30552 Satellite Geodesy – E20

Lecture 1

Geodetic coordinate systems and time systems

by
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Satellite geodesy

- What is satellite geodesy?
- What do you expect from a course in satellite geodesy?

- Discuss with you neighbour for 2-3 minutes
- Then we discuss in class

DTU

Geodesy

- Geodesy is the science of estimating the size and the shape of the Earth, and of Earth’s gravity field
- Involves both theoretical studies, studies in data analyses and algorithms as well as studies in technologies for data collection
- An important part of geodesy is to establish and maintain reference systems and reference frames for the Earth
- UN passed a resolution in 2016 entitled **A Global Geodetic Reference Frame for Sustainable Development** which underlines the importance having access to global geodetic reference frames

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DTU

Introduction to the course

- Practical information
  - Lecture room
  - Course plan – no teaching week 42
  - Assignments, lab reports, mini-project
  - Marks for the reports, no exam

- Presentation of teachers
- Presentation of students

Content of the lecture

- Motivation; Why learn about coordinate systems?
- Elements of geodetic cartesian coordinate system
- Inertial and terrestrial reference systems
- Global and regional systems: ITRS, WGS84, ETRS
- Local ENU coordinate system
- Height systems, geoid and map projections

- Time systems
- Stability of clocks and frequency standards

- Assignment 1

Motivation

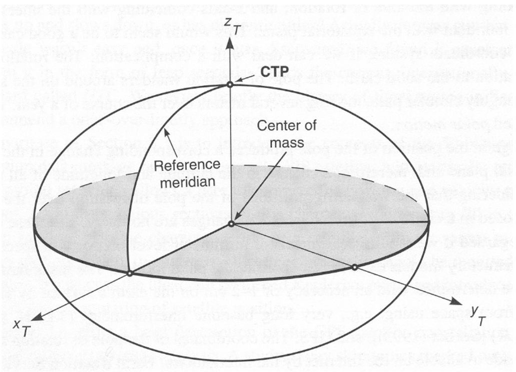
- The accuracy of a position is never better than the definition of the reference system or the reference frame

- Positions related to the Earth must be referenced to a coordinate system which is fixed to the Earth

- This is a complex matter since the Earth is dynamic, elastic, and the surface is continuously in motion (*geodynamics*)

Cartesian coordinate system

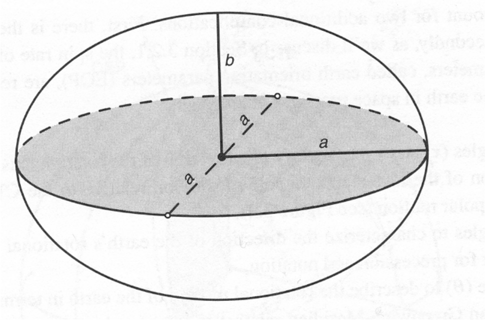
- Ideally:
  - Origo coinciding with center of mass of the Earth
  - Z-axis coincident with rotational axis
  - X-axis coincident with intersection of Greenwich meridian with equatorial plane
  - Y-axis to complete right hand coordinate system



- Figure from P. Misra and P. Enge, “Global Positioning System”, 2001

Ellipsoid of revolution

- An ellipsoid of revolution is the best mathematical model of the Earth
- Geodetic reference ellipsoids are defined by:
  - semi major axis, a
  - flattening, f
- Other variables:
  - semi minor axis, b
  - eccentricity, e



Useful relations between the variables:

$$e^2 = \frac{a^2 - b^2}{a^2} \qquad f = \frac{a - b}{a}$$

Figure: P. Misra and P. Enge, “Global Positioning System”, 2001

Cartesian and ellipsoidal coordinates

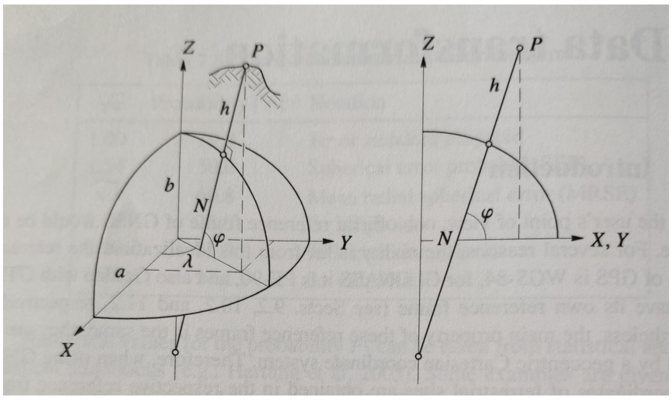


Figure: Hoffmann-Wellenhof, Lichtenegger and Wasle, ”GNSS …”, 2008

Polar motion

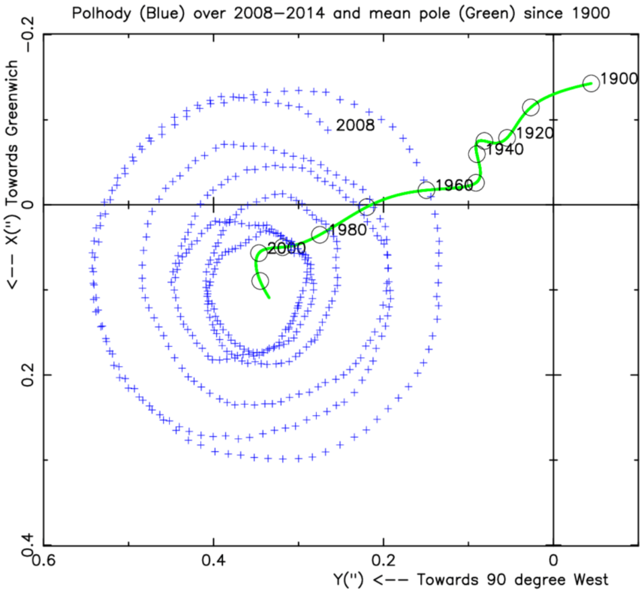
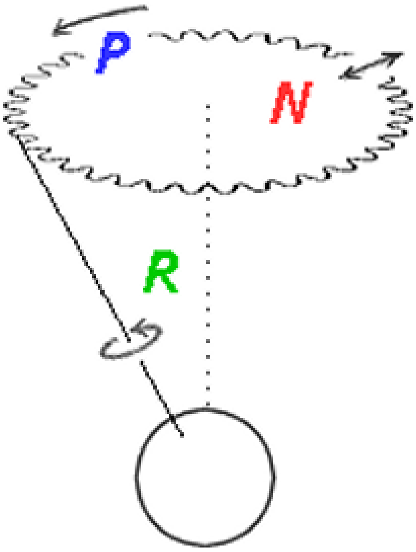


Figure: IERS Annual Report, 2013.

Precession and nutation

External forces cause the so-called *precession and nutation* – variations of the location of the rotational axis

- Figure from Wikipedia

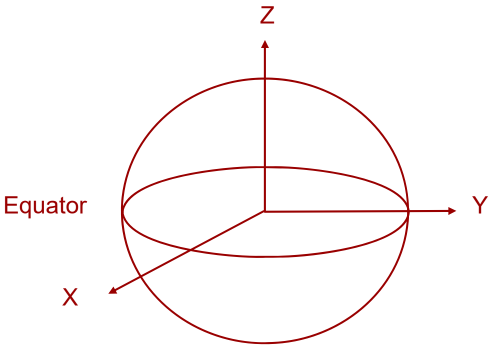


Orientation of rotation axis

- Orientation of the rotation axis is corrected by modeling both polar motion and the precession and nutation
- Coefficients for the models are determined by the International Earth Rotation Service (IERS) in Paris, and the coefficients are available on a monthly basis: http://www.iers.org/
- When converting coordinates between the inertial and an Earth fixed coordinate system, orientation of the axis of rotation must be considered
  - See equations in the GS-book, chapter 2.1.2.3

Conventional Inertial System (CIS)

- Ideally
  - Origo at mass center of Earth
  - Z-axis at rotational axis
  - X-axis in Equatorial plane towards vernal equinox
  - Y-axis in Equatorial plane to form right handed cartesian system

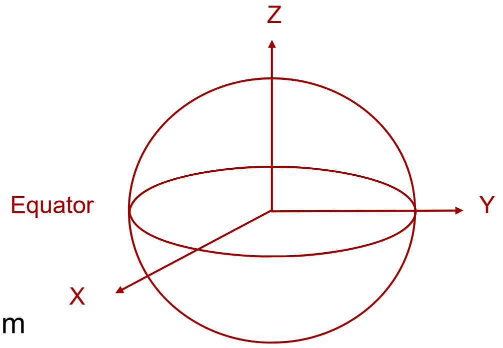


CIS is fixed in space (relative to distant stars and quasars) and does not rotate with the Earth

=> suitable for positioning of satellites

Conventional Terrestrial System (CTS)

- Ideally:
  - Origo at mass center of Earth
  - Z-axis through Conventional Terrestrial Pole
  - X-axis towards intersection of the reference (zero) meridian with the Equatorial plane
  - Y-axis in Equatorial plane to form right handed cartesian system



The CTS is an Earth Centered Earth Fixed (ECEF) reference system and it rotates with the Earth

Realization of a CTS

- A CTS is realized through *space geodetic observations* such as VLBI and SLR in combination with GNSS
- Consider a global *polyhedron*. Coordinates of points forming the polyhedron are accurately estimated on the surface of the Earth.
  - The more points and the better observation techniques, the better modeling of the shape of the Earth
- The CTS is realised by a set of points with coordinates given in the system. The points, their coordinates, and the system definition is called a geodetic ***reference frame***

Use of CTS

- The CTS is used for referencing positions given at, or near, the surface of the Earth
- CTS is not convenient for positioning of satellites and other objects in space because the CTS rotates with the Earth
- Examples of different CTS:
  - ITRS, WGS84, ETRS

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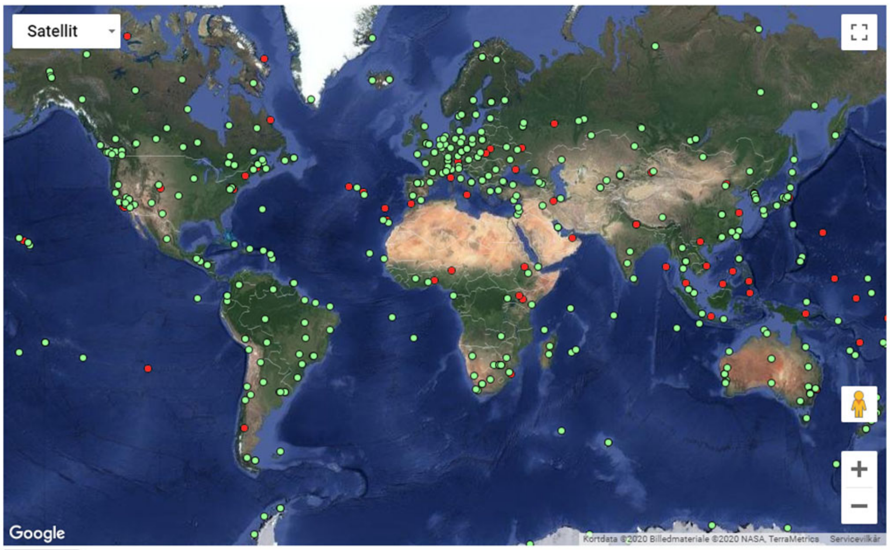
International Terrestrial Reference System (ITRS)

- The ITRS is the most accurate CTS defined to date
- Based on the GRS80 reference ellipsoid
- Defined and maintained by the IERS
- Realized by the reference frames, called ITRF
- New reference frames are defined when movements of the Earth’s surface cause older frames to be in-accurate

GRS80 ellipsoid

- The GRS80-ellipsoid is the most accurate reference ellipsoid defined to date
- Defined by the International Association of Geodesy (IAG)
- a = 6378137.0 meter
- f = 1 / 298.257222101
- The GRS80 ellipsoid is used for the ITRS

International GNSS Service (IGS) network of stations



International Terrestrial Reference Frame (ITRF)

- The various realizations of ITRF are named by year of definition. Most current is ITRF2014. Previous was ITRF2008. Work with ITRF2020 is initialised

- Even more detailed definitions are given by time epoch, for instance:
  - *ITRF2000 epoch 0.0* is the ‘name’ of the set of coordinates for the points in the reference frame valid at January 1. 2000 @ 00:00

- With this detailed level of definition it is possible to use the ITRF for geodetic applications with very high accuracy requirements e.g. monitoring of geodynamic effects

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World Geodetic System 1984 (WGS84)

- An ECEF developed and defined by the US Department of Defence for use with GPS

- Based on the WGS84 ellipsoid which is almost identical with the GRS80 ellipsoid

- WGS84 is a dynamic reference system, i.e. the system has been redefined five times. With the last redefinition WGS84 (G1762) is set to equal ITRF2008 at the level of approximately 1 cm (RMS)

WGS84 realization

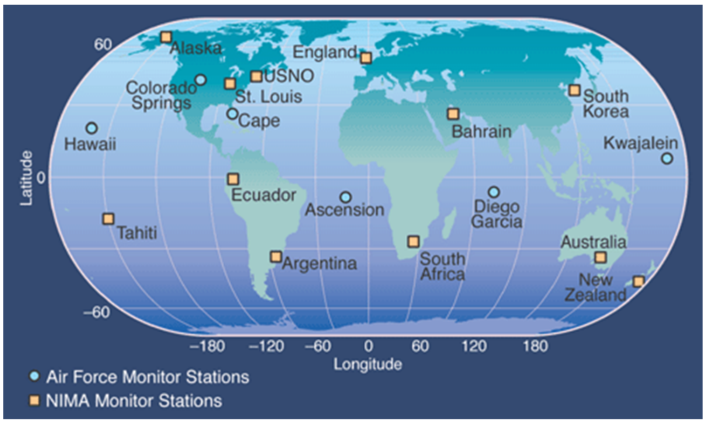

Figure from: The Aerospace Cooperation

Future reference systems and reference frames

In 2016 the UN general assembly passed a resolution entitled ”A Global Geodetic Reference Frame for Sustainable Development” underlining the importance of reference frames in all countries

Dynamic reference frames include surface velocity models. Therefore future coordinate transformations will also be dynamic including horizontal and vertical velocities

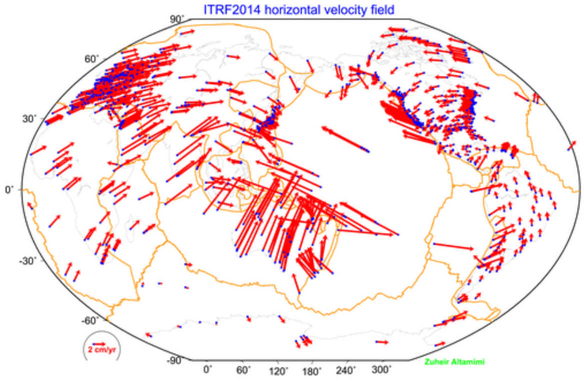

Figure: Z. Altamimi et al. (2016)

European Terrestrial Reference System (ETRS)

- ETRS is a European version of the ITRS, fixed to the Euro-Asia tectonic plate
- It has been decided to use ETRS89 for mapping and surveying in Europe - until further notice
- ETRS is realized with the ETRF (European Terrestrial Reference Frame)
- ETRS89 is realized using different ETRF in the European countries
  - In Denmark ETRS89 is realized by the ETRF92
  - Difference between ETRF92 and ITRF2014 is approximately 0.7 meter

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Traditional regional geodetic datum

- A geodetic datum is the traditional approach to geodetic reference systems
- Note the difference between modern *global* reference system and traditional *regional* datum
- ED50 (European Datum 1950) is a regional European Datum
  - Defined by NATO after WWII
  - Used until approx. 2000 for charting and mapping in most of Europe

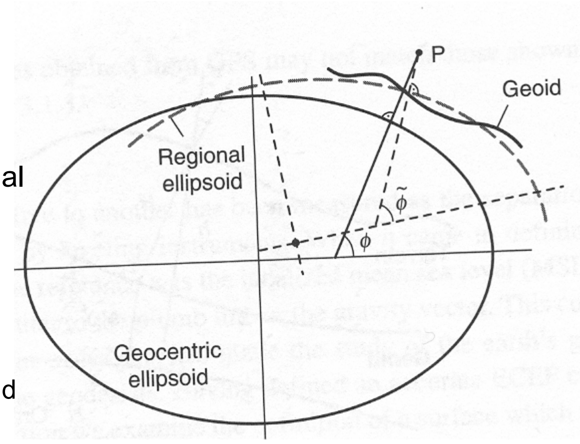

Figure: P. Misra and P. Enge, "Global Positioning System", 2001

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Local ellipsoidal coordinate system

- is defined by the ellipsoidal vertical in a point, P, where P is determined by its latitude and longitude

- The axes of the local system are defined so:
  - z_e is along the ellipsoidal vertical
  - x_e and y_e are in the plane perpendicular to the vertical

- Depending on the rotation angle used x_e and y_e are often directed towards geographical North and East

- The local system is therefore also referred to as an ENU (East-North-Up) or NEU (North-East-Up) system

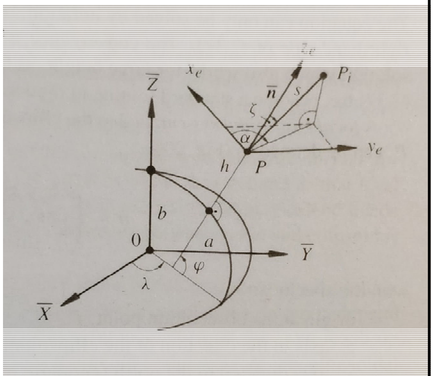

Figure 2.8. Global and local ellipsoidalsystems

Use of the local system

- A local ellipsoidal system can be used for providing information on the location of satellites *relative* to a point on the Earth
  - E.g. location of a satellite relative to a receiving ground based antenna or calculation of when a satellite is visible above the horizon

- With the definition of an ENU system we can define:
  - Azimuth, α
    - Angle relative to the North
    - Defined in the horizontal plane of the ENU system
  - Zenith angle, ζ
    - Angle relative to the Zenith (up)
    - Defined in the vertical plane of the ENU system

Transformation between cartesian systems (1)

- For conversion between two cartesian coordinate systems the transformation parameters needed are:
  - 3 rotations about the X, Y, and Z axes respectively, contained in the combined rotation matrix:

$$\mathrm{R} = R_1(\alpha)\cdot R_2(\beta)\cdot R_3(\gamma)$$

  - 3 translations along the X, Y, and Z axes respectively, contained in the translation vector: T= $\begin{bmatrix}\Delta X\\ \Delta Y\\ \Delta Z\end{bmatrix}$
  - 1 scale factor, m

Rotation matrices

- We need the rotation matrices for conversion of coordinates from one cartesian coordinate system to another cartesian coordinate system
  - The rotation angle must be pre-defined θ

$$R_1(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & \sin(\theta) \\ 0 & -\sin(\theta) & \cos(\theta) \end{bmatrix}$$

$$R_2(\theta) = \begin{bmatrix} \cos(\theta) & 0 & -\sin(\theta) \\ 0 & 1 & 0 \\ \sin(\theta) & 0 & \cos(\theta) \end{bmatrix} \qquad R_3(\theta) = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Conversion between cartesian systems (2)

- Given:
  - point P in one coordinate system $\mathrm{x_P} = \begin{bmatrix} x_P \\ y_P \\ z_P \end{bmatrix}$
- Wanted:
  - point P’ in another coordinate system $\mathrm{x_P'} = \begin{bmatrix} x_P' \\ y_P' \\ z_P' \end{bmatrix}$
- Expression: $\mathrm{X_p'} = m \cdot (R \cdot X_p) + T$

- This is needed for Assignment 1. The expressions are also provided in the GS-book with another notation

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Height systems

- In geodesy several definitions of height are applied. Some of the most common are:

- *Ellipsoidal height*, measure along the ellipsoidal vertical

- *Orthometric height*, measured along the plump line
  - Often related to height above local sea level
  - As reference for sea level a geoid model is often used

- The separation between the surface of the ellipsoid and the surface of the geoid is called the *geoid height* (or the *geoid undulation)*

Height systems

For many practical applications the following is assumed:

h = H + N

The assumption works in areas with a homogeneous gravity field e.g. Denmark

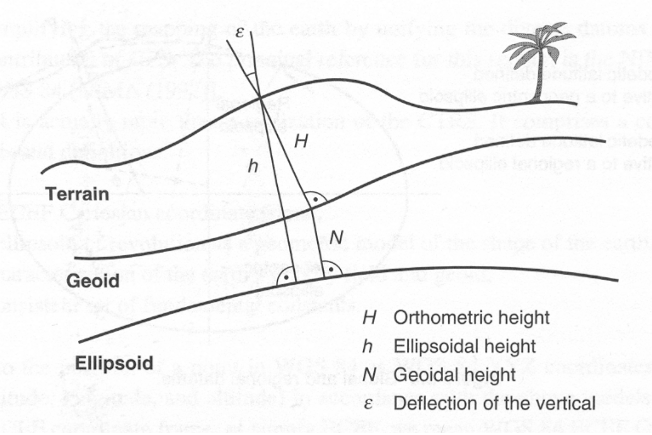

Figure: P. Misra and P. Enge, “Global Positioning System”, 2001

More on this in later lectures and in the course Physical Geodesy

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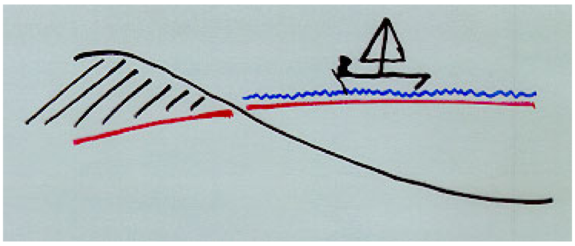
The geoid

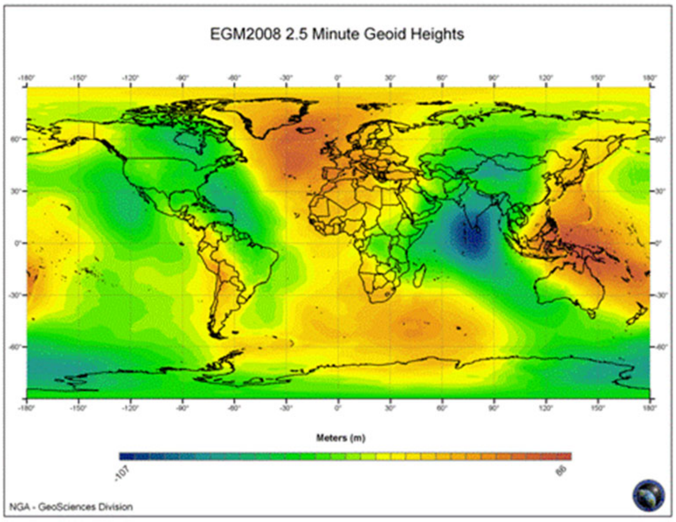
- The geoid is an equipotential surface in the Earth’s gravity field, i.e. gravity acceleration is constant along the surface
- The surface of the geoid is fitted to mean sea level (MSL)
- Geoid models are generated based on gravity observations and spherical harmonic models of the global gravity field
- The best global geoid model at present is considered to be the Earth Gravitational Model, EGM2008

The geoid in practise

- The location of the surface of the geoid is basically as the surface of the Earth would be, if the Earth was covered with water and only affected by gravity (no external forces)
- The geoid continues beneath the surface of the Earth



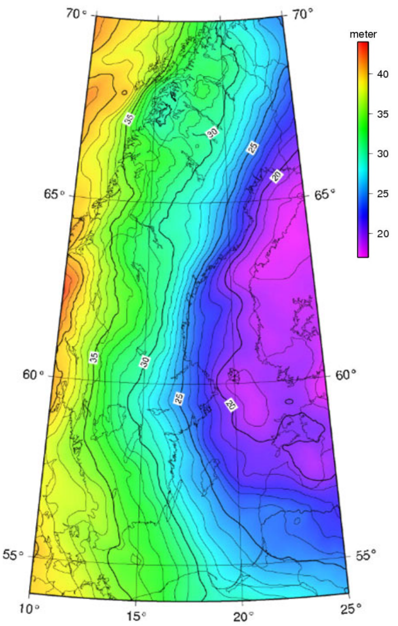
EGM2008



Global model, long wavelength, in Denmark off with a couple of meter

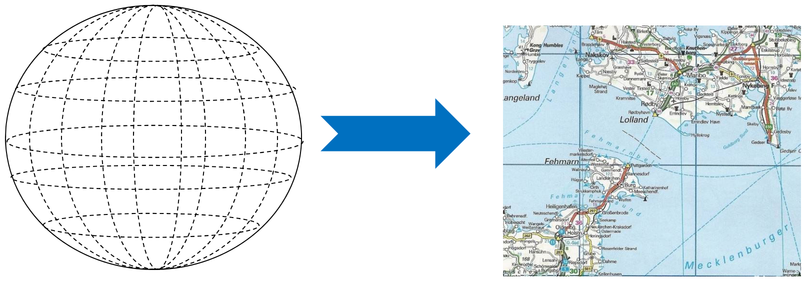
Swedish geoid model

- SWEN08_RH2000
- Geoid model fitted to SWEREF99 and to the Swedish national height system, RH2000
- Accuracy 10-15 mm within Sweden
- Similar national models exist in most countries



Plot from Lantmäteriet

Map projections



- A map projection is used for mapping the curved 3D surface of the Earth onto a plane 2D map
- Many different map projections exists. Examples are Transverse Mercator and the Universal Transverse Mercator (UTM) projections

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- Assignment 1

Motivation – why learn about time?

- Because we are working with different time scales depending on the applications
- Because our satellite geodetic observation methods can provide measures of physical changes with a very high precision
- Because with many observation methods precise time synchronization or time stamping is important
- Because the transmission time of a bit of a GPS satellite signal is approximately 0.078 seconds. To determine a position with GPS we need to know the time very well
- Etc.

Solar time

- Defined by Earth rotation with respect to the sun
  -> one rotation is 24 hours
- Since Earth rotation is not constant neither is the solar time reference. The length of a day varies with up to 16 minutes during a year
- In practise, a mean time is used:
- Greenwich Mean Time – also referred to as GMT or ZULU

Sidereal time

- Defined by Earth rotation with respect to distant stars
- 1 sidereal day (SID) = 23 hours and 56 minutes in solar time
  - or 1 solar day = 1.003 sidereal day
- Since Earth rotation is not constant neither is the sidereal time reference.
- In practise, a mean time is used: Mean sidereal time
- Interesting fact: The GPS constellation repeats itself after half a SID

Greenwich Apparent Sideral Time (GAST)

- GAST is defined as the angle between the reference meridian (Greenwich) and the X-axis of the CIS (i.e. the direction to the vernal equinox)

- GAST provides the current mean sideral time at the reference meridian

- GAST is defined as an angle, but related to time by considering the Earth rotation rate given as an angular velocity
  - Earth rotation within an hour is approx. 15°
  - The width of time zones are approx. 15°in longitude

- GAST indicates the rotation, about the Z-axis of the inertial coordinates system, of the ECEF coordinate system

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Atomic time (1)

- The current official SI-definition of the atomic time scale is since 1967 determined by cesium
  - 1 second = the time it takes for 9,192,631,770 periods of radiation of the cesium-133 atom

- Atomic time is homogeneous and continuous (no jumps) and therefore attractive for many satellite geodetic applications

- The fundamental atomic time scale is called *International Atomic Time* (abbreviated TAI)

Atomic time (2)

- Atomic time is not referenced to Earth rotation and over time it therefore drifts further and further away from solar time
- This has lead to the introduction of *Universal Time Coordinated*, UTC which is the official time used in many countries today
- The definition of TAI and UTC is controlled by the "*Bureau International des Poids et Mesures" (BIPM)* in Paris *http://www1.bipm.org*

Universal Time Coordinated (UTC)

- *Coordinated Universal Time* (UTC) is defined as:
  - 1 second UTC = 1 second TAI
- UTC time is corrected for changes in the rotation of the Earth by an integer number of seconds (leap seconds)
- Leap seconds are introduced, when considered necessary, on either June 30th or December 31st
  - Most recent leap second introduced Dec. 31st 2016 (previous was in June 2015)
- Current difference: TAI – UTC = 37 seconds
- UTC is homogeneous, but not continuous because of the leap seconds

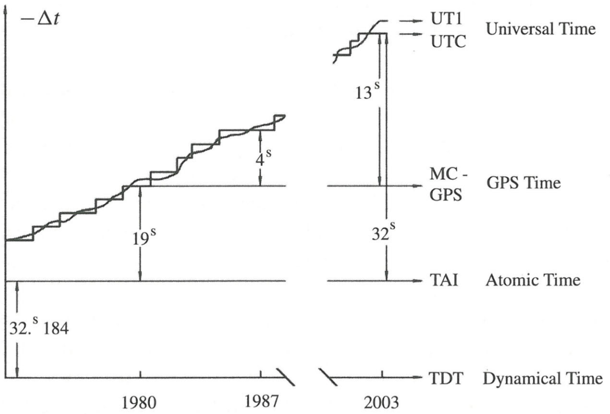
GPS time (1)

- For GPS-positioning it is crucial with a homogeneous *and* continuous time scale. Therefore a new time scale was defined to be used for GPS, called *GPS time*
- GPS time is defined as UTC but without the leap seconds
  - GPS time = UTC time at January 6th, 1980, @ 00:00 hours
  - GPS time in 2020 equals UTC + 18 seconds + a few nanoseconds
- GPS time is maintained by the US Naval Observatory
  http://www.navcen.uscg.gov

GPS time (2)

- A time epoch given in GPS time is referenced as:
  - Week number, counted since January 6th, 1980
  - Number of seconds within the week (counter is reset at midnight between Saturday and Sunday, i.e. from 604800 to 0 seconds)
- Current GPS week is: 2120
- For some applications, GPS time is also given as:
  - Week number, day of week, seconds of day
- In Assignment 2 we will work with GPS time

Summing up on time scales used in geodesy



Values in the figure are from 2003 and therefore not valid today.

For the current values see e.g.:http://www.leapsecond.com/java/gpsclock.htm

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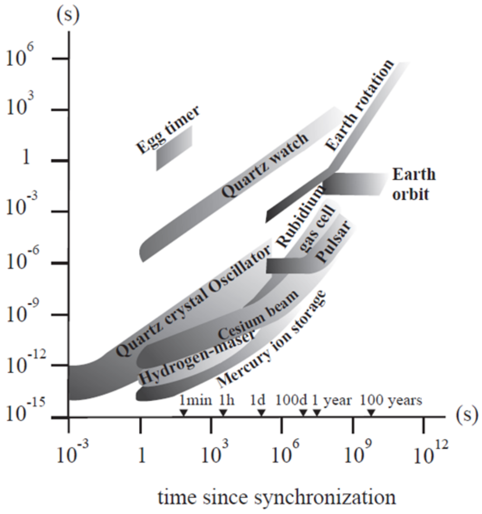
Stability of clocks and frequency standards

- A precise and well defined time reference, only makes sense if we have clocks sufficiently good to “keep the time”
- For example for GPS:

  An error of 1 microsecond in the GPS receiver clock induces an error of 300 meter in the range estimate to the satellite
- Estimation and modelling of both satellite and receiver clock errors is therefore important for geodetic applications
- Figure: G. Seeber, “Satellite Geodesy”, 2003



GPS satellite clock error (or instability)

- GPS satellites are equipped with two (or three) rubidium and two cesium atomic clocks that are monitored by the GPS control stations
- The current size of the satellite clock errors are between 2 and 750 microseconds
- Satellite clock errors are modelled by polynomials of second order. Coefficients are transmitted to GPS users via the navigation message from the satellites

GPS satellite clock error model (1)

- Satellite clock correction model valid at time t:

$$\delta t = a_{f0} + a_{f1}(t - t_0) + a_{f2}(t - t_0)^2 + \Delta t_r$$

- Where:
  - t_0 is the reference time epoch (January 1980)
  - a_{f0} is coefficient for clock offset (seconds)
  - a_{f1} is coefficient for fractional offset (sec/sec)
  - a_{f2} is coefficient for clock drift (sec/sec^2)
  - Δt_r is related to the relativistic effect

GPS satellite clock error model (2)

- There is also a relativistic effect of the satellite clocks since the satellites move with respect to the earth. Compensation is handled by running the satellite clocks a little slower

- When the model of satellite clock correction is applied to all satellites, their clocks are synchronized to within 5-10 nanoseconds

Practical considerations

- For many applications it is important to remember the difference between especially UTC and GPS time. For instance:
  - When logging GPS data to a PC which runs on UTC time
  - When integrating GPS with other sensors such as cameras, laser scanners or INS equipment that might be referenced to UTC

- Example:
  - An airplane, with a speed of 500 km/h, will move 2,500 meters in 18 seconds
  - When not correcting for the time offset between UTC and GPS time, an equivalent error in the position is introduced

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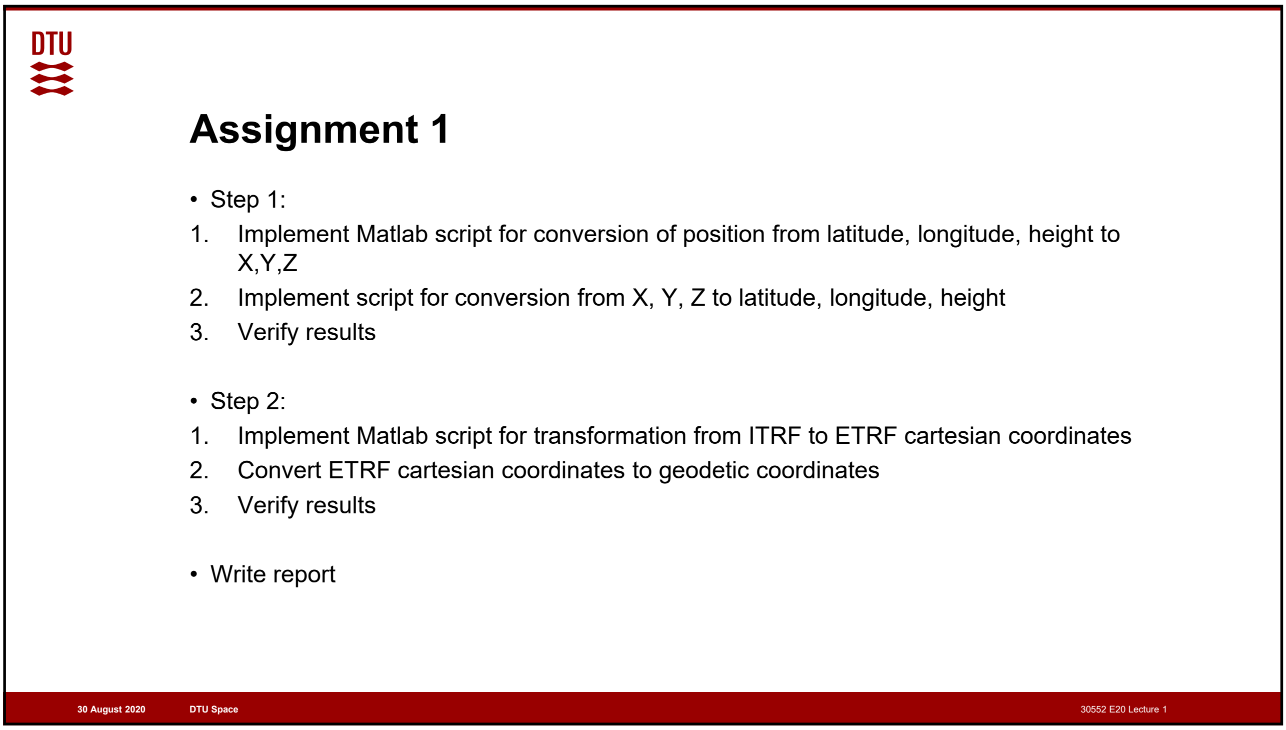
Assignments in this course

- 7 assignments during the course

- 6 assignments involve doing something with equations/calculations/estimations mainly using Matlab
  - These six assignments shall result in reports to be handed in
  - The reports will be marked

- 1 assignment is a mini project where you must present the results for the class
  - The presentation is marked by both content and form

- Final mark for the course will be the average of the marks for the assignments where the mark for presentation has a double weight
  - As the mark is given for assignment reports, there is no exam



Assignment 1

- Step 1:

1. Implement Matlab script for conversion of position from latitude, longitude, height to X,Y,Z
2. Implement script for conversion from X, Y, Z to latitude, longitude, height
3. Verify results

- Step 2:

1. Implement Matlab script for transformation from ITRF to ETRF cartesian coordinates
2. Convert ETRF cartesian coordinates to geodetic coordinates
3. Verify results

- Write report