Electrical Conductivity in the Earth's Mantle: Combined Inversion of Surface and CHAMP Observations

Jakub Velímský and Zdeněk Martinec

E-mail: jakub.velimsky@mff.cuni.cz WWW: http://geo.mff.cuni.cz/~velimsky/

Department of Geophysics, Faculty of Mathematics and Physics, Charles University in Prague, Czech Republic

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Summary

We present a modification of the time-domain, spherical harmonic-finite element approach (Velímský and Martinec, 2005) that is based on the combined use of vector magnetic field measurements from satellites and surface observatories. We use a physical model that consists of a spherical Earth with a heterogeneous conductivity distribution surrounded sequentially by

1. An insulating atmosphere;
2. An ionosphere approximated by an infinitesimally thin spherical sheet of horizontal currents;
3. Another insulating layer where satellite data are acquired; and
4. A distant current system in the magnetosphere.

The inverse problem is then formulated using two datasets. The first dataset contains CHAMP night-time data processed into track-by-track time series of zonal spherical harmonic coefficients of vertical and horizontal field. The second dataset comprises of the spherical harmonic coefficients of hourly means from surface observatories. The conductivity is sought in terms of layered 1-D spherically symmetric model with a laterally varying surface conductance map prescribed on top.

References

Everett, M.E., S. Constable & C. Constable, 2003. Effects of near-surface conductance on global satellite induction responses. *Geophys. J. Int.,* **153,** 277–286.

Velímský, J. & Z. Martinec, 2005. Time-domain, spherical harmonic-finite element approach to transient three-dimensional geomagnetic induction in a spherical heterogeneous Earth, *Geophys. J. Int.,* **161**, 81–101.

Velímský, J., Z. Martinec, & M.E. Everett. Electrical conductivity in the Earth's mantle inferred from CHAMP satellite measurements — I. Data processing and 1-D inversion, *Geophys. J. Int., in press.*

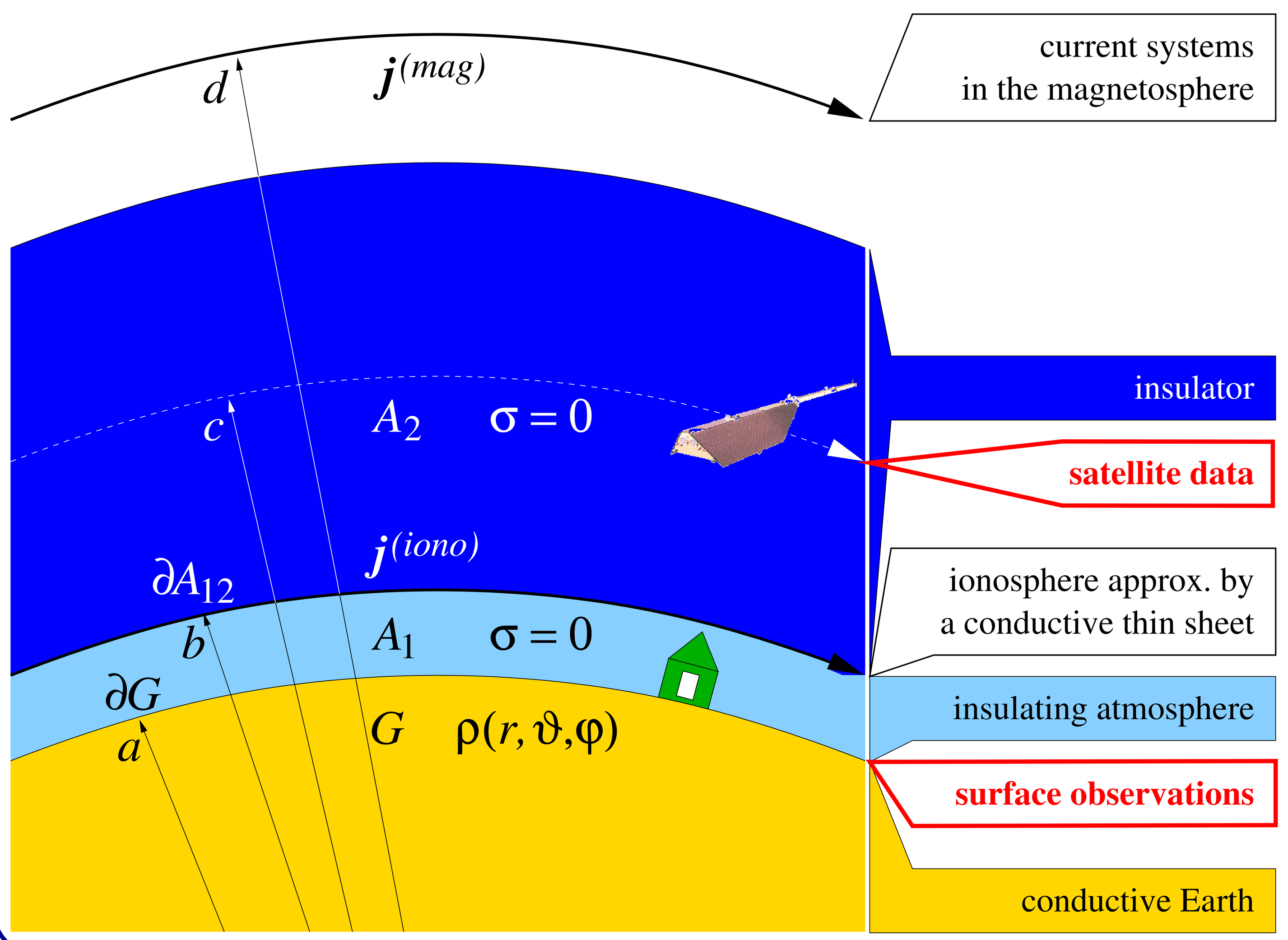
Physical configuration and computational domains

$d > 4a$

$c = a + h_{sat} \approx \begin{cases} a + 400\,\mathrm{km} & \text{for CHAMP} \\ a + 700\,\mathrm{km} & \text{for Ørsted} \end{cases}$

$b = a + 110\,\mathrm{km}$

$a = 6371\,\mathrm{km}$



Governing equations

Analytical solutions

in A_2: $\mathbf{B}(r,\vartheta,\varphi;t_i) = -\mathrm{grad}\,U^{(2)}(r,\vartheta,\varphi;t_i)$

$$U^{(2)}(r,\vartheta,\varphi;t_i) = c\sum_{j=1}^{j_{\max}}\sum_{m=-j}^{j}\left[{}^iG^{(e,2)}_{jm}\left(\frac{r}{c}\right)^j + {}^iG^{(i,2)}_{jm}\left(\frac{c}{r}\right)^{j+1}\right]Y_{jm}(\Omega)$$

on ∂A_{12}: $[\mathbf{n}\cdot\mathbf{B}]^+_- = 0$

in A_1: $\mathbf{B}(r,\vartheta,\varphi;t_i) = -\mathrm{grad}\,U^{(1)}(r,\vartheta,\varphi;t_i)$

$$U^{(1)}(r,\vartheta,\varphi;t_i) = a\sum_{j=1}^{j_{\max}}\sum_{m=-j}^{j}\left[{}^iG^{(e,1)}_{jm}\left(\frac{r}{a}\right)^j + {}^iG^{(i,1)}_{jm}\left(\frac{a}{r}\right)^{j+1}\right]Y_{jm}(\Omega)$$

on ∂G: $[\mathbf{B}]^+_- = 0$, $\mathbf{n}\times\delta\mathbf{B} = 0$

Spherical–harmonic finite–element approach

in G:

Find $\mathbf{B} \in H^h_{\mathrm{curl}}$: $\displaystyle\int_G \left[\mu_0\frac{\partial\mathbf{B}}{\partial t}\cdot\delta\mathbf{B} + \rho\,\mathrm{curl}\,\mathbf{B}\cdot\mathrm{curl}\,\delta\mathbf{B}\right]dV = 0\ \forall\delta\mathbf{B}\in H^h_{\mathrm{curl},0}$

$$H^h_{\mathrm{curl}} = \left\{\mathbf{B}\,\middle|\,\mathbf{B}(r,\vartheta,\varphi;t_i) = \sum_{j=1}^{j_{\max}}\sum_{m=-j}^{j}\sum_{\lambda=-1}^{1}\sum_{k=1}^{P+1}{}^iB^{(\lambda)}_{jm,k}\,\psi_k(r)\,\mathbf{S}^{(\lambda)}_{jm}(\Omega)\right\}$$

$$H^h_{\mathrm{curl},0} = \left\{\delta\mathbf{B}\,\middle|\,\delta\mathbf{B} = \sum_{jm}\left[\sum_{\lambda=-1}^{1}\sum_{k=1}^{P}\delta B^{(\lambda)}_{jm,k}\,\psi_k\,\mathbf{S}^{(\lambda)}_{jm} + \delta B^{(-1)}_{jm,P+1}\,\psi_{P+1}\,\mathbf{S}^{(-1)}_{jm}\right]\right\}$$

Implementation of boundary conditions

$$X^{(1|2)}(\mathbf{r};t_i) = \sum_{jm}\left[{}^iG^{(e,1|2)}_{jm}\left(\frac{r}{a|c}\right)^{j-1} + {}^iG^{(i,1|2)}_{jm}\left(\frac{a|c}{r}\right)^{j+2}\right]\frac{\partial Y_{jm}}{\partial\vartheta}(\Omega)$$

$$Y^{(1|2)}(\mathbf{r};t_i) = \frac{-1}{\sin\vartheta}\sum_{jm}\left[{}^iG^{(e,1|2)}_{jm}\left(\frac{r}{a|c}\right)^{j-1} + {}^iG^{(i,1|2)}_{jm}\left(\frac{a|c}{r}\right)^{j+2}\right]\frac{\partial Y_{jm}}{\partial\varphi}(\Omega)$$

$$Z^{(1|2)}(\mathbf{r};t_i) = \sum_{jm}\left[j\,{}^iG^{(e,1|2)}_{jm}\left(\frac{r}{a|c}\right)^{j-1} - (j+1)\,{}^iG^{(i,1|2)}_{jm}\left(\frac{a|c}{r}\right)^{j+2}\right]Y_{jm}(\Omega)$$

$${}^iX^{(1|2)}_{jm} = {}^iG^{(e,1|2)}_{jm}\left(\frac{r}{a|c}\right)^{j-1} + {}^iG^{(i,1|2)}_{jm}\left(\frac{a|c}{r}\right)^{j+2}$$

$${}^iZ^{(1|2)}_{jm} = j\,{}^iG^{(e,1|2)}_{jm}\left(\frac{r}{a|c}\right)^{j-1} - (j+1)\,{}^iG^{(i,1|2)}_{jm}\left(\frac{a|c}{r}\right)^{j+2}$$

$\forall jm$ there are $3(P+1)+4$ unknowns:

$$\left\{\left[{}^iB^{(\lambda)}_{jm,k}\right]^1_{\lambda=-1}\right\}^{P+1}_{k=1},\ {}^iG^{(e,1)}_{jm},\ {}^iG^{(i,1)}_{jm},\ {}^iG^{(e,2)}_{jm},\ {}^iG^{(i,2)}_{jm}$$

Galerkin method in G provides $3P+1$ equations using test functions:

$$\left\{\left[\delta B^{(\lambda)}_{jm,k}\right]^1_{\lambda=-1}\right\}^{P}_{k=1},\ \delta B^{(-1)}_{jm,P+1}$$

Continuity of $\mathbf{B}$ across ∂G provides:

$${}^iB^{(-1)}_{jm,P+1} + j\,{}^iG^{(e,1)}_{jm} - (j+1)\,{}^iG^{(i,1)}_{jm} = 0$$

$${}^iB^{(0)}_{jm,P+1} = 0$$

$${}^iB^{(1)}_{jm,P+1} + {}^iG^{(e,1)}_{jm} + {}^iG^{(i,1)}_{jm} = 0$$

Continuity of Z across ∂A_{12} provides:

$$j\left(\frac{a}{b}\right)^{j-1}{}^iG^{(e,1)}_{jm} - (j+1)\left(\frac{b}{a}\right)^{j+2}{}^iG^{(i,1)}_{jm}$$

$$-j\left(\frac{c}{b}\right)^{j-1}{}^iG^{(e,2)}_{jm} + (j+1)\left(\frac{b}{c}\right)^{j+2}{}^iG^{(i,2)}_{jm} = 0$$

$\forall jm$ and a time slice t_i are two boundary conditions that must be provided, one for the Earth's surface ∂G:

1A) ${}^iG^{(e,1)}_{jm}$ model of the external (ionospheric and magnetospheric) field based on surface observations or a-priori considerations

1B) ${}^iX^{(1)}_{jm}$ surface observations of the horizontal components

1C) ${}^iZ^{(1)}_{jm}$ surface observations of the vertical components

and one at the satellite's altitude:

2A) ${}^iG^{(e,2)}_{jm}$ model of the magnetospheric field based on satellite observations or a-priori considerations

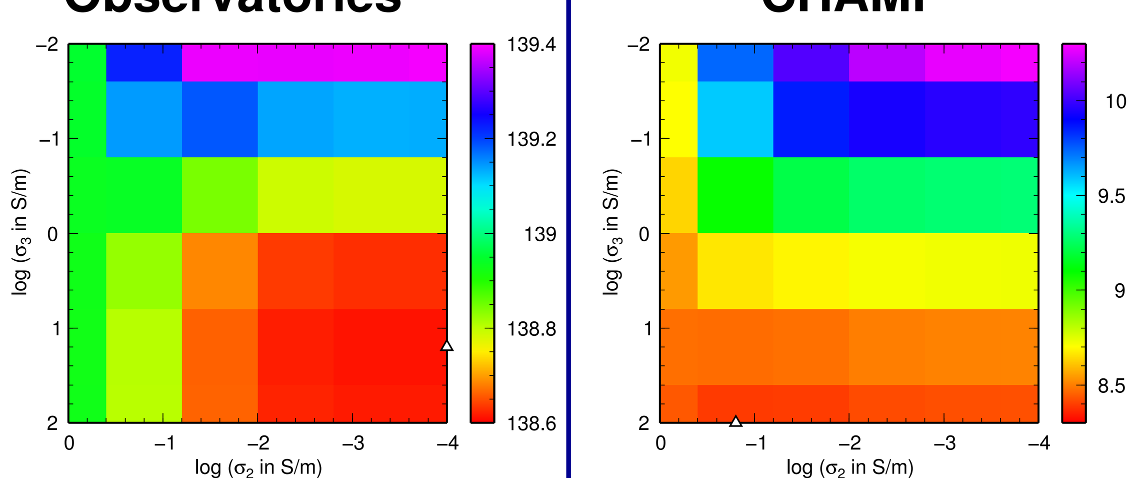
2B) ${}^iX^{(2)}_{jm}$ observations of the horizontal components by a satellite at radius c

2C) ${}^iZ^{(2)}_{jm}$ observations of the vertical component by a satellite at radius c

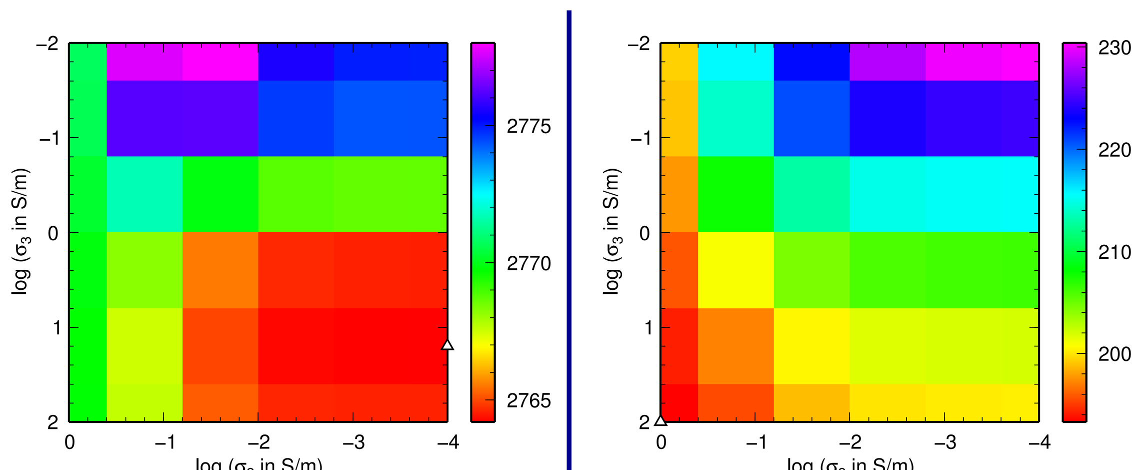
Any combination of the boundary conditions {1A,1B,1C} ⊗ {2A,2B,2C} is possible.

Results

Observatories **CHAMP**



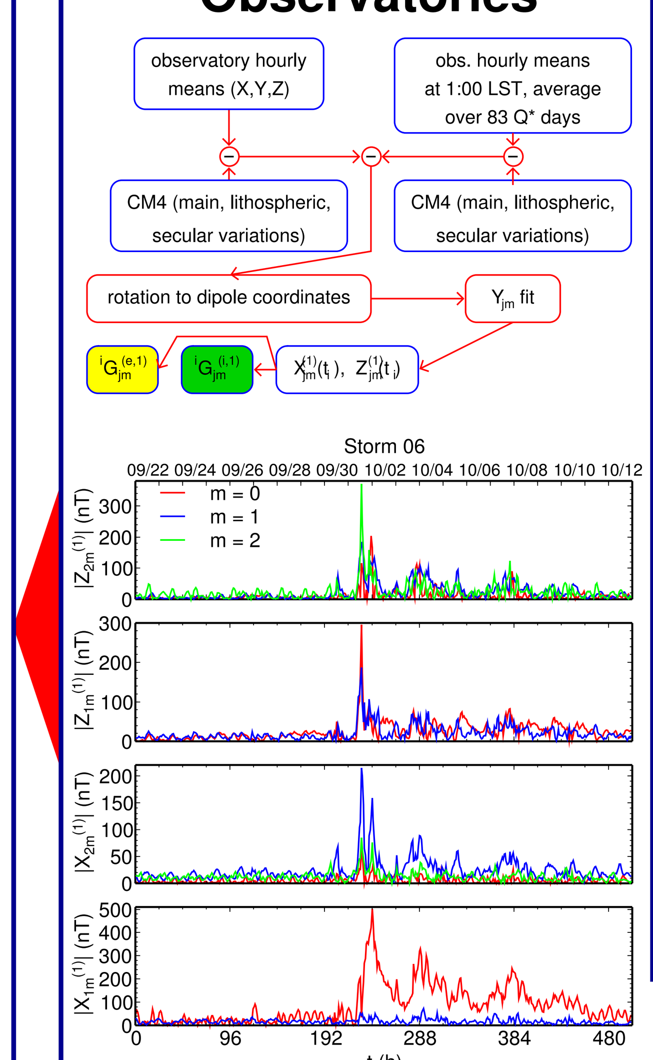
$$\chi^2_{L_2}(\sigma_2,\sigma_3) = \frac{1}{N_I}\sum_{i\in I}\sum_{jm}\left|{}^iG^{(i,1|2)}_{jm} - {}^iG^{(i,1|2)}_{jm}(\sigma_2,\sigma_3)\right|^2$$



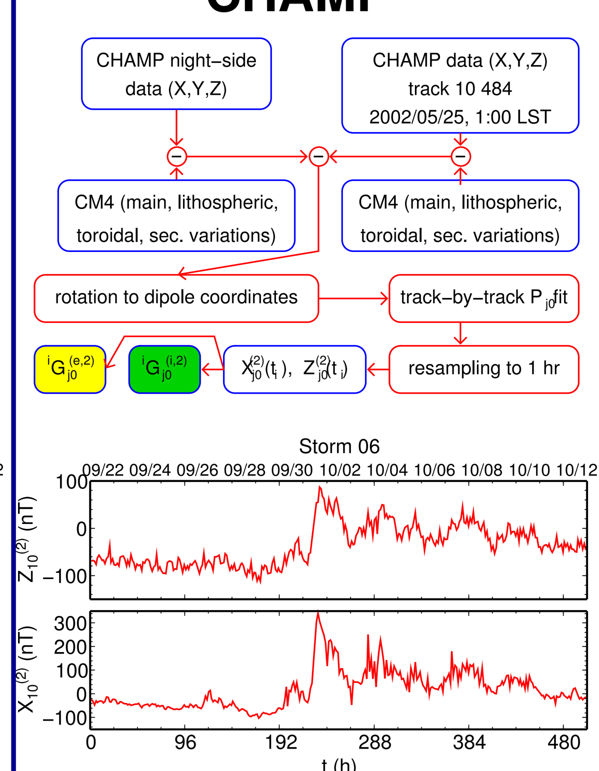
$$\chi_{L_1}(\sigma_2,\sigma_3) = \frac{1}{N_I}\sum_{i\in I}\sum_{jm}\left|{}^iG^{(i,1|2)}_{jm} - {}^iG^{(i,1|2)}_{jm}(\sigma_2,\sigma_3)\right|$$

Data processing

Observatories



CHAMP

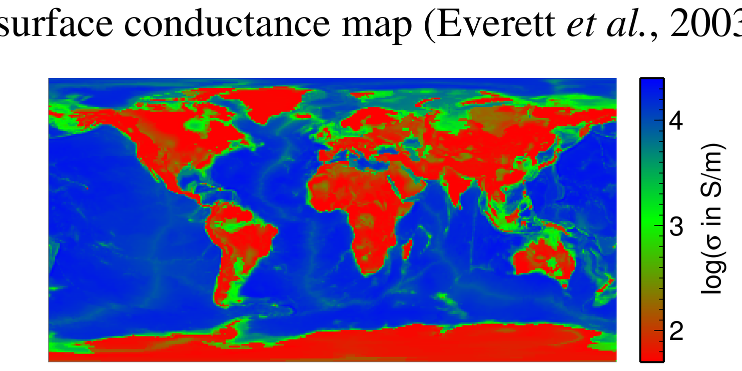


Database overview

Storm	Days		Obs.	CHAMP night-side tracks			
	From	To		First	Last	Total	Missing
1	2001/09/21	2001/10/07	71	6 669	6 931	263	0
2	2001/10/11	2001/10/27	74	6 978	7 240	262	1
3	2001/11/17	2001/11/29	78	7 550	7 751	197	5
4	2002/04/07	2002/04/26	89	9 738	10 047	306	4
5	2002/08/24	2002/09/12	84	11 897	12 207	311	0
6	2002/09/22	2002/10/12	89	12 348	12 673	324	2
7	2003/05/18	2003/06/01	88	16 048	16 280	233	0
8	2003/06/07	2003/06/21	88	16 359	16 591	233	2
9	2003/07/01	2003/07/24	94	16 732	17 105	374	1
10	2003/10/19	2003/11/03	88	18 445	18 694	250	12
11	2003/11/11	2003/11/25	88	18 804	19 037	234	6

Inverse problem setup

- Assumption of axially symmetric ring-current
- Zonal SH coefficients ${}^iG^{(e,2)}_{j0}$ are derived on track-by-track basis from night-side CHAMP vector data
- SH coefficients ${}^iG^{(e,1)}_{jm}$ are derived from observatory hourly means
- ${}^iG^{(i,2)}_{j0}$ and ${}^iG^{(i,1)}_{jm}$ are used to evaluate data misfit for various conductivity models
- we concentrate on data form 11 largest storms from 2000–3
- Model parameterization:
  - 0–50 km: surface conductance map (Everett *et al.*, 2003)



  - 50–670 km: σ_2
  - 670–2891 km: σ_3
  - core: $\sigma_4 = 10^5\,\mathrm{S/m}$
- parametric space of σ_2, σ_3 is explored using with sampling of 0.4 in the log scale