A LAYMAN LOOKS AT CURRENTS IN THE IONOSPHERE

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ABSTRACT/RESUME

This paper presents, in a simple manner, the basic physics that lies behind the '+diamagnetic' and 'gravity drift' magnetic field corrections that are beginning to be used in analysis of CHAMP data taken when it is in the night-time equatorial F-region ionosphere. The results are obtained by considering the detailed trajectories of ions and electrons subject to combined fields, as well as by the conventional force-density macroscopic approach. The resulting electric currents give magnetic perturbations outside as well as inside the ionosphere. It is also pointed out that magnetometers are calibrated for use in 'free space', so that using them inside the 'diamagnetic' ionosphere could lead to incorrect readings; the resulting errors could be of the same magnitude as that of the field perturbations themselves.

1. INTRODUCTION

In between collisions, ions and electrons have their trajectory determined by the force equation

$$\boldsymbol{F} = m\mathrm{d}\boldsymbol{v}/\mathrm{d}t = e\boldsymbol{v}\mathbf{x}\boldsymbol{B} + m\boldsymbol{g} + e\boldsymbol{E}. \qquad (1)$$

The resulting motion of each ion is complicated, and it would be difficult to determine the resulting magnetic field. However the contribution to the local $\boldsymbol{E}$ and $\boldsymbol{B}$ produced by any one ion will give no force on that ion, so these contributions can be ignored, making the equation linear in $\boldsymbol{v}$. The $m\boldsymbol{g}$ and $e\boldsymbol{E}$ terms make the equation inhomogeneous, but a general solution can be obtained by adding a *particular* solution of the *whole* equation to the *general* solution of the *homogeneous* part. It is therefore possible to add the velocity solutions of simpler sub-problems, which themselves have simple physical interpretations. In particular, once $\boldsymbol{B}$ is given, it is possible to treat separately the effects of adding the fields $\boldsymbol{g}$ and $\boldsymbol{E}$. And in the present context one can then simply add the magnetic fields given by each separate sub-problem.

In the next two sections I look at the trajectory of 'thermal' positive ions and electrons in a $\boldsymbol{B}$ field, and the corresponding magnetic field perturbation – the 'adiabatic' term. Then I look at the trajectories, and associated magnetic fields, for ions/electrons initially at rest in $\boldsymbol{B}$ and $\boldsymbol{g}$, or $\boldsymbol{B}$ and $\boldsymbol{E}$, fields. The magnetic field of the overall motion of thermal electrons/ions in $\boldsymbol{B}$, $\boldsymbol{g}$, and $\boldsymbol{E}$ fields is (in the present situation) simply the sum of these separate contributions

2. THE DIAMAGNETIC TERM ev□B

For an ion moving with velocity $\boldsymbol{v}$ in the presence of a magnetic field $\boldsymbol{B}$, in otherwise field-free space, the force equation is just $\boldsymbol{F} = e\boldsymbol{v}\square\boldsymbol{B}$. The magnetic force on the ion is perpendicular to its motion, so that $\boldsymbol{v}_{\mathrm{par}}$, the component of velocity $\boldsymbol{v}$ along $\boldsymbol{B}$, is unchanged. The component $\boldsymbol{v}_{\mathrm{perp}}$ of $\boldsymbol{v}$ perpendicular to $\boldsymbol{B}$ gives a circular motion in the perpendicular plane, having speed v_{perp}, and gyro-radius and period given by

$$r_{\mathrm{d}} = mv_{\mathrm{perp}}/eB, \qquad \tau = 2\pi m/eB. \qquad (2)$$

Thus the ion has a helical trajectory, with the axis of the (clockwise) circular motion anti-parallel to $\boldsymbol{B}$. (In this situation, the only combination of the relevant variables that has the dimension of time is (m/eB); most treatments work in terms of the (angular) frequency (eB/m). The velocity scale is given by the ion's v_{perp}.)

The circular part of the motion looks like an electric current of strength e/τ flowing round a loop of area πr_{d}^2. At distances large compared with its radius it therefore looks like a magnetic dipole of moment

$$p_{\mathrm{d}} = (e/\tau)\,\pi\, r_{\mathrm{d}}^2 = mv_{\mathrm{perp}}^2/2B, \qquad (3)$$

oriented anti-parallel to $\boldsymbol{B}$. (Note that the dipole moment cannot be increased indefinitely; as B decreases the gyro-frequency also decreases, and eventually the 'collisionless' approximation breaks down.)

Now consider all the ions in a region in which there are n ions per unit volume; the region is large compared with r_{d}, but small enough that $\boldsymbol{B}$ is uniform within it. Unless there is some deliberate injection of a stream of ions, the $\boldsymbol{v}_{\mathrm{par}}$ part of the ion velocities will be random in sign, so that on average they will give no resultant current. But the $\boldsymbol{v}_{\mathrm{perp}}$ parts of their velocities all give anti-parallel moments $\boldsymbol{p}_{\mathrm{d}}$. The ion 'gas' therefore looks (in some respects) like a diamagnetic material, having a magnetisation $\boldsymbol{M}$ (dipole moment per unit volume) given by

$$\boldsymbol{M} = -(1/B)(\tfrac{1}{2}nmV_{\mathrm{perp}}^2)\,\boldsymbol{b}, \qquad (4)$$

where V_{perp} is the rms value of $\boldsymbol{v}_{\mathrm{perp}}$, and $\boldsymbol{b}$ is a unit vector in the direction of $\boldsymbol{B}$. Treating the ions as a gas in thermal equilibrium, the initial ion velocities will be random in direction, so V_{perp}^2 will be $(2/3)V^2$, where V is the total (3-dimensional) rms ion speed. We also have $\tfrac{1}{2}mV^2 = 3kT/2$, where k is Boltzmann's constant, and T

is the ion 'temperature'. Alternatively we can put $P = (nmV^2/3)$, where P is the ion 'gas pressure' P. So we have the various forms

$$\boldsymbol{M} = -(1/B)(\tfrac{1}{2}nmV_{perp}^2)\,\boldsymbol{b} = -(1/B)\,nkT\,\boldsymbol{b}$$
$$= -(1/B^2)\,nkT\,\boldsymbol{B} = -(1/B)\,P\,\boldsymbol{b} = -(1/B^2)\,P\,\boldsymbol{B}. \quad (5)$$

Note that the direction of the magnetisation is independent of (the sign of) the charge e, so that both ions and electrons give the same anti-parallel direction of $\boldsymbol{M}$. For electrons the 'pressure' $P = nmV^2/3$ will be comparable to that for the ions, so both ions and electrons contribute to the diamagnetism.

For simply shaped regions of uniform magnetisation, the resultant field perturbation can be obtained analytically. However for more complicated geometries, the magnetic field has to be determined by numerical integration. For an individual dipole moment $\boldsymbol{p}$ at the origin, it is simple to calculate the resulting magnetic field at position $\boldsymbol{r}$:

$$\boldsymbol{B} = \mu_0[\boldsymbol{p} + 3(\boldsymbol{p}\bullet\boldsymbol{r})\boldsymbol{r}/r^2]/4\pi r^2. \quad (6)$$

Similarly, for a distribution of magnetisation $\boldsymbol{M}$, integration over the distribution can be used to give the magnetic field *outside* the distribution. But this integral is *not* uniquely convergent *inside* the distribution; and while the integrals for magnetic scalar potential φ, or vector potential $\boldsymbol{A}$, *are* convergent inside, differentiating them to give $\boldsymbol{H}$ or $\boldsymbol{B}$ leads to the same non-convergent integral. One way of obtaining a convergent integral is to use an appropriate (purely mathematical) vector transformation on the integrand. One such transformation is *equivalent* to replacing the distribution of magnetisation $\boldsymbol{M}(x,y,z)$ by a distribution of current density (current/unit area) $\boldsymbol{j}_d(x,y,z)$,

$$\boldsymbol{j}_d = \text{curl}\,\boldsymbol{M}, \quad (7)$$

and then to calculate the vector potential $\boldsymbol{A}$, and hence magnetic field $\boldsymbol{B}$, given by this $\boldsymbol{j}_d$; see any standard text, or e.g.[1]. (In the ionosphere this is equivalent to using Ampere's law for $\boldsymbol{B}$ rather than for $\boldsymbol{H}$, as the current density is assumed to be in free space – see the next paragraph.)

When dealing with conventional microscopic magnetisation (coming from nuclear and electron spins and/or orbits) this equivalent macroscopic current density is fictitious, so does not contribute to **curl** $\boldsymbol{H}$. However, in the ionosphere the orbits are themselves of *macroscopic* scale. In a region of uniform $\boldsymbol{M}$, at any one instant the ions are moving in random phase in their orbits, so there is no overall linear current. But in a region where $\boldsymbol{M}$ is varying in space, the cancellation is not perfect, giving a local **curl** $\boldsymbol{M}$ *real* current density.

Using the gas pressure approach, and making the assumption that $\boldsymbol{B}$ is uniform in strength and direction, we finally get

$$\boldsymbol{j}_d = (1/B)\,\mathbf{curl}(-P\boldsymbol{b}) = -(1/B)(\mathbf{grad}\,P)\times\boldsymbol{b}$$
$$= -(1/B^2)(\mathbf{grad}\,P)\times\boldsymbol{B}. \quad (8)$$

This current $\boldsymbol{j}_d$ flows in planes perpendicular to $\boldsymbol{B}$, in closed loops along the contours of P in that plane. The integrated current flow (ampere/metre) in going from outside to the centre of the ionised region is P/B. Where **grad** P is large (e.g. top boundary), this current is concentrated into a thin region of high current density, but where **grad** P is small (side boundaries) the current flows in a thick region of low current density. (If there is a discontinuous change $\delta\boldsymbol{M}$ in $\boldsymbol{M}$, then the volume current density **curl** $\boldsymbol{M}$ is replaced by a surface current density $\delta\boldsymbol{M}\times\boldsymbol{n}$.)

If we know, or can model, P, and hence deduce $\boldsymbol{j}_d$, a straightforward, if messy, integration can be used to calculate the resultant $\delta\boldsymbol{B}$ everywhere, both outside and inside the ionosphere. Ideally this would mean that for *each* measurement time an integration would be needed over the *whole* of the ionised region (which would probably involve using a model). So far, to avoid such an integration, an approximate calculation has been used, with the advantage that only the *local* ion pressure needs to be known, but unfortunately it is a rather poor approximation. This approximation, used by [2], is to assume that the sum of the ion 'pressure' P and the magnetic 'pressure' $\tfrac{1}{2}B^2/\mu_0$ is constant.

Putting $\boldsymbol{B} = \boldsymbol{B}_0 + \delta\boldsymbol{B}$, where $\delta\boldsymbol{B}$ is the perturbation caused by the diamagnetism, and remembering that $\delta B \ll B_0$, the approximation gives $\boldsymbol{B}.\delta\boldsymbol{B} = -P\mu_0$. Assuming that $\delta\boldsymbol{B}$ is anti-parallel to $\boldsymbol{B}$, this gives $\delta B = -\mu_0 P/B$ for the reduction in B when going from outside the ionosphere to a point inside where the ion pressure is P. (For the night-time F-region values used by [2], M is about 2×10^{-3} A/m, corresponding to P about 6×10^{-8} Pa, giving $\mu_0 M = \mu_0 P/B = 2.5$ nT.) That $\delta\boldsymbol{B}$ is anti-parallel to $\boldsymbol{B}$ is probably a good approximation inside the ionosphere. But the discussion below on the magnetostatics of simple bodies shows that for a finite (uniformly) magnetised region this value for δB is the *maximum* possible, and would be valid only for a 'long thin' region.

Another problem is that this magnetic pressure approach completely ignores the corresponding field change *outside* the ionosphere; for the geometry of the CHAMP orbit this will be a *gradual* reduction in field intensity as the plasma region is approached, and *not* an abrupt drop at the boundary. Of course the real ionosphere does not have sharp boundaries, but the approximation inevitably predicts zero field perturbation outside the ionosphere.

3. FIELD PRODUCED BY VOLUME OF DIAMAGNETIC MATERIAL

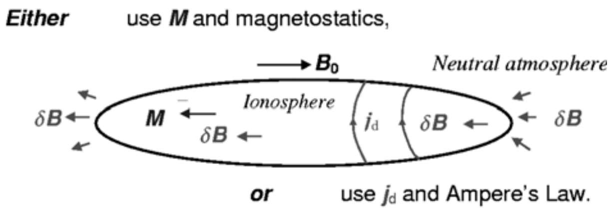

Figure 1. Magnetic field is produced both inside and outside the diamagnetic ionosphere.

For some simply shaped regions of uniform magnetisation, the resultant field perturbation can be obtained analytically. For example, think of inserting a uniform diamagnetic ellipsoid (having one axis parallel to $\boldsymbol{B}_0$) into a previously uniform $\boldsymbol{B}_0$ in a vacuum – Fig. 1. Inside this ellipsoid there is now a uniform backward $\boldsymbol{M}$, and so the $\boldsymbol{B}$ inside is reduced. But this is not the only effect. Because $\boldsymbol{B}$ is continuous at a normal boundary, this means that $\boldsymbol{B}$ is also reduced *outside* the ellipsoid on the axis parallel to $\boldsymbol{B}_0$. Conversely, around the 'equator' of the ellipsoid, the $\boldsymbol{B}$ outside is slightly increased. We can think of the diamagnetic body as repelling the lines of force of $\boldsymbol{B}$, and diverting some of them to now flow outside the body. Quantitatively, we have to everywhere add to the original uniform $\boldsymbol{B}_0$ the $\delta\boldsymbol{B}$ produced by the induced magnetisation. Inside the ellipsoid this is a uniform backward field, while outside it resembles that of a (backward) dipole. The reduction of internal $\boldsymbol{B}$ field will be in the range $(0\text{-}1)\mu_0 M$; the fraction is large for an ellipsoid which is long and thin with respect to the direction of $\boldsymbol{B}_0$, (2/3) for a sphere, and small for a short fat ellipsoid. The larger the reduction of internal $\boldsymbol{B}$, the larger the reduction of on-axis external $\boldsymbol{B}$. Clearly the real ionosphere is not ellipsoidal, but whatever its shape the situation will be similar.

This discussion has been in terms of the field $\boldsymbol{B}$ (measured in tesla), for which it does not matter if the backward 'magnetisation' $\boldsymbol{M}$ is produced in a physical material filling the ellipsoid (magnetisation that can be replaced/represented by 'fictitious' surface currents), or by *real* surface currents flowing on the surface of the ellipsoid (with there being a vacuum both inside and outside the ellipsoid); in both cases $\boldsymbol{B}$ is decreased inside the ellipsoid. But if what we measure is the field $\boldsymbol{H}$ (ampere/metre) there *is* a difference; for a material diamagnetic, $\boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M})$, so $\boldsymbol{H}$ is *increased* inside the material, but for our, real-current, 'vacuum', ionosphere, $\boldsymbol{B} = \mu_0\boldsymbol{H}$ everywhere, and $\boldsymbol{H}$ is *decreased* inside. In this context, saying that the plasma is diamagnetic can be confusing!

Inside the plasma, the relation between $\boldsymbol{M}$ and $\boldsymbol{H}$ is non-linear, but in a *particular* situation we can define an *effective* susceptibility χ_e, valid for that particular point on the M/H curve, as

$$\chi_e = M/H = (\text{very nearly})\ \mu_0 M/B = -\mu_0 P/B^2; \quad (9)$$

for our ions χ_e is about $-8\text{x}10^{-5}$, which is about 10 times that of water.

4. THE GRAVITATIONAL TERM *mg*

If we introduce gravity, the force equation for our ion is now

$$\boldsymbol{F} = e\boldsymbol{v}\times\boldsymbol{B} + m\boldsymbol{g}. \quad (10)$$

To investigate the effect of adding gravity, it is convenient to consider a situation in which the ion is initially at rest. The gravitational field $\boldsymbol{g}$ is vertical; for the time being assume that the magnetic field $\boldsymbol{B}$ is horizontal and northward. The ion starts to accelerate downwards with acceleration g, but as its downward speed builds up it is deflected more and more towards the east, until it is travelling horizontally. It then travels a mirror image path, eastwards and upwards, until it is at rest again at its original height, but displaced eastward, see Fig. 2. This is a cycloid, the path of a point on the rim of a wheel rolling eastward under a horizontal line through the starting point.

Figure 2. Cycloidal trajectory of a positive ion initially at rest.

For a more general direction of $\boldsymbol{B}$, we can resolve $\boldsymbol{g}$ into parts parallel and perpendicular to $\boldsymbol{B}$, as $\boldsymbol{g} = \boldsymbol{g}_{par} + \boldsymbol{g}_{perp}$. The $\boldsymbol{g}_{par}$ part is not affected by the magnetic field, so that, in our approximation of 'free' ions, the ions will experience uniform acceleration in this direction. (In practice, the acceleration will end at a collision.) But the electrons will experience the same acceleration, so there will be no net electric current. The $\boldsymbol{g}_{perp}$ part will lead to a sideways drift $(m/eB)\boldsymbol{g}_{perp}\times\boldsymbol{b}$, together with a circular motion.

The circular part of the sideways ion motion again corresponds to 'diamagnetic' magnetisation, but for our weak gravity this is completely negligible compared with the diamagnetic dipole moment p_d of Section 2.

However the average sideways drift corresponds to a *linear* current which is much more efficient at producing a magnetic field. For an ion density n, the average 'eastward' drift gives a current density of

$$\boldsymbol{j}_{\rm g} = (nm/B)\, \boldsymbol{g}_{\rm perp}\times\boldsymbol{b} = (nm/B^2)\, \boldsymbol{g}_{\rm perp}\times\boldsymbol{B}. \qquad (11)$$

Note that (like the diamagnetic magnetisation/current density) this current density is independent of the magnitude or sign of the charge. Negative charges drift westward, giving a conventional current eastward. But now, because of the isolated factor *m*, the contribution from the electrons is trivial.

But this situation is much more complicated than that in the diamagnetic case. Their, once the distribution of *P* (or nV^2) is given, the equivalent $\boldsymbol{j}_{\rm d}$ is immediately defined uniquely. The boundary of the ionosphere matters only in that it is inherent in the way the number density *n* varies in space. But for the gravity term, while we can think of the corresponding $\boldsymbol{j}_{\rm g}$ as 'source' or 'driving' currents, these $\boldsymbol{j}_{\rm d}$ are in themselves *not* consistent with the ionosphere being finite in extent, and having boundaries. In practice, charge distributions are built up (*very* quickly) on the boundaries (more generally, in regions where there is a gradient of electrical conductivity), the electrostatic field of which is such as to deflect the currents into closed paths. So before the resultant magnetic field perturbation $\delta\boldsymbol{B}$ can be calculated, the resulting, compromise, current distribution has to be determined. (In some respects the situation is analogous to that of the electric currents induced in the oceans by the interaction of ocean velocities with the geomagnetic field; in that case ($\boldsymbol{v}\times\boldsymbol{B}$) acts like a local forcing voltage gradient, but the eventual current distribution depends also on the ocean/land boundaries where there are conductivity contrasts. But in the ionosphere there is the extra complication that the electrostatic field, like the gravitational forcing term, results in perpendicular currents.) And then the resulting magnetic field has to be determined by integrating over the current system.

So far only a comparatively crude approximation of the resulting current flow has been tried [3], but the field perturbation (a few nT) produced by this approximate current distribution has been calculated both inside *and* outside the ionosphere.

5. THE ELECTROSTATIC TERM *eE*

If instead of introducing gravity we introduce ***E***, we have

$$\boldsymbol{F}= e\boldsymbol{v}\Box\boldsymbol{B} + e\boldsymbol{E}. \qquad (12)$$

This is essentially the same situation as in Section 4, but with $m\boldsymbol{g}$ replaced by $e\boldsymbol{E}$.

The part $\boldsymbol{E}_{\rm par}$ of ***E*** which is parallel to ***B*** will accelerate the ions (until the next collision), and look like a (varying) current in the direction of $\boldsymbol{E}_{\rm par}$. Electrons will be accelerated (greater acceleration, but more frequent collisions) in the opposite direction, giving a current in the same direction; so there could be a significant field-aligned current. But this contribution really needs to be considered in a larger-scale context, where collisions are taken into account; it does explain why the macroscopic electrical conductivity along the magnetic field lines is so much larger than the transverse conductivity.

The part $\boldsymbol{E}_{\rm perp}$ of ***E***, in the plane perpendicular to ***B***, will give the same type of cycloidal motion as above. The average sideways drift speed is $E_{\rm perp}/B$, and the corresponding current density is

$$\boldsymbol{j}_{\rm E} = (ne/B)\, \boldsymbol{E}_{\rm perp}\times\boldsymbol{b} = (ne/B^2)\, \boldsymbol{E}_{\rm perp}\times\boldsymbol{B}. \qquad (13)$$

This current density is independent of *m*, so has the same magnitude (but opposite sign) for electrons as for ions, and this magnitude is *very* much greater than for the gravity induced drift. However the very fact that the electrostatic forces are so comparatively large does mean that the number density of ions and electrons are essentially identical, and as the corresponding currents are in opposite directions, there is no net magnetic field.

Although the electrostatic field does not in itself produce a magnetic field, it is an important factor in determining the actual current flow (and hence magnetic field), produced by gravity. There is also the complication that, because of the very high electrical conductivity along the magnetic field lines, conjugate points north and south of the equator tend to have the same electrostatic potential.

6. THE MACROSCOPIC FORCE-DENSITY EQUATIONS

So far I have deduced the magnetic field contributions from the 'diamagnetic' and gravitational-drift effects by considering the motion of individual 'free' or 'collisionless' ions and electrons. But the individual trajectories are complicated, and most workers prefer to work with the *macroscopic* situation, starting with the force *density* (force per unit volume), obtained by taking an average over a volume small compared with the mean free path between collisions, but large compared with the gyro-radius. Equating this force density to zero for equilibrium for a particular ion species, then gives (see e.g.[4], eqn. (2.34), though this approximates **grad** *nkT* by *kT* **grad** *n*)

$$0 = -\mathbf{grad}\, P + nm\boldsymbol{g} + ne\boldsymbol{E} + ne(\boldsymbol{U}\mathbf{x}\boldsymbol{B}); \qquad (14)$$

here ***U*** is the *average* (arithmetic mean) local velocity of the ions, and it is assumed that this adjusts itself until the $ne(\boldsymbol{U}\mathbf{x}\boldsymbol{B})$ Lorenz force exactly cancels all the other forces. (I am ignoring any overall convective drift velocity of the atmosphere, carrying the plasma with it.) If this force density equation is solved for $\boldsymbol{U}_{\rm perp}$ (by taking the vector product with ***B*** throughout), we get for the average perpendicular current $ne\boldsymbol{U}_{perp}$ the same expression as is given by the 'trajectory' treatments above:

$$ne\boldsymbol{U}_{perp} = (-\mathbf{grad}\, P + nm\boldsymbol{g} + ne\boldsymbol{E})\mathbf{x}\boldsymbol{B}/B^2. \qquad (15)$$

In Eq. (15), the origin of the last two terms in the parentheses is obvious, but that of the first, diffusive, term is perhaps more subtle. If *P* were the pressure in a *neutral* atmosphere, then the balance between the upward diffusion driven by -**grad** *P*, and the downward gravitational $nm\boldsymbol{g}$, is what gives the conventional scale-height exponential reduction of density with height. For the ions in the F-region ionosphere, there is a similar situation in the higher parts, above the region of ion production. But there can be no such balance at the bottom and side 'boundaries' of the ionosphere. In a region of uniform ion density (more strictly **grad** *P* = 0) the ion density is given by an equilibrium between *local* ion production (from incoming radiation of various sorts) and *local* ion/electron recombination via collisions; any diffusion of ions into and out of any part of this region will cancel. If the ionisation rate changes with position, then, to first order, the ion density will change appropriately. But in a region where the ion density changes with position, there will be now be a *net* diffusion of ions (and electrons) down the density gradient; it is this net, macroscopic, diffusive motion which is being driven by the -**grad** *P* force density. Then, as with the other components of the perpendicular force density, this results in a (-**grad** *P*)**x*B*** type sideways current.

So the force density approach gives the same current distribution as the trajectory approach. But the force-density approach gives no *physical* insight (to me at least) as to why forces perpendicular to ***B*** give rise to sideways velocities; these velocities just arise in the mathematics. And in this approach the 'diamagnetic' behaviour of the plasma is not mentioned explicitly.

7. MEASURING MAGNETIC FIELD INSIDE THE IONOSPHERE

There is another major complication! As was pointed out in [5], even if magnetometers used only perfectly non-magnetic material, they are (in effect) calibrated as though they were working in a vacuum. If they are placed in a permeable medium they will displace the medium, and the field value they give will depend slightly on the permeability of the medium and the shape of the magnetometer. At that time the only medium of interest was water, of susceptibility only about -10^{-5}, and we were measuring to about 1 nT, so this did not matter. But now we are flying magnetometers through a medium with effective susceptibility about -10^{-4}, so the effect *will* be significant; the following discussion suggests that the effect will be comparable to the magnitude of the $\delta\boldsymbol{B}$ produced by the adiabatic ionosphere!

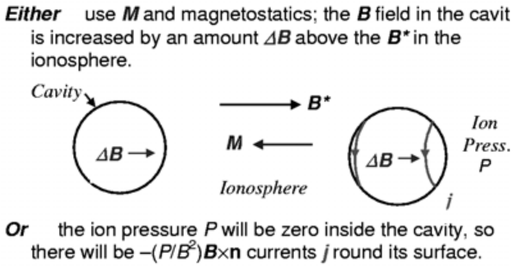

Figure 3. If a magnetometer (in a cavity) is inserted into the 'diamagnetic' ionosphere, the field inside the cavity will differ from that in the ionosphere.

The algebraic theory of [1,5] assumed a magnetic medium which had a linear *M*/*H* curve, so cannot be applied directly here. But qualitatively there will be a similar effect. For simplicity, think of the magnetometer as a physically small detector at the centre of a vacuum container (small detector, so that its own shape and materials do not affect the field near the surface of the container). Then put this magnetometer in the ionosphere, in a region where there is uniform ionisation, and hence uniform magnetisation ***M***, which is excluded by the magnetometer - see Fig. 3.

For the time being assume that the gyro-radii are small compared with the size of the magnetometer.. The field seen by the magnetometer can then be obtained in two ways.

7.1 Magnetisation approach

The magnetometer will exclude the ionospheric plasma. As a first, crude, approximation assume that the change from the ***M*** in the main part of the ionosphere to zero inside the magnetometer is abrupt; magnetically this is equivalent to adding a material having magnetisation -***M*** inside the magnetometer. So the field inside the magnetometer is now the original field $\boldsymbol{B}^*$ (= $\boldsymbol{B}_0 + \delta\boldsymbol{B} = \boldsymbol{B}_0 + (0\text{-}1)\mu_0\boldsymbol{M}$) in the ionosphere, together with the field given by the reverse magnetisation inside. This is simply the situation of Section 3 again, though with reversed sign. The field inside the magnetometer will be $(\boldsymbol{B}+\Delta\boldsymbol{B})$ where $\Delta\boldsymbol{B}$ is in the range $-(0\text{-}1)\mu_0\boldsymbol{M}$, with the factor being large for a magnetometer which is long and thin with respect to the direction of ***B***, (2/3) for a sphere, and small for a short fat magnetometer. (The same argument can be presented in terms of the equivalent (real) surface current ***M***□***n*** on the outside of the magnetometer.)

7.2 grad *P* approach

On this approach the currents producing the diamagnetic $\delta\boldsymbol{B}$ are some distance away from this region of uniform magnetisation, so are not affected by the magnetometer. But the ion/electron pressure is zero inside the magnetometer, so there must be local **-grad** *P* forces and currents around the surface of the magnetometer (for our assumed abrupt density change these will give the same ***M***□***n*** surface current density as above), and it is these local currents which will give the $\Delta\boldsymbol{B}$ field change inside the magnetometer

7.3 Discussion of magnetometer results

The above discussions use a very crude approximation, which also ignores the fact that the detector might occupy most of the 'container', and also the effect of the magnetometer materials. However it does indicate that the fact that the magnetometer excludes the plasma means that the field seen by the magnetometer could differ by a significant amount from the ambient field. An accurate calculation could be made for an idealised spherical magnetometer of uniform material, e.g. the water of a proton precession magnetometer. (The situation is complicated by the fact that for small *differential* changes of $\boldsymbol{H}$ the *differential* susceptibility is $\chi_d = \partial M/\partial H = +\mu_0 P/B^2$; note the change of sign!) But I suspect that the only way to estimate the effect of the shape and materials of the magnetometer will involve laboratory experiments with the actual magnetometers being immersed in fluids of different susceptibilities.

However Section 7.1 assumed that the magnetic effect of the local plasma corresponded to a local 'magnetisation', and Section 7.2 assumed that because the ion pressure P was zero inside the magnetometer this gave rise to surface currents. These assumptions are very dubious for the ions, which have a gyro-radius of about 6 m, considerably larger than the size of the magnetometer. So the concept of a local 'magnetisation' is probably not valid. Similarly, while there will certainly be no ions inside the magnetometer, so there will be a local -**grad** *P*, the conversion from a -**grad** *P* force to a -(**grad** *P*)x$\boldsymbol{B}/B^2$ average current density is probably valid only for length-scales large compared with the gyro-radius.

The electrons in this region have a much smaller gyro-radius, about 30 mm, so it is more likely that the arguments of Sections 7.1/7.2 are valid to some extent; the larger the mechanical/thermal shielding round the magnetometer, the more nearly these arguments will hold.

At present I have no idea how the presence of the spacecraft affects the trajectories of local ions and electrons. The spacecraft orbital speed is large compared with the ion thermal speeds, but small compared with the electron thermal speeds. There must have been a lot of work on the electrostatic charging of spacecraft when flying through the ionosphere, and the effect this has on ion trajectories; has any of this work considered the effect on local magnetic fields?

7.4 The effect of the gravity-drift current

My *guess* is that there will not be any error in measuring that part of the field coming from the $m\boldsymbol{g}$ and $e\boldsymbol{E}$ forces. These have (essentially) all their sources far from the magnetometer; I think that the resultant currents will simply be slightly deflected around the spacecraft, and that this deflection will not produce significant magnetic field perturbations.

7.5 The effect of the main spacecraft body

This provides another, much larger, nearby 'vacuum' cavity, in the medium. Its volume is about 5 m^3, and it is replacing a magnetisation of about 0.002 A/m by zero, so its effective dipole moment is 0.01 Am^2. At a distance R this will give a field of $(2/R^3)$ nT, so this should not be a problem.

8. DISCUSSION

I have shown how a simple approach can lead to the 'adiabatic' and 'gravity drift' electric currents in the ionosphere. But the calculation of the resulting magnetic field disturbance is significantly more complicated than the rather crude approximations used so far.

A problem that does not seem to have been appreciated so far is that the spacecraft magnetometers are (in effect) calibrated for use in 'free space' conditions. When they are flying through the 'adiabatic' ionosphere, and excluding the gyrating electrons (and possibly ions), it is likely that the field they indicate is not quite the same as the field actually present in the ionosphere.

8. REFERENCES

1. Lowes, F.J., The torque on a magnet, *Proc. R. Soc. Lond.*, **A337**, 555-567, 1974.
2. Lühr, H., Rother, M., Maus, S., Mai, W. & Cooke, D., The diamagnetic effect of the equatorial Appleton anomaly: its characteristics and impact on geomagnetic modelling, *Geophys. Res. Letters*, **30**, (17), 1906-1909, 2003.
3. Maus, S. & Lühr, H., A gravity-driven electric current in the Earth's ionosphere identified in CHAMP satellite magnetic measurements, *Geophys. Res. Letters*, **33**, L02812, doi:10.1029/2005GL024436, 2006.
4. Kelley, M.C., *The Earth's Ionosphere*, Academic Press, London, 1989.
5. Lowes, F.J., Do magnetometers measure B or H?, *Geophys. J. R. astr. Soc.*, **37**, 151-155, 1974.