Electrical Conductivity Modeling and Inversion of Satellite Induction Data

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Recovery of electrical conductivity in the earth is possible from satellite magnetic data provided that the earth response to induction by magnetospheric sources can be characterized. The satellite records a continuous time series of the geomagnetic field that mixes the time and spatial varying part of the transient field. A constellation of satellites such as SWARM provides at same instant t more than one estimate of the field. A time and space spectral expansion of the data provides linear combinations of the external and internal spherical harmonic spectrum of the field. The internal transient field is the result of induction in the conductive earth. Different modeling strategies are tested to obtain quantities that may be used in an inverse approach to derive the electrical conductivity of the earth crust and mantle. Results based on synthetic magnetic data computed for the SWARM constellation are presented.

MODELING

Each sample $(B_r, B_\theta, B_\phi)_i$ measured at UT instant t_i and co-ordinates (r_i, θ_i, ϕ_i) is characterised by its SHE and its discrete Fourier expansion:

$$(1)\quad B_p(r_i,\theta_i,\varphi_i,t_i)=\sum_{\omega\geq 0}\sum_{l,m}\left[\eta^l_p(r_i)I^m_l(\omega)+\varepsilon^l_p(r_i)E^m_l(\omega)\right]Q^m_{p,l}(\theta_i,\varphi_i)e^{j\omega t_i}\Delta\omega$$

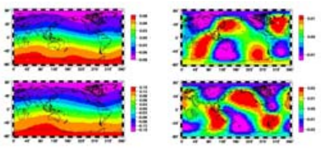
η, ε and Q depend on the component p (r, θ, ϕ). Q is the appropriate spherical harmonic for component p. I and E are respectively the internal (INT) and external (EXT) coefficients of the magnetic potential to be determined for each l and m and each frequency ω.

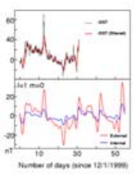
ω varies from DC to Nyquist

The degree l varies from 1 to l_{max} and the order m from –l to l.

For a perfect set of data, I(ω) and E(ω) can be determined uniquely.

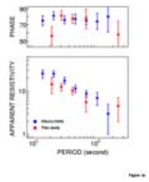
In order to get I and E, either least-square (LS) or integral approach (IE) work. For IE:

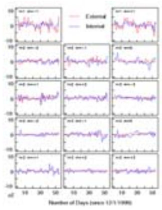




It is now well established that the mean one-dimensional (1-D) response of the Earth may be obtained from satellite data. Using the main component of the magnetospheric source field (the external dipole), the internal/external ratio is obtained at a number of periods (right figure). It may then be inversed to obtain a mean global 1-D conductivity profile.

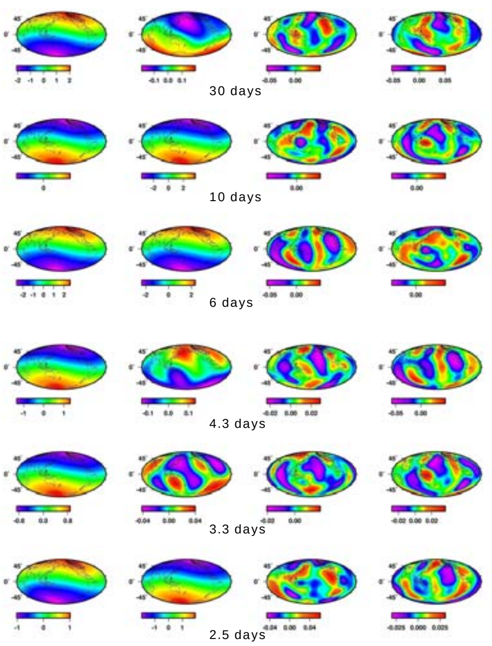
While there is a great value to use satellite data to obtain such a 1-D mean reference conductivity profile, the real challenge in front of us is to obtain the tri-dimensional (3-D) response of the earth in such a way that 3-D conductivity models of the earth's mantle may be calculated.





Above: The external and internal potentiel field of the magnetospheric dipole has be obained from the Oersted data. For comparison the external potential is shown on top of the Dst index (top).

To the right: External and internal potentials for different degrees and orders of the spherical expansion of the magnetic field. While significantly smaller than the main dipole term, they indicate that both the external and internal fields contain energy at wavelength smaller than the degree 1 and order 0 considered above.



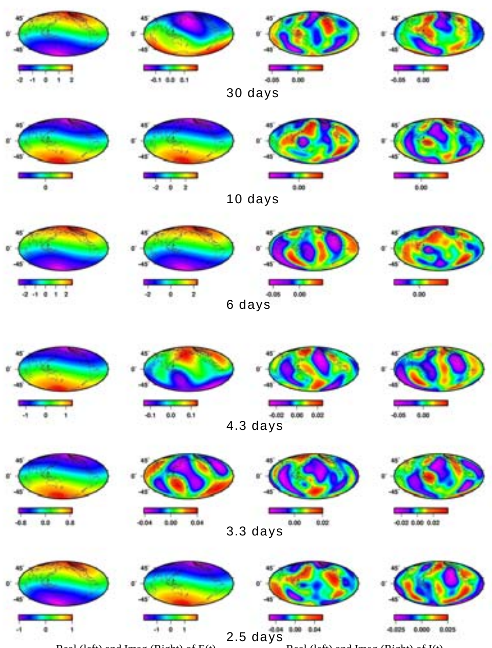
Real (left) and Imag (Right) of E(t) Real (left) and Imag (Right) of I(t)

Equation (1) has been used on the E2E simulator induction data simulated for different SWARM constellations. The synthetic data used are calculated for a 1-D mantle and a thin heterogeneous upper shell simulating the ocean / sediment / continent contrast (see the E2E report). The objective of the inversion presented here is to obtain the correct distribution of conductivity in this upper shell, at the given limited wavelengths l=1-7

On each panel to the left and to the right, the external (E) and internal (I) potentials have been computed for a SHE up to l=3 for E and l=7 for E and up to 40 periods. The real and imaginary part of the resulting coefficients are displayed in the space domain for a choice of 6 periods for different tests (using 30 days of constellation 3 and 4 to the left and one year to the right).

The external potential (E) shown on both panels at a given period is the sum of l=1-3 and is dominated by the l=1 dipole. The internal potential (I) is the sum of l=2-7 to emphasize the visibility of wavelengths shorter than l=1.

The potential coefficients extracted from the synthetic satellite SWARM data and displayed on both panels are used in a full 3-D inversion approach. The data modelled here consist of SHE coefficients from l=1-3 for the external field and l=1-7 for the internal field. I used 15 periods from 30 days to 2.14 days.



Real (left) and Imag (Right) of E(t) Real (left) and Imag (Right) of I(t)

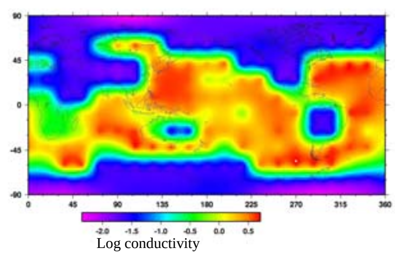
The full 3-D inverse procedure uses the external E coefficients as forcing terms at all periods. Here we limited the demonstration to an upper heterogeneous layer and a stack of homogeneous shells. Note that in general any number of heterogeneous shells may be used.

The inversion is carried out to minimize a weighted misfit function between the internal potentials obtained from the satellite data and the internal potential calculated with my 3-D forward solver. In the case treated here, the upper shell was divided into 10 (latitude) by 20 (longitude) meshes. The iterative inverse procedure seeked the optimum distribution of conductivity within the upper shell to minimize the misfit.

In order to accelerate the procedure, a preliminary 1-D model was seeked using both E and I coefficients at l=1-3. A 1-D model close to the 1-D model used to synthetize the data was obtained. Then the 3-D inversion was run starting from the 1-D model obtained.

The result of the inversion is given in the figure to the right. The result is quite satisfactory. Both the shape and the amplitude of the conductivity anomalies are correctly retrieved. Note that because a SHE of the data limited to l=7 was used, no fine detail may be recovered here.

Upper shell structure obtained from the inversion of the I data Forced by the E data through a 3-D induction solver



Conclusion:

The result presented in this poster shows that:

1/ It is possible to obtain from SWARM data set a series of induced data at different wavelengths and periods along with the inducing external field

2/ The external field obtained may be used as a forcing term in 1-D and 3-D modelling to recover the electrical conductivity

3/ The 3-D inversion of these data sets is now working

In practice:

There is obviously no need to recover the coast effect from the data and procedure either correcting or taking into account this effect should be used as a preconditionner to the data inversion. I intend to use a subsequent set of data calculated during the E2E project to inverse for the mantle structure and study the best approaches to optimize the SWARM constellation.