frac{di_{nmpq}}{dt} &= \frac{GMa_e^n F_{nmpq} G_{npq} S'_{nmpq}}{\overline{n} a^{n+3}\sqrt{1-e^2}\sin i}((n-2p)\cos i - m), \\
\frac{d\omega_{nmpq}}{dt} &= GMa_e^a \left(\frac{\sqrt{1-e^2}}{e} F_{nmp} G'_{npq} - \frac{\cot i}{\sqrt{1-e^2}} F'_{nmp} G_{npq}\right)\frac{S_{nmpq}}{\overline{n} a^{n+3}}, \\
\frac{da_{nmpq}}{dt} &= \frac{2GMa_e^n F_{nmp} G_{npq} S'_{nmpq}}{\overline{n} a^{n+2}}(n-2p+q), \qquad (3.117) \\
\frac{de_{nmpq}}{dt} &= \frac{GMa_e^n F_{nmp} G_{npq} S'_{nmpq}}{\overline{n} a^{n+3} e}((1-e^2)(n-2p+q) - \sqrt{1-e^2}(n-2p)), \\
\frac{d\overline{M}_{nmpq}}{dt} &= \frac{GMa_e^n F_{nmp} S_{nmpq}}{\overline{n} a^{n+3}}\left(2(n+1)G_{npq} - \frac{1-e^2}{e} G'_{npq}\right) + \overline{n}.
\end{aligned}$$

Periodic behaviour of the elements

- Effects of the perturbations can be grouped into having a linear (secular), short or long term periodic behaviour
- Perturbations caused by Earth's gravity field are very important, especially for satellites in low orbit
- Largest secular perturbations are caused by the zonal harmonic coefficient C_{20} i.e. with degree n=2 and order m=0 which is affecting the elements Ω, ω and i
  - Also referred to as the J_2 effect
- Other coefficients cause other types of perturbations which affect the Kepler elements in different ways



Parameter	secular perturbations	long-period perturbations	short-period perturbations
a	-	-	×
e	-	×	×
i	-	×	×
Ω	×	×	×
ω	×	×	×
$\bar{M}$	×	×	×

Size of perturbations cause by Earth's gravity field

- Effect on a GPS satellite after 4 hours – 20 000 km orbit height

Kepler Element	Deviation of Earths gravity field from a sphere	Variations in Earth gravity field
a	2600 m	20 m
e	1600 m	5 m
i	800 m	5 m
Ω	4800 m	3 m
ω + M	1200 m	4 m

From: Seeber, G. (2003). *Satellite Geodesy*. 2nd edition. Walter de Gruyter

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Perturbations caused by Sun and Moon

- Forces caused by the gravitational attraction of the Sun and Moon induced on a satellite motion are significant and must be considered in orbit determination for satellites used in geodesy

- The perturbations caused by Sun and Moon are, however, much smaller than the effects caused by Earth's gravity field
  - Sun and Moon are considered point-masses in the modelling

- Example of acceleration induced on GPS satellite by:
  - the Moon: $5 \cdot 10^{-6}$ m/s^2
  - the Sun: $2 \cdot 10^{-6}$ m/s^2
  - the Planets: $3 \cdot 10^{-10}$ m/s^2

Perturbations caused by Ocean and solid Earth tides

- The Sun and the Moon cause both ocean tide and solid Earth tide effects
- This induces a movement of masses and thereby changes the gravitational potential of the Earth
- For satellites in low orbit, the effect is important to consider in orbit calculations
  - Effect is largest on Ω and i
- Models for ocean and solid Earth tide are used to estimate the mass variations. These are then related to the orbit perturbations through the spherical harmonic model of Earth's gravity field

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Perturbations caused by atmospheric drag

- Atmospheric drag is causing the most important non-gravitational perturbation
- The aerodynamic forces acting on a satellite depend on the satellite geometry, velocity and orientation. Also on the density, temperature and composition of the atmosphere
- The perturbing effect is largest for satellites in low orbit (< 2000 km) where the density of the atmosphere is largest
- Geometrical models of the satellites and models of the atmosphere density and composition are necessary to obtain a good estimation of the perturbing effect
- At high altitudes, solar activity and geomagnetic activity affect the composition of the atmosphere and these factors therefore also contribute to the atmospheric drag
- More on the atmosphere in Lecture 5

Perturbations caused by solar radiation

- Radiation from the Sun affect the satellite directly and also in-directly when it is reflected from the surface of the Earth (the albedo)
- Size of the perturbation effect is dependent on the solar activity (amount of radiation from the sun) and the geometry and surface material of the satellite
- In estimating the perturbation effects, the transition of the satellite between being in direct sunlight and in total shadow of the Earth is a challenge
- In estimating the in-direct radiation effect, surface models of the Earth must be utilized
- In-direct radiation is normally < 10% of the direct solar radiation, for satellites in low orbit

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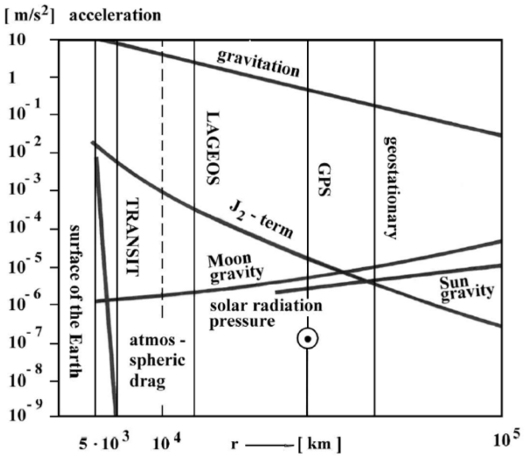
Other effects

- For very precise estimates of satellite orbits also a number of minor perturbing forces must be included such as:
  - Friction by charged particles (ions and electrons) in the upper atmosphere
  - Thermal radiation of the satellite itself
  - Electromagnetic interaction with the geomagnetic field
  - Interplanetary dust
- In case of satellite maneuvers where the satellite is moved or turned in its orbit, the orientation of the satellite will change and this must be considered in the models used to estimate the perturbations

Perturbations for various orbit types



From: Seeber, G. *Satellite Geodesy*. 2nd edition, 2003. Walter de Gruyter

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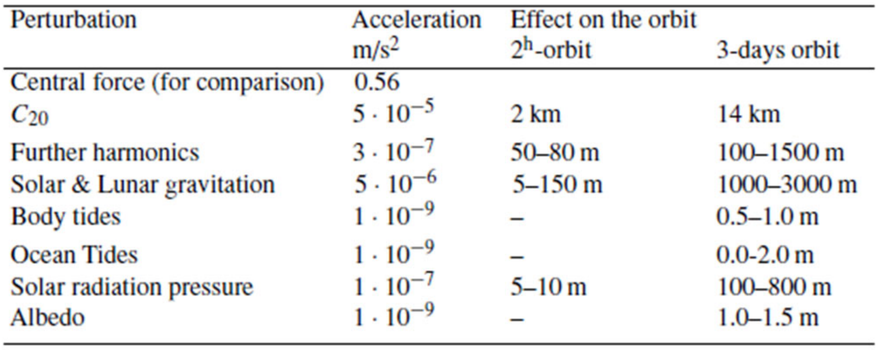
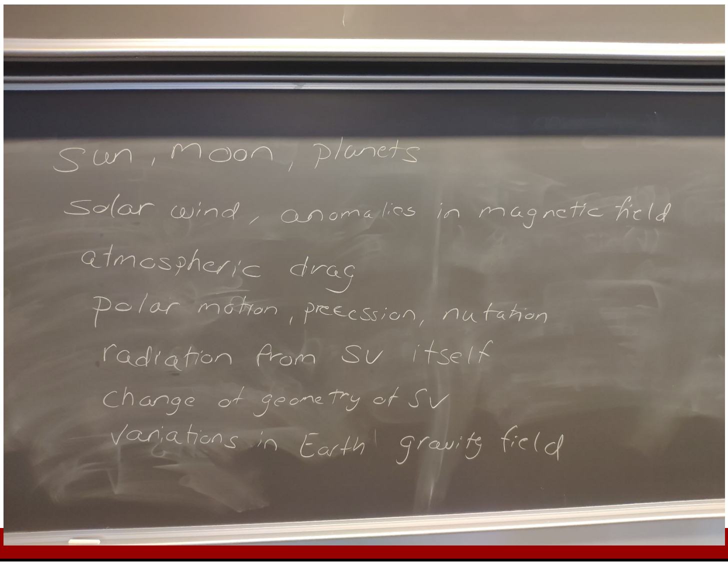
Perturbations on GPS satellite orbit



Perturbation	Acceleration m/s^2	Effect on the orbit 2^h-orbit	3-days orbit
Central force (for comparison)	0.56		
C_{20}	$5 \cdot 10^{-5}$	2 km	14 km
Further harmonics	$3 \cdot 10^{-7}$	50–80 m	100–1500 m
Solar & Lunar gravitation	$5 \cdot 10^{-6}$	5–150 m	1000–3000 m
Body tides	$1 \cdot 10^{-9}$	–	0.5–1.0 m
Ocean Tides	$1 \cdot 10^{-9}$	–	0.0-2.0 m
Solar radiation pressure	$1 \cdot 10^{-7}$	5–10 m	100–800 m
Albedo	$1 \cdot 10^{-9}$	–	1.0–1.5 m

From: Seeber, G. *Satellite Geodesy*. 2nd edition, 2003. Walter de Gruyter

Factors causing perturbations - from last lecture



Sun, moon, planets

solar wind, anomalies in magnetic field

atmospheric drag

polar motion, precession, nutation

radiation from SV itself

change of geometry of SV

variations in Earth' gravity field

Orbit determination – how to do it

- Un-disturbed orbit (purely theoretical):
  - Calculation of satellite positions from Kepler elements:
    1. Calculate eccentric anomaly from mean anomaly
    2. Calculate true anomaly
    3. Calculate distance between satellite and centre of gravity
    4. Calculate cartesian coordinates of satellite in orbit system
    5. Calculate geocentric coordinates of satellite in ECEF
  - Roughly the same as Assignment 2

- Disturbed orbit (in practise):
  - Perform integration of the equation: $\ddot{\boldsymbol{r}} = -\frac{GM}{r^3}\dot{\boldsymbol{r}} + \boldsymbol{k}_s$
    - where $\mathbf{k}_s$ is the resulting vector of all perturbing forces
  - Integration is carried out numerically or analytically

Orbit determination – post processing

- Orbit determination with post processing makes use of observations of the satellite
- Basically all current satellites in orbits lower than 20 000 km have GPS receivers onboard for orbit determination in (near) real-time or by post processing
- For precise orbit determination, GPS positioning may not be enough, therefore use other techniques:
  - For example: DORIS, satellite laser ranging (SLR) or inverse techniques based on observations collected by the satellites
  - Star cameras are used for orientation of satellites
- If only a short part of the orbit is needed, e.g. for remote sensing, a more simple representation of the orbit can be applied such as a polynomial approximation

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Precise orbits for GPS (1)

- Precise GPS satellite positions are determined based on several days of data collected from the satellites
- The various perturbation effects are modelled independently, and an orbital fit is carried out to estimate the Kepler elements
- Each Kepler element is modelled independently, and by combination of the models, final positions of the satellites are estimated

Precise orbits for GPS (2)

- The International GNSS Service (IGS) coordinates an international cooperation on estimation of precise satellite positions based on data from approx. 400 permanent GPS reference stations distributed globally
- Different orbit products:
  - Precise: Accuracy ~2.5 cm available 12-18 days
  - Rapid: Accuracy ~2.5 cm available 17-48 hours
  - Ultra-Rapid (observed half): Accuracy ~3 cm available 3-9 hours
  - Ultra-Rapid (predicted half): Accuracy ~5 cm available real-time
  - Broadcast: Accuracy 100 cm available real-time
- Available in the sp3 format here:
- https://kb.igs.org/hc/en-us/articles/115003935351

IGS network

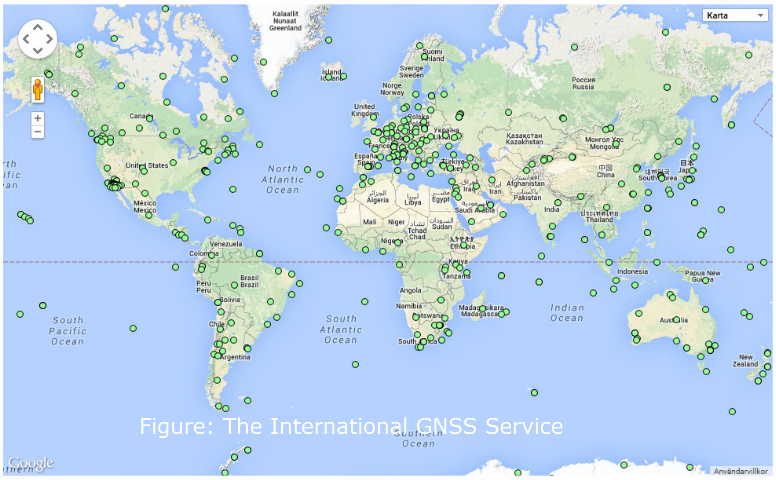

Figure: The International GNSS Service

The sp3 data format - example

```
#cP2006  5 29  0  0  0.00000000      96 ORBIT IGb00 HLM  IGS
## 1377  86400.00000000   900.00000000 53884 0.0000000000000
+   29   G01G02G03G04G05G06G07G08G09G10G11G13G14G15G16G17G18
+        G19G20G21G22G23G24G25G26G27G28G29G30  0  0  0  0  0
+          0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
+          0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
+          0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
++         3  3  3  3  3  3  3  3  3  3  3  3  3  3  3  3  3
++         4  3  3  4  3  4  3  3  3  4  3  3  0  0  0  0  0
++         0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
++         0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
++         0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
%c G  cc GPS ccc cccc cccc cccc cccc ccccc ccccc ccccc ccccc
%c cc cc ccc ccc cccc cccc cccc cccc ccccc ccccc ccccc ccccc
%f  1.2500000  1.025000000  0.00000000000  0.000000000000000
%f  0.0000000  0.000000000  0.00000000000  0.000000000000000
%i    0    0    0    0      0      0      0      0         0
%i    0    0    0    0      0      0      0      0         0
/* FINAL ORBIT COMBINATION FROM WEIGHTED AVERAGE OF:
/* cod emr esa gfz jpl mit ngs sio
/* REFERENCED TO IGS TIME (IGST) AND TO WEIGHTED MEAN POLE:
/* CLK ANT Z-OFFSET (M): II/IIA 1.023; IIR 0.000
*  2006  5 29  0  0  0.00000000
PG01  14003.983504 -20531.964425   9495.875403     55.833061  9 10  6 152
PG02  -4857.749099  14526.560859  21435.621040      0.595589 12 11 10 188
PG03   7573.446990 -22332.880868 -12077.575640    102.321813 11 10 11 144
PG04 -17065.511518   8524.720000  18582.647681    241.170978 12 11  9 172
PG05  13397.115513  21415.123417   8278.836680     56.878513  8  8 10 129
Etc.
```

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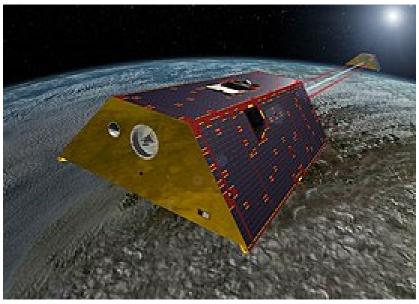
Satellite orbit types and terms



- Satellite *constellation* means a number of satellites operating together or being used together
  - – For instance the GPS satellite constellation (all GPS satellites)

- Satellite *mission* often means one or more satellites launched and operated for a specific purpose. E.g.
  - – GRACE (two satellites)
    - Upper figure from NASA/JPL-Caltech
  - – SWARM (two satellites)
    - Lower figure from ESA

  - – The satellite *payload* are the instruments onboard the satellite needed to generate data or collect data according to the purpose of the satellite

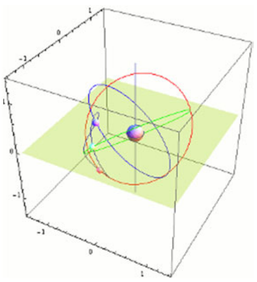
Orbit types - characterization by orbit height

- LEO – Low Earth Orbit: < 2000 km
  - Most satellites for Earth observation (images, radar, laser etc.)
  - Newer communication satellites
    - E.g. Starlink from SpaceX (currently 700 of 1440 satellites are in orbit)

- MEO – Medium Earth Orbit: 5000 – 20 000 km
  - E. g. navigation satellites (GPS, Galileo, GLONASS etc.)

- GEO – Geostationary satellites: 36 000 km
  - Always in same location relative to a point on Earth
  - Traditional communication satellites
  - No coverage of the Earth's polar regions

Other orbit types



- IGSO – Inclined Geo-synchronous Orbit
  - Differ from GEO by having an inclined orbit
  - Orbit ground track on Earth is 8-shaped
  - E.g. Japanese Quasi Zenith Satellite System (QZSS), a GPS augmentation system
    - Illustrations from Wikipedia

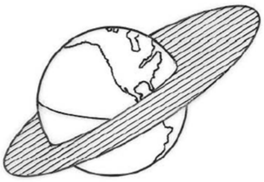
- HEO – Highly Elliptical Orbit
  - Very elliptical shape of orbit, e.g. for communication services at high latitudes
- Polar orbit: Inclination of 90°, orbit is fixed in space
- Other orbits types also exist

Orbit types

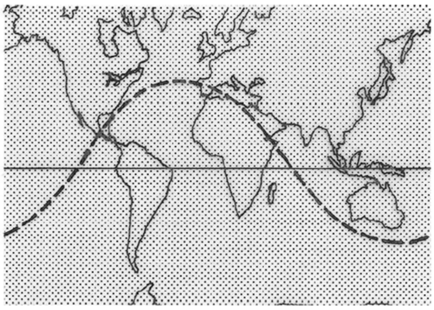
r [km]	h [km]	v_c [km/s]	U [min]	Examples
6 378	7	7.91	84.49	near Earth's surface
6 770	400	7.67	92.57	space station, gravity field missions
7 400	1 000	7.34	105.6	Earth observation satellites
7 730	1 360	7.18	112.9	TOPEX/POSEIDON
10 000	3 600	6.31	165.6	PAGEOS
12 300	5 900	5.69	226.2	LAGEOS
26 600	20 200	3.87	12h	GPS
42 160	35 790	3.07	23h 56m	geostationary satellite
384 400		1.02	27d 08h	Moon

r is orbit radius, h is mean orbit height, v is average velocity, U is period (called T in the lecture slides)

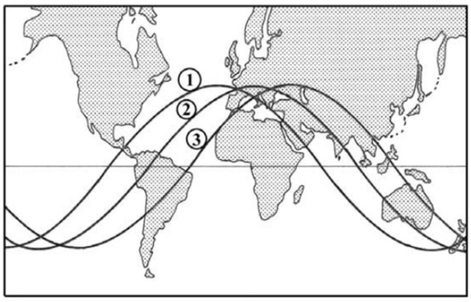
Orbit types



Orbital plane



Sub-orbital track (or ground track)



Ground track displacement with Earth rotation

Outline

✓M and E from second lecture
✓GPS time from first lecture

✓Orbit perturbations
✓Perturbations caused by Earth' gravity field
✓Perturbations caused by other forces
✓Orbit determination
✓Example: Precise orbits for GPS
✓Types of satellite orbits: LEO, MEO, GEO, IGSO, HEO
- Assignment 3

Assignment 3

- After the theoretic stuff in Assignment 1 and 2 it is time to work with real satellites
- With Assignment 3 you must characterize and describe a satellite / satellite mission / satellite constellation
- Each student works with one satellite / satellite mission / satellite constellation and presents this to the rest of the class on September 28th
- See the list of satellites / satellite missions / satellite constellations in the Assignment text in DTU Inside (or next slide)
- If you have specific wishes regarding which satellites to work with, send an email to Anna by Sunday September 20th at the latest. Preferably list three choices with priority

DTU

List of satellites/missions/constellations:

- Cryosat-2
- Galileo
- GLONASS
- GOCE
- GPS
- GRACE
- Hy-2A/Hy-2B
- IceSat-2
- Jason-CS/Sentinel-6
- SARAL/AltiKa

- Sentinel 1A/1B
- Sentinel 3A/3B
- SWOT (not yet operational)
- TOPEX/Poseidon+Jason-1/2/3

- DORIS (ground based tracking, not a satellite)
- SLR (ground based tracking, not a satellite)
- VLBI (ground based tracking, not a satellite)

DTU

Before we leave:

Disinfect table and chair
Maintain your distance to others
Wash or sanitize your hands
Respect guidelines and restrictions outside