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Trajectory Approach to Diamagnetic and Drift Currents

Ignore any convection of the ions and electrons with the neutral atmosphere.

Assume ions and electrons are collision-free.


Consider only forces (& motions) perpendicular to B_0 , and look at the resulting trajectory for an individual ion or electron.

Easiest to break the problem into three parts:

(1) Only field is $B=B_0\mathbf{x}$. Initial velocity $\mathbf{v}=(v_y, v_z)$

$B \otimes$ (into the paper) Ions Electrons

Trajectory is a circle in the (y,z) plane



(smaller but faster)

Giving dipole moment d \odot $m v^2 / 2 B_0$ \odot

(drift current zero)

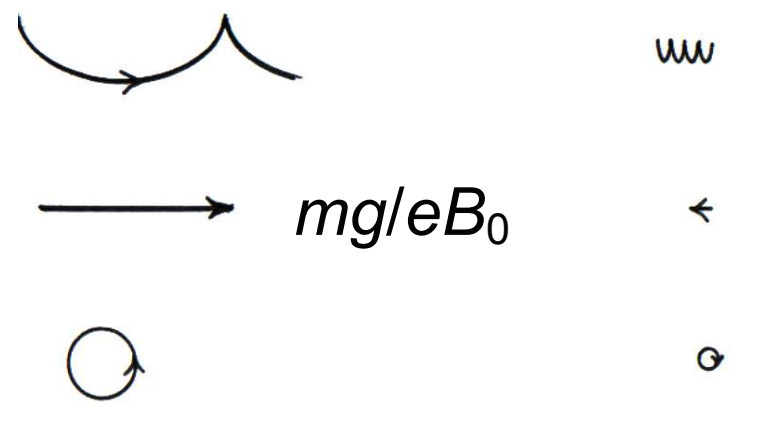
Both ions and electrons give dipole moment, opposite to B

This is the 'diamagnetic' effect

(2) Fields $B=B_0\mathbf{x}$ and $g=gz$. Initial velocity zero

$B \otimes$ $g \downarrow$ Ions Electrons

Trajectory is a cycloid = uniform drift + circle



Giving drift current j_g + dipole moment

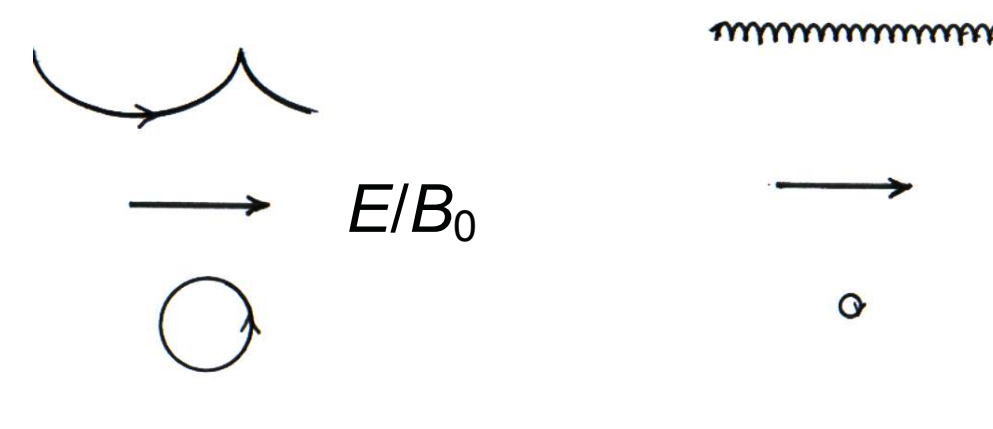
Only ions give significant drift current
Dipole moments are insignificant

This is the gravity drift current

(3) Fields $B=B_0\mathbf{x}$ and $E=Ez$. Initial velocity zero

$B \otimes$ $E \downarrow$ Ions Electrons

Trajectory is a cycloid = uniform drift + circle



Giving drift current j_E + dipole moment

The ion and electron drift currents CANCEL
Dipole moments are insignificant

(E-field adjusts itself to confine other currents)

(4) For a particle having an initial velocity \mathbf{v} in combined fields B, g, E , just add the dipole moment of (1) to the gravity drift current of (2).

(5) Overall effect

When summed over the n ions per unit volume, the individual dipole moments of (1) give a volume magnetisation $M = nd = -(nmV^2/2B^2)\mathbf{B}$ where V^2 is the ms perpendicular velocity. In terms of the ion pressure P this becomes $M = -PB/B^2$. Similarly for the electrons, which have about the same pressure.

(To calculate the resultant magnetic field, it is sometimes easier to work with the equivalent volume current density $j_d = \text{curl } M$, which (for uniform B) becomes $j_d = -(\text{grad } P) \times B/B^2$. For a sharply bounded region, this becomes a surface current $\delta M \times n = -(\delta P/B^2)B \times n$ at the boundary.)

Similarly, the individual ion drifts of (2) give a volume current density $j_g = (nm/B^2)g \times B$

The Resulting Magnetic Field, and its Measurement

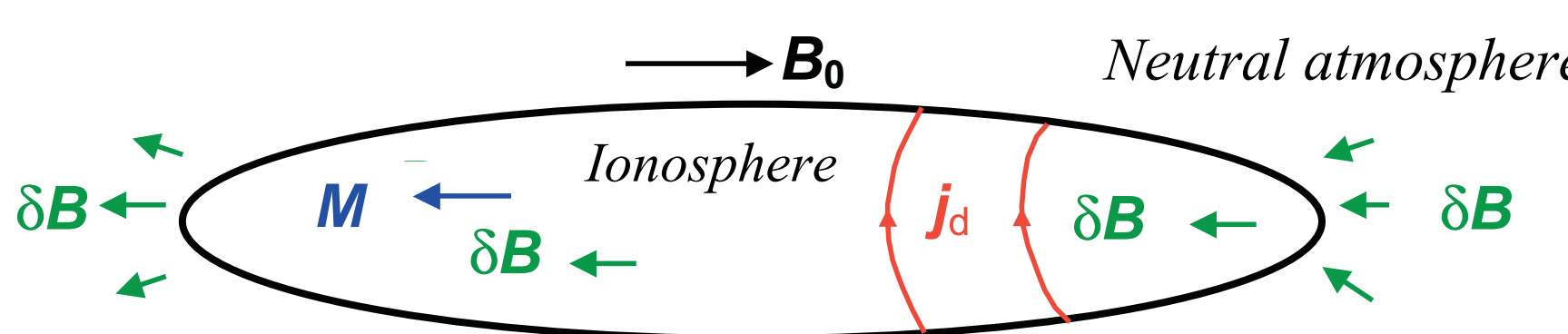
The 'diamagnetic' M (or the equivalent current density j_d) in the ionosphere will produce a small magnetic field $\delta B(r, \theta, \lambda)$ both inside AND outside the ionosphere. This δB will be (nearly) opposite to B_0 inside the ionosphere, and also along the CHAMP orbit north and south of the ionosphere. See (6) below.

Outside the ionosphere, the spacecraft magnetometer (which is calibrated for use in 'free space') will accurately measure the ambient $B^*=(B_0+\delta B)$. (For the CHAMP orbit, $|B^*| \leq |B_0|$.)

BUT INSIDE the 'diamagnetic' ionosphere, the spacecraft magnetometer will measure a **SLIGHTLY LARGER FIELD** than the ambient $B^*=(B_0+\delta B)$. See (7) below.

(6) Magnetic field δB produced by M (or j_d)

Either use M and magnetostatics,



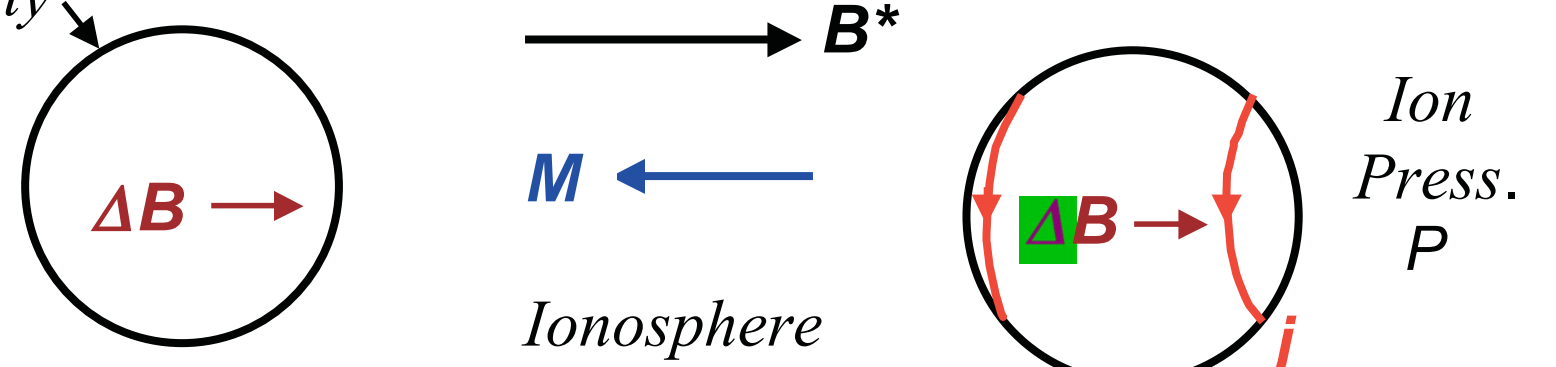
or use j_d and Ampere's Law.

The strength and geometry of the resulting $\mu_0 B$ will depend on the shape of the ionosphere, but $|\delta B|$ will be less than $\mu_0 M = \mu_0 P/B$.

(7) Measuring the field inside the ionosphere

Think of the magnetometer as being (part of) a non-magnetic body or cavity, which is inserted into, and displaces, the 'diamagnetic' plasma of the ionosphere.

Either use M and magnetostatics; the B field in the cavity is increased by an amount ΔB above the B^* in the ionosphere.



Or the ion pressure P will be zero inside the cavity, so there will be $-(P/B^2)B \times n$ currents j round its surface.

What the magnetometer measures is $(B^*+\Delta B)$, with $|B^*+\Delta B| \geq B^*$.