CONTRIBUTION OF REGIONAL MODELLING TECHNIQUES TO THE *SWARM* MISSION

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ABSTRACT/RESUME

A primary objective of the Swarm mission is to resolve the lithospheric field with the best possible accuracy. It will provide unprecedented accurate and high resolution magnetic data that will complete our magnetic data collection. The Swarm mission will detect various secondary magnetic fields generated by ocean tidal flows, small scale ionospheric or induced fields that superimpose with the genuine lithospheric field. In this paper, we illustrate how these scientific problems can be approached using the regional modelling philosophy. In conjunction with a concerted effort to compile a World Digital Magnetic Anomaly Map from independent aeromagnetic surveys, the Swarm mission will close the gap and provide our first ever complete 'view' of the magnetic crust at regional scales.

1. INTRODUCTION

The beginning of the International Decade of Geopotential Research provided the scientific community with magnetic data obtained at different altitudes and epochs. Three missions, particularly important for geomagnetism were launched in 1999 (Ørsted), and in July and November 2000 (CHAMP and SAC-C respectively). Many results were achieved during this new era of magnetic measurements and these three missions demonstrated the capability of satellites to improve the spatio-temporal resolution of magnetic models ([1], [2], [3]). They proved, in particular, that weak magnetic fields, like the lithospheric part [4], or fields due to ocean tidal flow [5], could also be monitored. A model of the crust's magnetization was also compared with satellite data [6], and heat fluxes associated with weak lithospheric magnetic field were detected [7]. Despite these important achievements, many scientific issues remain about the internal magnetic field, for which the future Swarm mission has been specifically designed to address.

The Swarm mission was selected in 2004 as the 5th mission in the ESA's Earth Explorer Program. The mission will provide an outstanding survey of the magnetic field and its temporal variation. The constellation of three satellites will result in a unique prospecting inside the Earth and its scientific payload will supply with the observations necessary to disentangle the signals from various magnetic sources ([8], [9]). In particular, the Swarm mission will help to access the entire magnetic crust by filling the spectral gap between degrees 60 and 150 of the spherical harmonics. Moreover, we expect to detect oceanic stripes for the first time [10]. The new mission already initiated new efforts and intensely focused magnetic research for a better 'view' of the Earth at a global scale. The major purpose of this paper is to show that new tools, developed to represent regionally the magnetic field, will be useful in the Swarm context as they will give a new opportunity to detect and study secondary magnetic fields of small scales.

Several regional techniques were proposed in the past in order to analyze the magnetic data at regional scales. The polynomial modeling, the rectangular harmonic analysis [11] or the spherical cap harmonic analysis [12] gave important results (e.g. [13]) but have strong physical approximations. Considering the high accuracy of the most recent data and the forthcoming Swarm satellite measurements, these methods are thus of little use.

New efforts to improve the situation were made during the preparation and evaluation of the Swarm mission and some methods, still under development, are now bridging the gap between global and regional representations. Poisson wavelet analysis ([14], [15]) or a special case of band limited wavelets [16] are two such examples and may play an important role in the near future. More recently, Simons and Dahlen [17] proposed a regional representation using localized and band limited functions on some closed portion of the sphere. These methods are theoretically powerful but still need to find their way from the applied mathematician's desk to the geophysicist practitioners'. In this paper, the results were all obtained with the Revised Spherical Cap Harmonic Analysis [18], which is also a technique purposefully designed to represent accurately the magnetic field at regional scales. After a brief summary of its basic principles, we discuss the ability of the technique to invert simultaneously near-surface and satellite data. Then, we show that it can be used to piecewise modelling the magnetic field at a global scale and to identify and isolate weak and small-scale magnetic signals usually ignored. Throughout the paper, we also report on the remaining work to be done before fully meeting the Swarm's objectives.

2. BASIC PRINCIPLES OF THE SPHERICAL CAPS HARMONIC ANALYSIS

The estimation of potential fields, such as the gravitational or magnetic potential, from noisy observations taken over an incomplete portion of the globe is a classic example of an ill-posed inverse problem. A possible alternative to the Spherical Harmonics is to build up a basis functions having the appropriate orthogonal and convergence properties within the domain under study. If we postulate that between ground and satellite altitudes the magnetic field is curl and source free, Laplace equation is satisfied. In that case, we assume that ionospheric currents lying at 110 km altitude are removed by a preliminary data processing. The appropriate basis functions will be obtained after solving, either analytically or numerically, a complete boundary value problem. Laplace equation must thus be associated with boundary conditions applied on each surface of the domain shown in Fig. 1. In addition, the choice of the boundary conditions guarantees the continuity between the magnetic field inside and outside the domain.

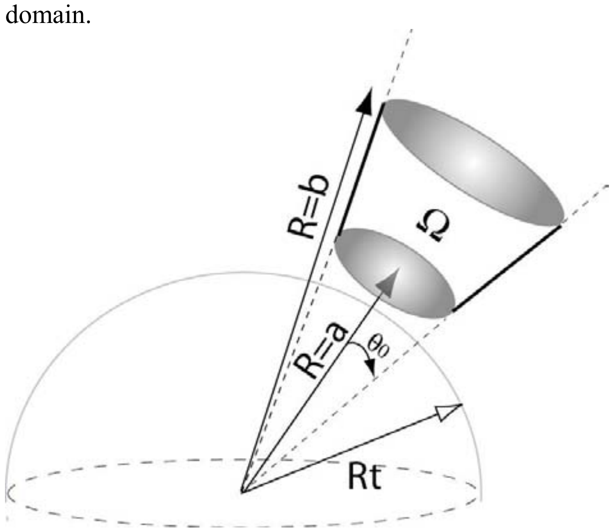

Figure 1: The magnetic field is modeled regionally inside the domain Ω bounded by three surfaces. Two surfaces are at radii R=a and R=b (shaded surfaces) and one is lateral (bold line).

The Revised Spherical Cap Harmonic Analysis is original in the sense that, contrary to other regional techniques using the restriction of global functions, it uses basis functions with local support. The implementation of temporal varying field to include secular variation is straightforward and local parameters can be time-dependent. It is thus very convenient for studying the field at regional scales. Nevertheless, three main fundamental difficulties remain. The first is to disentangle the internal from the external field by simply studying the shape of the basis functions, the second is to perform a spectral analysis and the third relates to downward/upward continuation. In the future, the first problem may be tackled by solving the inverse problem in a regional comprehensive way ([1], [9]). Nevertheless, although we may imagine splitting the initial domain into sub-regions having different physical properties, the problem cannot be fully alleviated. Indeed, sources may lie anywhere outside the domain, not only below or above, and we have a natural indetermination.

Since the real meaning of the spherical cap harmonics is not yet fully established, the definition of spectrum at regional scales is also difficult [18]. If there are analytical relationships between spherical harmonics and spherical cap harmonics [18], the converse is untrue because a unique potential inside a given domain may be represented by an infinite spherical harmonic potentials equal to the regional representation but different outside [19]. One possibility is to invert these relationships numerically and to analyze the results in terms of conventional spherical decomposition at each integer degree *n,* but the resulting spectrum will be aliased.

Finally, the downward continuation to the ground of models obtained with data measured only at satellite altitudes is also critical: in addition to the intrinsic instability of the downward continuation, edge effects contribute to the field distortion.

It is worth noting that most of the drawbacks are theoretical. Even if a rigorous demonstration remains to be done, any regional modelling earlier or later raises these important questions. Wavelets or band-limited functions applied over one region only, not at the global scale, do not reliably separate internal from external field. Edge effects and noise also exist during the downward continuation process. Moreover, if a spectrum can be analytically derived in spherical harmonics from these representations, it is non-unique and flawed since the field outside is not considered or tapered to zero, which is unrealistic. Whatever the technique, only *ad hoc* solutions to these fundamental problems may be proposed. All of these techniques are in their infancy and thus not yet well established. New theoretical insights are needed as well as real data situations with observational errors that both will help to testify them.

The principal properties of spherical caps were qualitatively reviewed in this section and the focus was brought on their drawbacks. From now on, it will be shown that, provided we keep in mind its limitations, in several situations this new method can do a much better job than spherical harmonics. Although results presented in the following sections are obtained with spherical caps, other emerging methods may reach or even surpass the following results in the future.

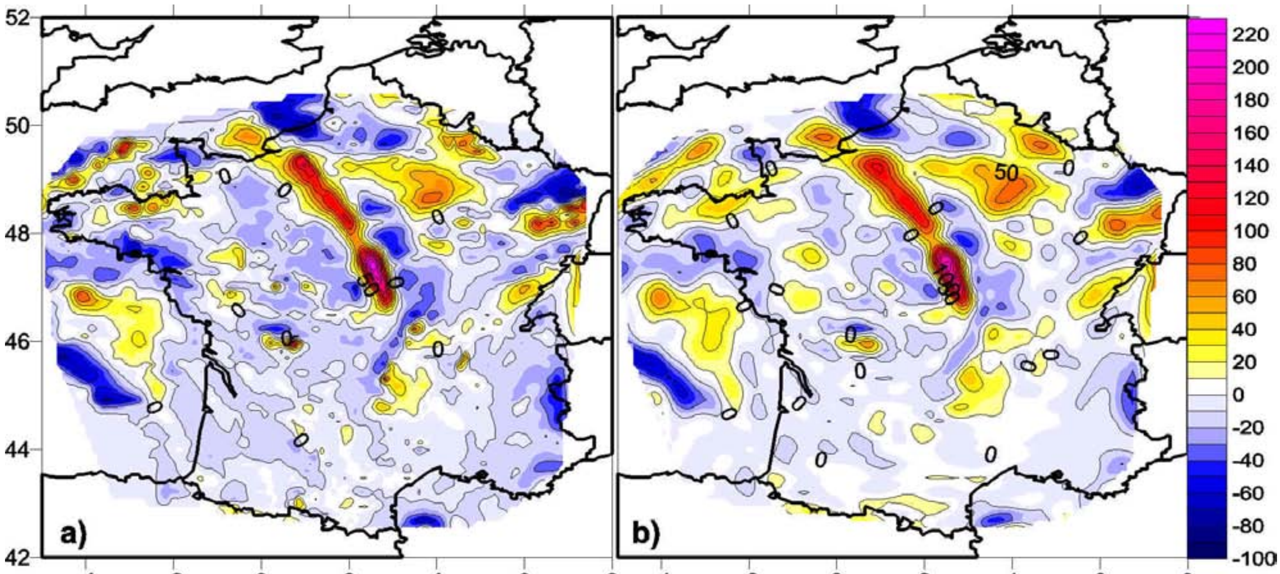

Figure 2: *Comparison between aeromagnetic anomaly intensity (a) and predicted anomaly intensity using Revised Spherical Cap modeling (b).*

3. MERGING GROUND AND SATELLITE DATA

The Swarm mission will help to close the spectral gap and, combining this new level of information with aeromagnetic and ground survey data, it will provide our first ever top-to-the bottom view of the magnetic crust. Considering a typical homogeneous 5 km aeromagnetic grid resolution, the maximum necessary spherical harmonic degree would be 8000, requiring more than 6 million Gauss coefficients to be found by inverse problem. Actual data distribution and computer facilities do not yet permit to deal with this issue and other strategies must be found. The main advantage of regional modelling is its ability to consider data over a limited area and to merge them together, from the near surface to satellite altitudes. As a result, a high resolution can be attained by regional modelling with a few hundreds regional parameters, depending on the region's size. Thébault et al. [20] obtained a regional model over France combining aeromagnetic data at 5 km altitude and CHAMP satellite data between 380 km and 450 km altitude at a spatial resolution of 40 km with 1166 parameters. Prior to the inversion, an external field model was estimated and removed from the satellite data using a global representation. Data were reduced to a same epoch and a common core field model was removed from the aeromagnetic and the satellite data using a comprehensive model [1]. Additional corrections were applied to satellite data, including star camera misalignment, polar electrojet or tidal effect (see [4] for data selection and reduction). The reduced data are ultimately considered to mostly contain the lithospheric field and, considering the inability of regional models to distinguish between internal and external field, pre-processing and pre-selection are necessary to remove the unwanted contributions. Here, we focus on the lithospheric field for geological applications but we may also derive a regional model for the core field.

Fig. 2 compares the aeromagnetic intensity anomaly with the predicted intensity anomaly. The agreement is satisfactory and residuals can be entirely explained by high-resolution anomalies contained in the data but not represented in the model. Interestingly, the modeling also yields the three components of the lithospheric field and it is possible to calculate the forward problem from ground to satellite altitudes. The reliability of the magnetic components obviously depends on how well the model is constrained by the data. It is often assumed that model's accuracy principally relies on data distribution but, in fact, when representing the field in three dimensions, the spectral content of magnetic data is also essential. Aeromagnetic data detect small wavelengths and shallow sources while satellite data are sensitive to large scales and deep sources. Therefore, we may have either incomplete data, with a spectral gap noticeable for some wavelengths, or inconsistent intermediate wavelengths that are neither well detected by near-surface data nor by satellite data. Typically, such a problem arises between 250km and 400km wavelengths and leads to a loss of power in the spectrum. In addition to this phenomenon, in most cases aeromagnetic and satellite data have inconsistent spectra, rendering difficult the modelling. As a result, the model will be poorly constrained around 300km as shown in Fig. 3. The circular anomalies, unrealistic, are often put to the credit of edge effects but since they do not occur in the synthetic case, we should see here the appearance of the spectral gap. The Swarm constellation will provide information about the horizontal gradient of the magnetic field and will alleviate this issue by better constraining the intermediate wavelengths. The need of intermediate altitudes observation will therefore be obsolete and regional modelling, taking into account the gradient of the magnetic field, will be of great interest in order to reconstruct the full available signal.

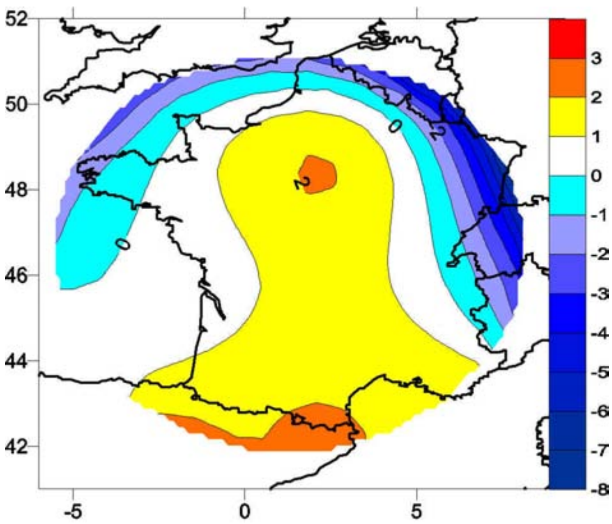

Figure 3: Map of the Z component of the lithospheric field at 300 km altitude. The model is poorly constrained by the data and the modeling creates spurious circular signal. Swarm data will better constrain the intermediate altitudes and help to stabilize the models.

4. USING SATELLITE DATA OVER A REGION ONLY OR AT GLOBAL SCALE

The present data distribution conditions at satellite altitudes are particularly favorable for the modelling at high resolution. Indeed, six years of CHAMP mission provided a near homogeneously data distribution all over the Earth, except near the geographic poles, between 350km and 480km altitude. It is thus now technically possible to derive core field, secular variation, or lithospheric field models with a better resolution than IGRF models over a special region of interest. Thanks to the amount of data, the continuity between adjacent regional models may also be warranted and the entire Earth can be covered with a patchwork of regional models. Although is deserves refinements, Thébault [21] showed that a lithospheric field models could be reliably found by such a technique. In the same vein, Lesur and Maus [22] applied band-limited functions for the same purpose. The advantage of this procedure for the lithospheric field modelling is twofold. Firstly, it is possible to constraint some regions where magnetic field disturbances are important independently from the others. It is thus allowed to represent independently polar and mid latitude regions ([16], [21]) and therefore to reduce the noise in Polar Regions [22]. Secondly, we can represent the field with a high resolution and detect very small features usually filtered out by truncated spherical harmonics. The main issue is to delineate the signal from the noise or other artifacts, but interesting characteristics come out and are worthy of discussion.

In the ideal case of complete, exact and noise-free data, all representations are equivalent and should give the same unique solution. In practice, data are incurred with errors and the noise, not randomly distributed, produces slight differences between representations. These differences may be analyzed when converting a representation to another. Analyzing the difference between modelling CHAMP dataset using spherical cap and using spherical harmonics is instructive [16]. The residuals contain noise, external field, core field, but also genuine lithospheric signal and other physical induced field of various origins. The pure random signal averages out over the six years and is barely noticeable. To the contrary, the line-leveling error is responsible for the East-West oscillation and is the main source of error [4]. This effect is seen in Fig 4. Near the polar cap, the regional models are probably contaminated by the polar electrojet, and the residuals between Antarctica and Australia lie precisely in the region where the magnetic core field intensity is maximum in the Southern hemisphere. Some residuals lie over genuine lithospheric anomalies, at the collision zone between India and China for instance [21]. Other larger signals are intriguing and are shown in Fig. 4. Over Bangui or Northern Brazil, for instance, some signal remains un-modeled in spherical harmonics but is detected by regional representation. Its origin is not clear and deserves more attention as it could indicate induction effects produced by the lithospheric anomalies below. Other signals over oceans could be produced by regional ocean tidal effects. Hawe and Holme [23] recently isolated a very weak signal over the Argentine Basin that seems to be in good agreement with the anomaly seen in Fig. 4 (although it is not exactly located at the same place, [Hawe, personal communication]). The main residual in the Pacific Ocean lie over the Mid-Pacific Mountains and could also be due to ocean flow.

Lots remain to be done to assert the origin of these signals, which does not seem to be purely coincidental. Of course, we cannot dismiss the eventuality of other artifacts and the same work should be carried out with different satellite data. Nevertheless, using six years of data indicates that some residuals are always located in space, which also suggests a sort of regularity with time. Some signals are also coincidentally too close to geological units. Swarm will provide high accurate desynchronized data with a better spatial resolution. This will help to test the robustness of the residuals shown in Fig. 4. The mission will improve the signal-to-noise ratio, reduce the line-leveling problem, and remove some artifacts due to periodicity or quiet-time data selection in polar region [24]. The satellite constellation will also open new ways to better modelling the external field currents and help to perform a more accurate data pre-processing. Even if delineating the signal origin remains intricate, new accurate data will provide an access to it and regional modelling will be required to analyze the data. The main challenging task will be then to find the proper set of parameters, physically consistent, in order to model these data in space and time.

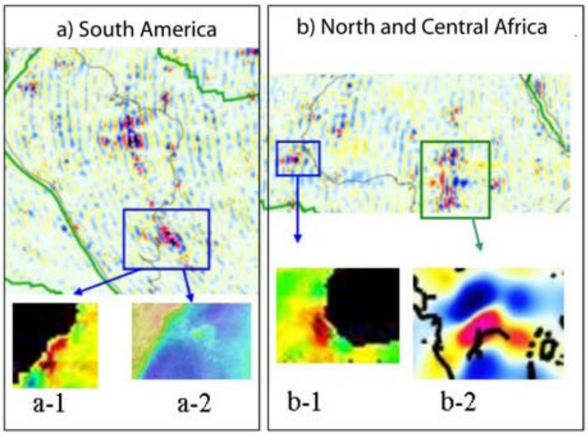

Figure 4: Signal remaining after the removal of all known magnetic field contributions over South America and Central Africa. Some signatures have unknown origins like over Brazil but other may be correlated with ocean velocity (a-1 or b-1) and bathymetry (a-2).

5. CONCLUSION

The future of regional modelling is bright. Regarding the present data distribution at ground and satellite altitude, regional applications yield insights previously unobtainable with standard cartesian or global spherical harmonics methods. In conjunction with a concerted effort to compile a World Digital Magnetic Anomaly Map from independent aeromagnetic surveys, the Swarm mission will allow us to represent the magnetic field from a few thousand to a few kilometers resolution. These will particularly improve our knowledge about the lithospheric field and its radial variation with altitude.

In addition, this new mission will initiate new efforts of intensely focused magnetic research on secondary magnetic field. Small and weak ionosphere currents, regional tidal ocean flow or other secondary magnetic field effects are apparently detectable in CHAMP data. Using desynchronized data, such as Swarm measurements, will help delineating the origin of the signal observed in the residuals and decide which ones are artifacts due to single and periodic satellite measurements. Regional techniques still need theoretical developments to be applied to the full vector field and the complexity of its space and temporal variations, for a comprehensive approach. The need for effective tools to collect and represent, analyze and model the data over the sphere is shared by the geophysical disciplines like geodesy, planetary sciences and geomagnetism and the improvements brought by the Swarm mission will also serve the scientific community to a much larger extent.

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