Evaluating spherical Earth magnetic gradients from SWARM by Gauss-Legendre quadrature integration

M.F. Asgharzadeh[1] (asgharzadeh.1@osu.edu)
R.R.B. von Frese[1] (vonfrese@geology.ohio-state.edu)
Hyung Rae Kim[2] (kimhr@core2.gsfc.nasa.gov)

1. Department of Geological Sciences, The Ohio State University, Columbus, OH, USA
2. Planetary Geodynamics Laboratory, NASA / GSFC, Greenbelt, MD, 20771, USA

Abstract:

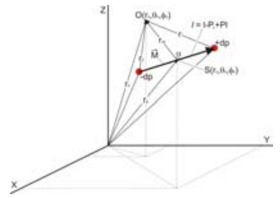
The SWARM constellation of satellites will provide magnetic observations from which spatial anomaly gradients can be recovered for geological analysis. In assessing the geological significance of the SWARM data, great need exists for computing theoretical anomalous magnetic fields from geologic models in spherical coordinates. In the present study, we explicitly develop the elegant Gauss-Legendre quadrature formulation for numerically modeling the complete magnetic effects (i.e., potential, vector and tensor gradient field) of the spherical prism. We also use these results to investigate the utility of satellite magnetic anomaly gradients for studies of the Earth's crust.

Ampère's Law: $\nabla \times \mathbf{B_o} = \mu_0 \mathbf{J_t}$

$\nabla \times \mathbf{B_o} = 0$

Scalar Potential: $\mathbf{B_o} = \nabla P_o$

EPS Magnetic Effects:

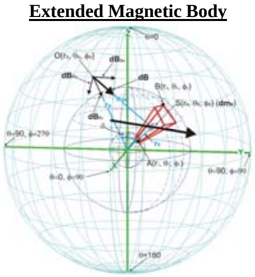


$$dP_{os} = \frac{\mu_o}{4\pi} \cdot dp \cdot (\frac{1}{r_1} - \frac{1}{r_2})$$

$$dP_{os} = \frac{\mu_o}{4\pi} \cdot \frac{\mathbf{dm} \cdot \mathbf{R_{os}}}{{R_{os}}^3} = \frac{\mu_o}{4\pi} \cdot \frac{dm \cdot cos\alpha}{{R_{os}}^2}$$

$$cos\alpha = \frac{dm_x \cdot R_{os,x} + dm_y \cdot R_{os,y} + dm_z \cdot R_{os,z}}{dm \cdot R_{os}}$$

Generalized Magnetic Effects for an Extended Magnetic Body



$$dP_{os} = \frac{\mu_o}{4\pi} \cdot (M_s dv_s) \cdot (\frac{cos\alpha}{{R_{os}}^2})$$

$$\mathbf{dB_{os}} = \frac{\mu_o}{4\pi} \cdot (M_s dv_s) \cdot (\nabla(\frac{cos\alpha}{{R_{os}}^2}))$$

$$\nabla(\mathbf{dB_{os}}) = \frac{\mu_o}{4\pi} \cdot (M_s dv_s) \cdot (\nabla(\nabla(\frac{cos\alpha}{{R_{os}}^2})))$$

$$P_o = \int_V dP_{os} = \frac{\mu_o}{4\pi} \int_V (\frac{cos\alpha}{{R_{os}}^2}) \cdot M_s dv_s$$

$$\mathbf{B_o} = \int_V \mathbf{dB_{os}} = \frac{\mu_o}{4\pi} \int_V (\nabla(\frac{cos\alpha}{{R_{os}}^2})) \cdot M_s dv_s$$

$$\nabla(\mathbf{B_o}) = \int_V \nabla(\mathbf{dB_{os}}) = \frac{\mu_o}{4\pi} \int_V (\nabla(\nabla(\frac{cos\alpha}{{R_{os}}^2}))) \cdot M_s dv_s.$$

$$\int_{\phi_s=\phi_1}^{\phi_2} \int_{\theta_s=\theta_1}^{\theta_2} \int_{r_s=r_1}^{r_2} f(r_o, \theta_o, \phi_o, r_s, \theta_s, \phi_s) dr_s d\theta_s d\phi_s$$

The GLQ Procedure

$$\frac{(r_2 - r_1)(\theta_2 - \theta_1)(\phi_2 - \phi_1)}{8} \times \sum_{nk=1}^{K} \sum_{nj=1}^{J} \sum_{ni=1}^{I} (A_{ni} A_{nj} A_{nk}) \times f(r_o, \theta_o, \phi_o, \hat{r}_{ni}, \hat{\theta}_{nj}, \hat{\phi}_{nk})$$

$$\hat{r}_{ni} = \frac{r_{ni} \cdot (r_2 - r_1) + (r_2 + r_1)}{2}$$

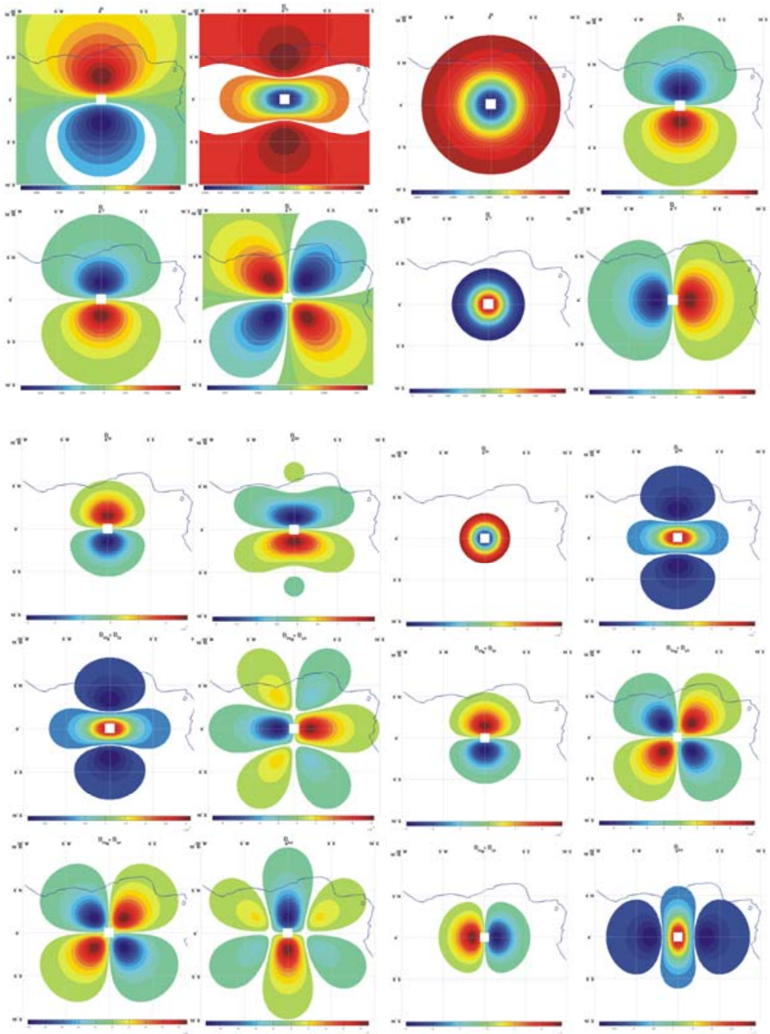
$$\hat{\theta}_{nj} = \frac{\theta_{nk} \cdot (\theta_2 - \theta_1) + (\theta_2 + \theta_1)}{2}$$

$$\hat{\phi}_{nk} = \frac{\phi_{nk} \cdot (\phi_2 - \phi_1) + (\phi_2 + \phi_1)}{2}$$

GLQ Magnetic Estimates of Spherical Prism

M = 0.5 A/m

Dec = 0, Inc = 0 | **Dec = 0, Inc = 90**



RTP Magnetic Field Components from CHAMP Observations