A global lithospheric magnetic field model with reduced noise level in the Polar Regions

Vincent Lesur [lesur@gfz-potsdam.de]
GeoForschungsZentrum Potsdam
Telegrafenberg
D-14473 Potsdam
Previously in:
British Geological Survey

Stefan Maus [Stefan.Maus@noaa.gov]
CIRES, University of Colorado,
David Skaggs Research Center
NOAA/NGDC E/GC1
325 Broadway
Boulder, CO 80305-3328 USA

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Theory

The geomagnetic field **B** can be represented as the negative gradient of the sum of $(L \quad 1)(2L \quad 1)$ functions $F_{ij}^{L}(\ , \ )$ that have their centres at the positions $(\ _i, \ _j)$. As long as the $f_l \quad 0$ this representation is strictly equivalent to the usual representation in spherical harmonic (SH) up to a maximum degree L. We have established two direct formulae (equations 1 and 2) to transform the new representation into a SH representation and conversely.

$$\mathbf{B} \quad \sum_{i \ 1 \ j \ 1}^{L \ 1 \ 2L \ 1} \widetilde{g}_{ij} \, F_{ij}^{L}(\ , \ ,r)$$

$$F_{ij}^{L}(\ , \ ,r) \quad \sum_{l \ 0 \ m \quad l}^{L \quad l} f_l(r) \, Y_l^m(\ _i, \ _j) Y_l^m(\ , \ ) \quad \sum_{l \ 0}^{L} f_l(r) \, P_l(\cos \quad) \qquad f_l(r) \quad 0$$

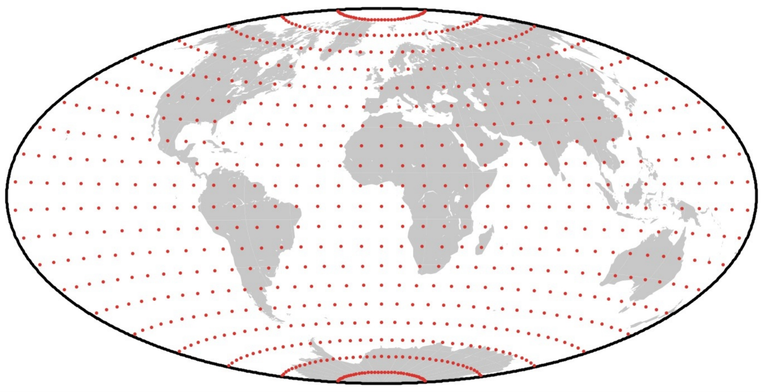
1 $$\widetilde{g}_l^m \quad \sum_{i \ 1 \ j \ 1}^{L \ 1 \ 2L \ 1} \widetilde{g}_{ij} \, f_l \, Y_l^m(\ _i, \ _j)$$

2 $$\widetilde{g}_{ij} \quad \sum_{l' \ 0 \, m' \quad l}^{L \quad l'} \frac{\widetilde{g}_{l'}^{m'}}{f_{l'}} \frac{2l' \ 1}{2(2L \quad 1)} w_i^{L \ 1} Y_{l'}^{m'}(\ _i, \ _j)$$

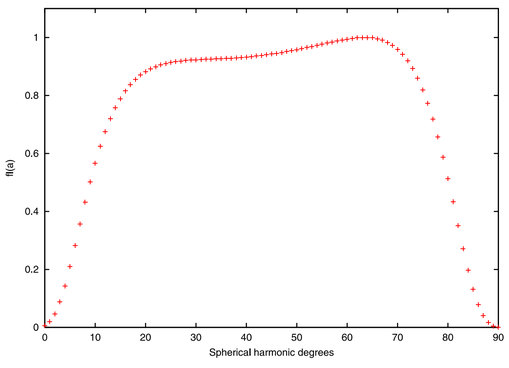
$$\mathbf{B} \quad a \sum_{l \ 1 \ m \quad l}^{L \quad l} \widetilde{g}_l^m \left(\tfrac{a}{r}\right)^{l \ 1} Y_l^m(\ , \ )$$

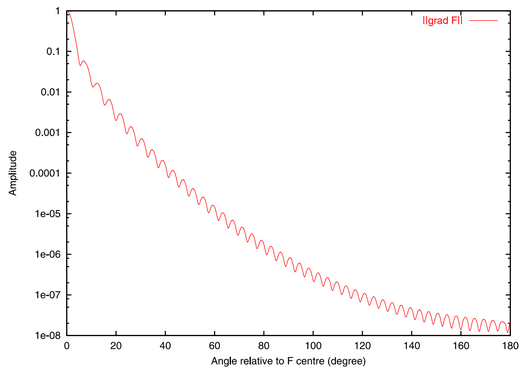
$\theta_i \quad \arccos(x_i)$; x_i is the ith zero of the Legendre polynomial $P_{L \ 1}(x)$.

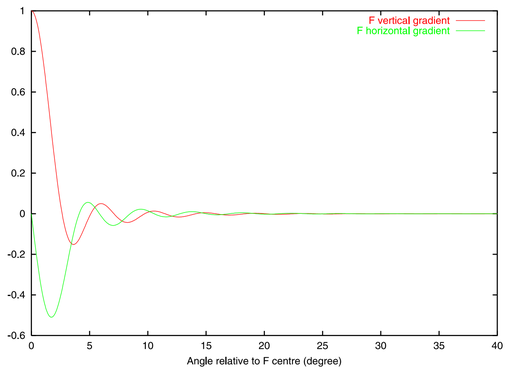
$_j \quad 2\pi j / (2L \quad 1)$



The $f_l(a)$ $l=0,...,L$ can be optimised such that the horizontal and vertical gradients of the functions $F_{ij}^{L}(\ , \ )$ decrease rapidly as the angular distance from their centres $(\ _i, \ _j)$ increases. If we consider a magnetic field model with a maximum spherical harmonic degree of $L \quad 90$, the gradient of $F_{ij}^{L}(\ , \ )$ is less than 1% of the value at its centre for a 14^o angle.







We built a model of the Earth's lithospheric magnetic field using localised functions with a maximum SH degree 90, where

1- The model is smoothed in the East-West direction at mid and low latitudes

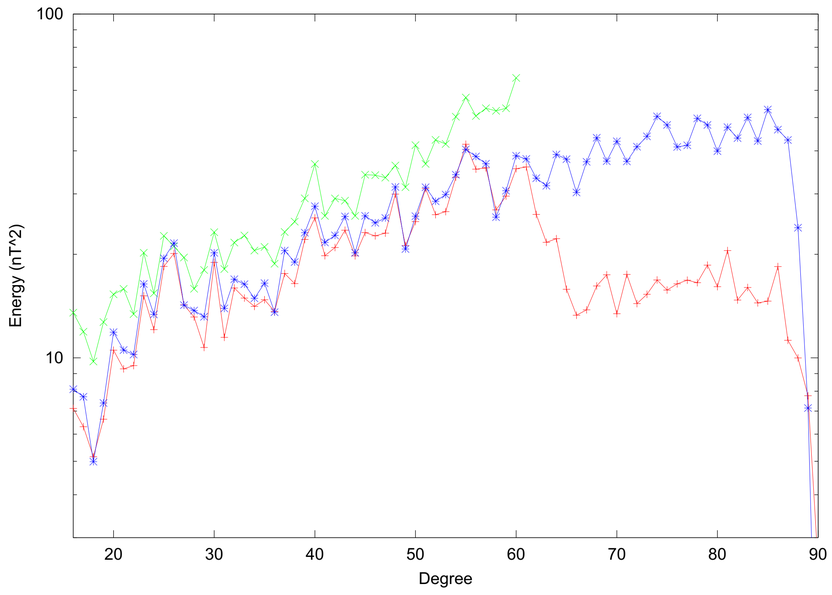
2- The localised function maximum SH degree is only 60 over the poles

The data set used in this study is exactly equivalent to the one which was used to produce the MF4 crustal field model (Maus et al., 2006)

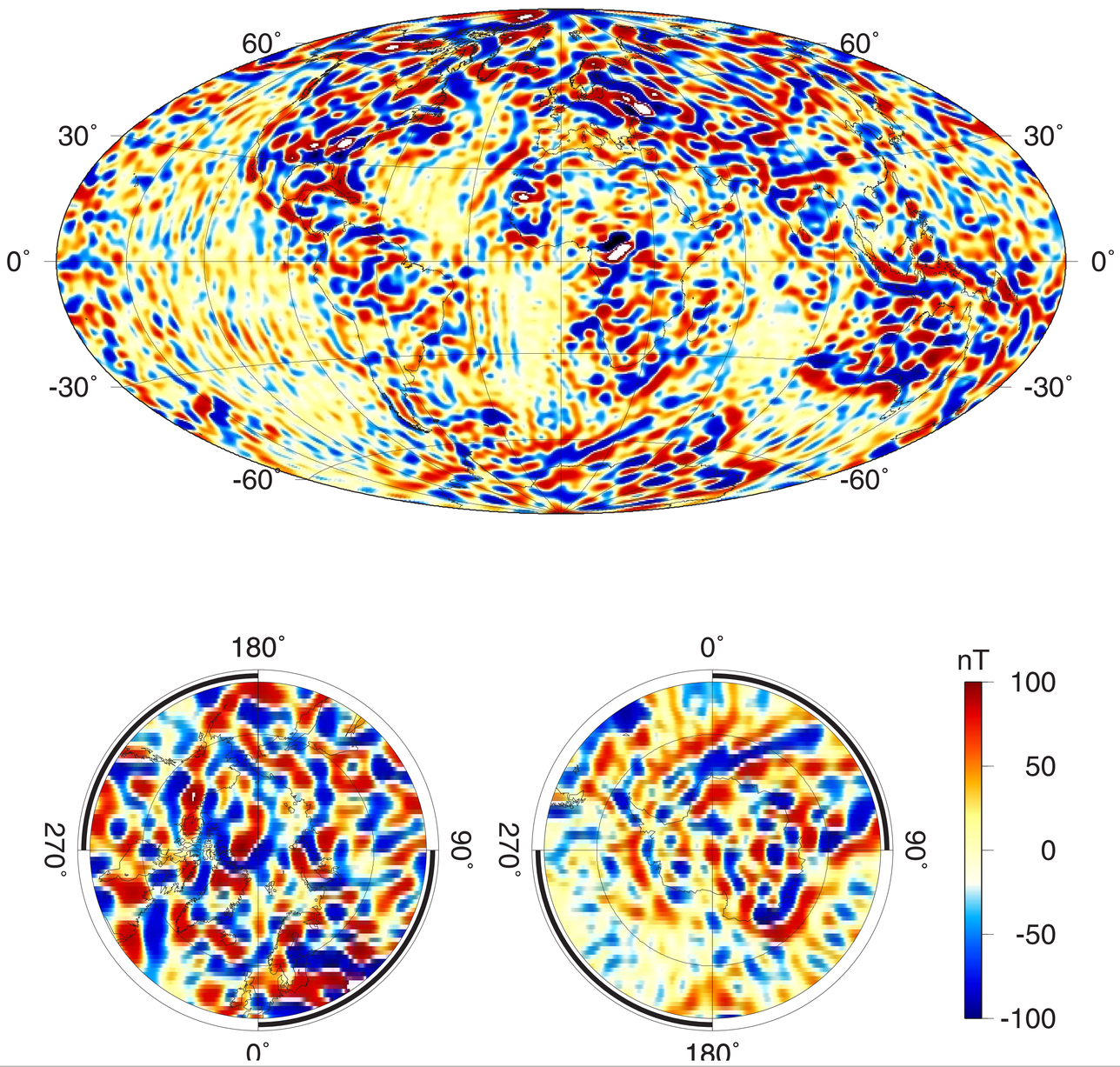
The power spectrum of the model is not realistic as it decreases significantly from degree 60. However, this model downward continued to the Earth's equatorial radius does not present very large spurious signals as other models do. By comparing the new model with the MF4 model and the MF4 truncated to maximum SH degree 60, it becomes obvious that we have successfully reduced the noise level in our model over the poles by limiting its maximum SH degree.

Maus, S., Roher, M., Hemant, K., Stolle, C., Luhr, H., Kuvshinov, A., & Olsen, N., 2006. Earth's lithospheric magnetic field determined to spherical harmonic degree 90 from Champ satellite measurements, *Geophys. J. Int., In print*

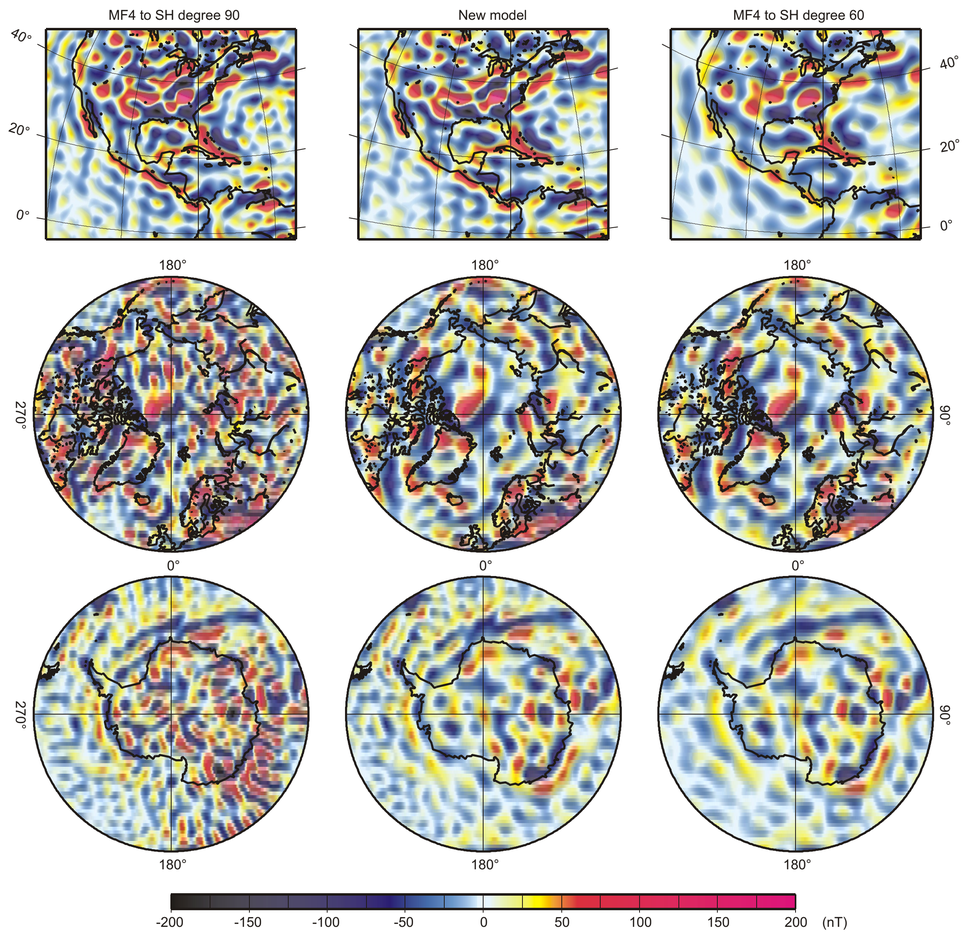
Power spectrum of our model (in red). Also shown are the CM4 (in green) and MF4 (in blue) power spectra.



Vertical component of the lithospheric magnetic field model at Earth's equatorial radius for SH degree 16 to 90.



Vertical magnetic field component at Earth's equatorial radius. Left column the MF4 at full resolution, central column the new model, right column MF4 truncated to maximum SH degree 60 .



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