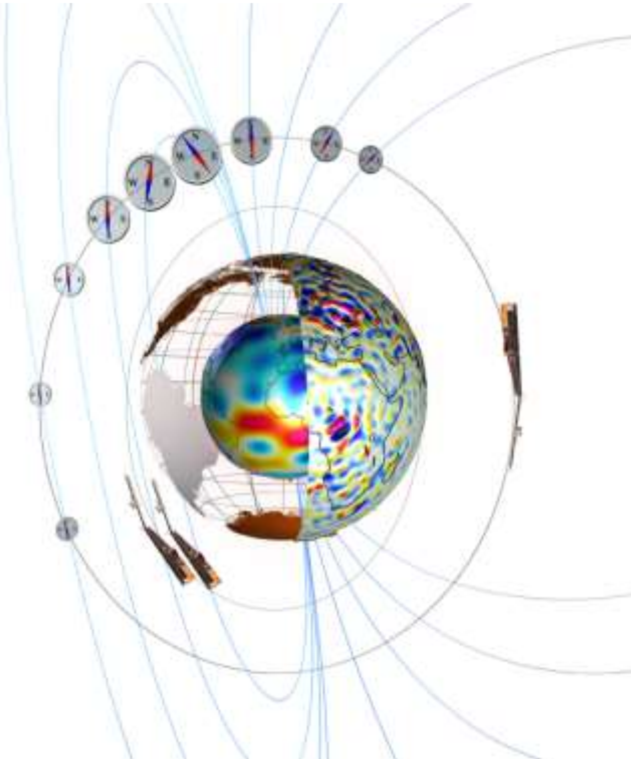
Proposal for Update of Level 1b Algorithms: Enhancement of B-spline Interpolations



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Record of Changes

Reason	Description	Rev	Date
Initial vers.	Released	1 dA	27 May 2016
Updated in response to comments from GMV	Updated references to [RD-1] sub-sections (Sections 3 and 4). Corrected formulas for error computation (Section 4.1.2). Clarified knot computation for smoothing B-splines (Section 4.1.3.2) Corrected typo in Section 5.	1 dB	8 June 2016
Update of knot trimming	Update of knot trimming code (in Section 4.1.3.2)	1 dC	15 June 2016

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1 Introduction

1.1 Scope and applicability

This document is a constituent of the Design Definition file and is provided as a formal request to change the Swarm Level 1b Algorithms for what concerns the B-spline interpolations of the Absolute Scalar Magnetometer (ASM) and Star Tracker (STR) data. The background for the change is the request to enhance the signal spectral content transferred in the interpolation of the ASM data.

2 Applicable and Reference Documentation

2.1 Applicable Documents

None.

2.2 Reference Documents

The following documents contain supporting and background information to be taken into account during the activities specified within this document.

[RD-1] SW-RS-DSC-SY-0002 Swarm Level 1b Processor Algorithms

2.3 Abbreviations

Acronym ***or abbreviation***	***Description***
ASM	Absolute Scalar Magnetometer
CCDB	Characterisation and Calibration DataBase
DISC	(Swarm) Data, Science, and Innovation Cluster
DTU	Technical University of Denmark, DK
ESA	European Space Agency
JIRA	Atlassian JIRA internet based tool for tracking issues with server located at DTU https://jira.spacecenter.dk/
L1b	Level 1b (satellite data)
Swarm	Constellation of 3 ESA satellites, http://www.esa.int/Our_Activities/Observing_the_Earth/Swarm/Introducing_Swarm
STR	Star Tracker
VFM	Vector Field Magnetometer

3 Algorithm Update Summary

The proposed update of the Swarm Level 1b processing algorithm relates to Sections 6.2.4.5, "Cubic B-Spline Interpolation" (ASM), and 6.4.2.4, "Interpolation of Attitudes" (STR), and Appendix D of [RD-1]. Specifically, the proposed update represents a simplification of the B-spline knot generation, including an addition of the specific case of a *non-smoothing* interpolation, which is made possible via a simple modification of the data sequence being interpolated. In addition, two CCDB parameters are added to control the computation of F_{error} and q_{error} respectively.

4 Detailed Description of Algorithm Update

This section contains the detailed description of the changes to the Swarm Level 1b Algorithm document, [RD-1], Section 6.2.4.5, "Cubic B-Spline Interpolation" (of ASM data), Section 6.4.2.4, "Interpolation of Attitudes" (from STR), and Appendix D. The overall scheme of the update entails a modification of the data sequences being interpolated (by linear levelling w.r.t. the end-points) combined with a simplification of the generation of B-spline knots. The existing, rather complex, procedure for generating the knot sequence exists to avoid end effects, seen as oscillations of the B-spline near the ends of the data sequence; however by revising the knots at the ends, there oscillations are avoided. For the special case of a non-smoothing B-spline interpolation instabilities arise in the design matrix inversion; this is avoided by reducing the number of end-knots which in combination with the levelling of the data yields the desired results.

In addition, CCDB parameters are introduced to control the computation of the estimated uncertainties of the scalar measurements and attitude information, F_{error} respectively q_{error}.

4.1 Algorithm Updates

The non-smoothing B-spline interpolation is characterized by setting the Knot_Space parameter to zero, where Knot_Space is the parameter of the Spline_Pars group of the CCDB, e.g. CCDB.L1BP.ASM.Spline_ Pars.Knot_Space for the ASM resampling or CCDB.L1BP.STR.Spline_Pars.Knot_Space for the STR interpolation.

4.1.1 Levelling of Data Sequences

The estimation of the B-spline functions to the sequences of measurements ("data sequences" or "data segments") is susceptible to large deviations at the end points; this effect vanishes if the values at the end points are zero. This can be accomplished easily by linear levelling of the data sequence. Figure 4-1 shows an example of levelling of ASM data from Swarm Alpha. The blue curve shows the original ASM data, $F^{(4)}$, the red, dashed line shows the straight line between the end-points of $F^{(4)}$, denoted F_{Lin}. Then the levelled data is simply computed as $F^{(4)}$ - F_{Lin} shown in green. The B-spline interpolations described in Sections 6.2.4.5 (ASM) and 6.4.2.4 (STR) of [RD-1] are performed on the levelled data (e.g. on $F^{(4)}$ - F_{Lin}) and the final results are obtained by adding the linear interpolation (e.g. F_{Lin}) to the B-spline approximation f (e.g. f + F_{Lin}).

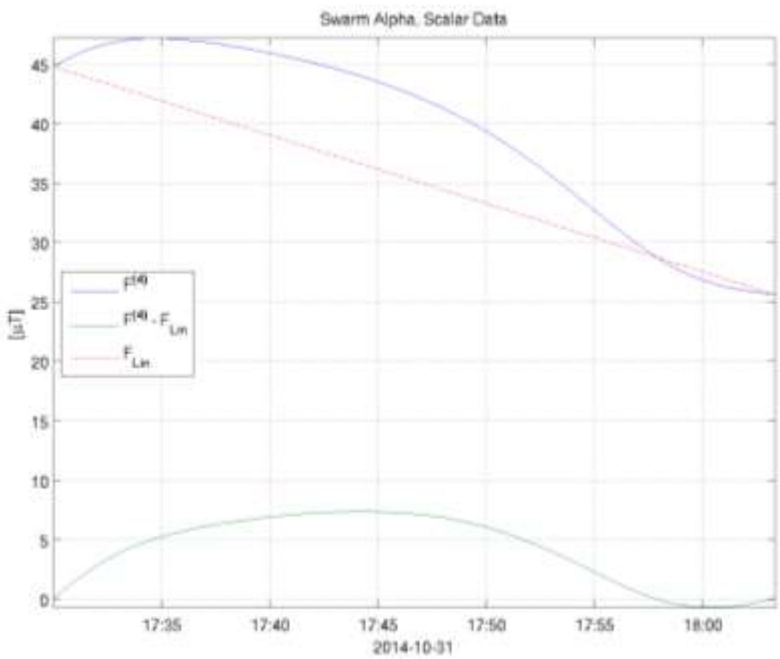

Figure 4-1 Example of Levelling ASM Data

Specifically, given a data sequence $y_0, y_1, \ldots, y_{n-1}$ with corresponding time instants $t_0, t_1, \ldots, t_{n-1}$, generate the levelled data sequence y':

$$y'_i = y_i - \text{interpolate}_{Lin}([t_0, t_{n-1}], [y_0, y_{n-1}], t_{i1}) \quad \text{(Eq. 4-10)}$$

Perform the B-spline interpolation as described in Sections 6.2.4.5 and 6.4.2.4 of [RD-1] on y′ yielding *f*′, then the final, interpolated values, *f*(t), are::

$$f(t) = f'(t) + \text{interpolate}_{Lin}([t_0\ t_{n-1}], [y_0, y_{n-1}], t) \qquad \text{(Eq. 4-20)}$$

4.1.2 Computation of F_{error} and q_{error}

In addition to the modification of the B-spline interpolation itself, a control of the computation of the related error estimates F_{error} and q_{error} is introduced. Specifically, the two equations (6.2-240) and (6.4-260) in Sections 6.2.4.5 respectively 6.4.2.4 of [RD-1] are modified as follows:

$$F_{err}^{(5)} = \sqrt{\left(\left(F_{err}^{(4)}\right)^2 + \lambda\left(F^{(4)} - f\right)^2\right) \Big/ (1+\lambda)} \qquad (6.2\text{-}240)$$

$$q_{error}^{new} = \sqrt{\left(\text{Level 1b}_{Inst}.q_{error}^2 + \lambda\eta^2 \sum (q_i - f_i)^2 / 4\right) \Big/ (1+\lambda)} \qquad (6.4\text{-}260)$$

In both equations a parameter, λ, is introduced as a multiplier on the second factor inside the square-root. λ is a scalar between 0 (zero) and 1 (one) to be added in the CCDB as CCDB.L1BP.ASM.Spline_Pars.L_error and CCDB.L1BP.STR.Spline_Pars.L_error for equation (6.2-240) respectively (6.4-260).

4.1.3 Computation of Knot Sequences for B-splines

The generation of knot sequences for B-spline interpolations described in subsection “***Knots***” of Appendix D in [RD-1] is – more or less – fully reworked. For smoothing B-splines, the knots consist of a number of repeated knots at each end and equidistantly spaced *interior* knots. For non-smoothing B-splines, i.e. when CCDB…Spline_Pars.Knot_Space = 0, the knots are identical to the time-instants of the data samples with a reduced repetition at each end. This is described in detail in the following.

Generally, let:

Table 4-1 B-spine Knot Legend

Element	Description
ks	Given knot spacing (= CCDB…Spline_Pars.Knot_Space)
$t_0,\ldots,t_{n-1}$	Time-instants of data samples (including possible extensions)
T	Nominal sampling rate (= CCDB…Spline_Pars.T_nom)
r	Number of end-knot repetitions (see below)
$k_{-r}, k_{-r+1},\ldots, k_{m+r-2}, k_{m+r-1}$	Time-instants of knots. $k_0,\ldots,k_{m-1}$ are the internal knots, i.e. $t_0 \le k_0$, $k_i < k_{i+1}$ for i = 0, 1, …, m-2, and $k_{m-1} \le t_{n-1}$; the remaining knots are repeated end-knots, i.e. $k_i = t_0$ for i < 0 and $k_j = t_{n-1}$ for j ≥ m.

4.1.3.1 *Non-smoothing B-spline Knots*

The B-spline knots for the non-smoothing B-spline, i.e. when ks = 0, are simply the time-instants of the extended data sequence. Hence, m = n and $k_i = t_i$ for i = 0, 1, …, n-1. The number of end-knot repetition, r, is one. In summary, the knot sequence is: t_0, t_0, t_1, t_2, …, t_{n-1}, t_{n-1}.

4.1.3.2 *Smoothing B-spline Knots*

For smoothing B-splines, the knot sequence consists of three-fold end-knots, i.e. r = 3, and interior knots are computed as follows:

Table 4-2 Smoothing B-spline Knot Generation

k_{rem}	Remainder from given knot spacing: $k_{rem} = \mathrm{mod}(t_{n-1} - t_0, ks)/2$
k_{offset}	Offset of first, interior knot: $k_{offset} = \begin{cases} k_{rem} + ks/2 & \text{if } k_{rem} + ks/2 \geq T \\ k_{rem} + ks & \text{otherwise} \end{cases}$
k_i	$= t_0 + k_{offset} + i\,T$ for i = 0, 1, …, m_0-1, iff n > 4 (if n ≤ 4 there are no interior knots)

The knot sequence has to be trimmed in case there are (small) gaps in the data sequence. First, all knots between t_0 and t_1 and between t_{n-2} and t_{n-1} are removed, that is, knots for which $t_0 < k_i < t_1$ or $t_{n-2} < k_i < t_{n-1}$ are removed leaving $m \leq m_0$ interior knots. Next, the existing procedure for removing knots described at the end of the ***Knots*** subsection of Appendix D ([RD-1]) is executed – in a slightly updated version given below (changes and one correction marked in blue) – to guard against too many knots compared to the number of data points:

```
lead = 0
i = 0
j = -3
j0 = j
while((i < n) and (j < m+3))
  if(t_i ≥ k_j)
    j = j + 1
    lead = lead + 1
  else
    if(lead > 3)
      % There are lead-3 knots too many.
      Replace the knots k_j0,…,k_j-1 by p=(j-j0-lead+3) new knots
      computed as the means of k_j0,…,k_j-1 divided in p parts
      (if p=3 and mod(j-j0,3)=1 make middle section of k's one
       larger, if mod(j-j0,3)=2 make outer sections one larger)
      lead = 2
    elseif(lead > 0)
      % lead can not be negative
      lead = lead - 1
    endif
    j0 = j
    i = i+1
  endif
endwhile
if(lead + m + 3 - j > 3)
  Repeat the above backwards,
  i.e. start with i = n-1 and j = m+2 and count these downwards
  if(lead > 3) by the end of the while-loop
    replace the middle[1] lead-2 interior knots by one knot = their mean
  endif
endif
```

[1] Towards lower indices, e.g. the "middle" 3 of 8 samples (numbered 1 through 8) would be samples 3, 4, and 5.

5 Validation

Validation of the method is demonstrated in Figure 5-1 and Figure 5-2 below. Both figures show the residuals of the B-spline interpolations and the ASM data themselves, i.e. $F^{(4)} - f$, for various knot spacings (0, 1.25, 3, 7, and 12 seconds) for ASM data from Swarm Alpha on 31. October 2014; the ASM data are the same as shown in Figure 4-1. Figure 5-1 shows the full data-sequence on a logarithmic scale from 10^{-5} to 1 nT, this demonstrates the increase in residuals as the knot spacing is increased (the residuals of the non-smoothing B-spline interpolation in blue are below 10^{-5} nT and hence not seen in Figure 5-1). The periods of elevated residuals correspond to periods of plasma bubbles occurring, hence where the spectral contents of the magnetic field intensity contains significant power at high frequency.

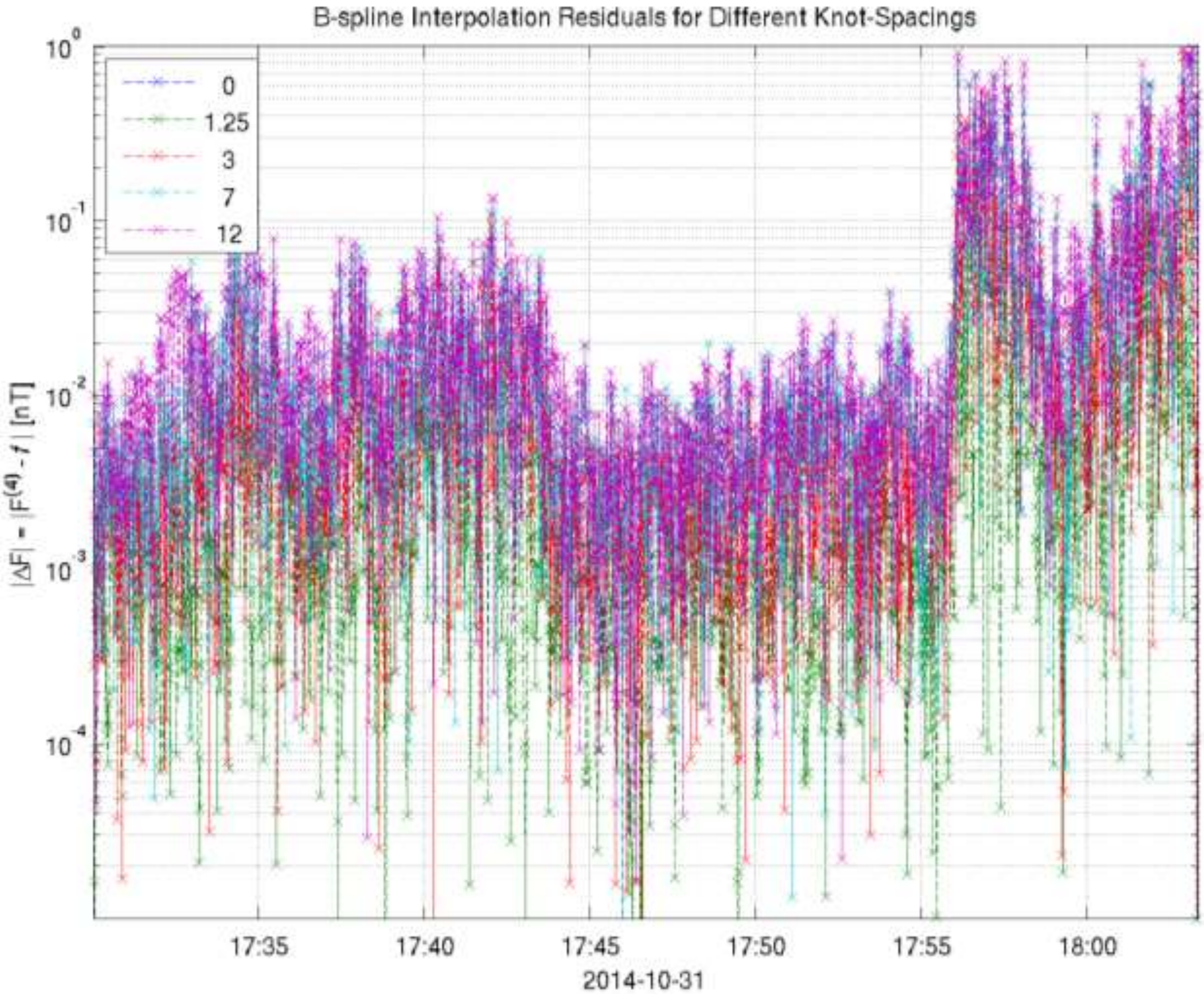

Figure 5-1: Residuals of B-spline Interpolations

Figure 5-2 show the details of the residuals near the ends of the data sequence. The scales are linear but the units are pT on the left and nT on the right. These plots show the perfect fit of the non-smoothing B-spline (in blue) and the increase in residuals as the knot spacing is increased. The plots also demonstrate the perfect fit to the end points, hence the former edge effects are eliminated.

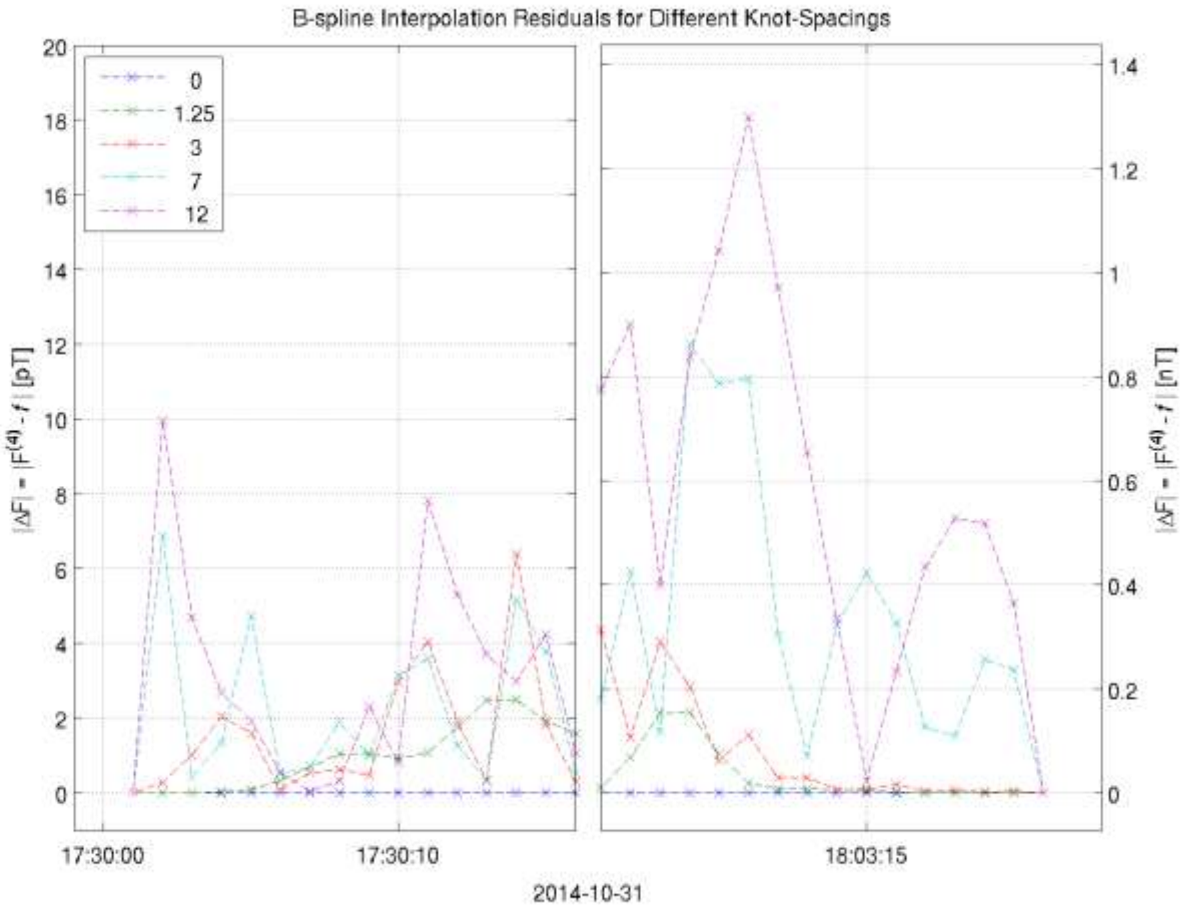

Figure 5-2: Residuals of B-spline Interpolations – End Details