Geomagnetic Data Assimilation with MoSST Core Model

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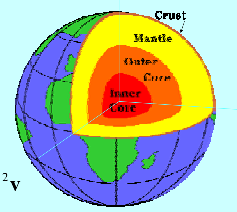
Introduction

Data assimilation is the methodology by which observations of a physical system are combined with the system model output to get an improved estimate of the state of the system. An optimal estimate can potentially be achieved by using Bayesian techniques, provided that good estimates of observation and model error statistics are available. We have begun to develop the means to estimate error statistics of MoSST core dynamics model, a geodynamo model in which spherical harmonics are used on spherical surfaces, and a finite difference algorithm is used in the radial direction. Surface observation can only provide information on poloidal magnetic field up to degree L ≈ 13 (in spherical harmonic expansion). Provided knowledge on observation error statistics, we could use our results on the error statistics of MoSST core dynamics model for an optimal geomagnetic data assimilation system.

We are developing a geomagnetic data assimilation system in which the error covariances are estimated using an ensemble of model runs. The covariances provide information on correlations in space and across different state variables, which can then be used to correct not only the observable part of the poloidal magnetic field, but also other unobservable state variables (e.g. the rest of the poloidal field, the toroidal field, the velocity field and the density anomaly) inside the core with respect to the surface observations. This "balanced" approach to correct the core state is less likely to cause non-physical responses. In particular, it corrects the non-observed state variables directly, thus shortening the spin-up time. This is very important, considering that the observation record is very short compared to the geological time scales. Our first objective is to use synthetic data to test the impact of error correlation length on spin-up time.

MoSST core dynamics model:

A numerical model describing convective flow in the Earth's outer core.



Equations of the Model

$$R_0\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla\right)\mathbf{v}+\hat{\mathbf{z}}\times\mathbf{v}=-\nabla p+\mathbf{J}\times\mathbf{B}+R_{th}\Theta\mathbf{r}+E\nabla^2\mathbf{v}$$

$$\frac{\partial\mathbf{B}}{\partial t}=\nabla\times(\mathbf{v}\times\mathbf{B})+\nabla^2\mathbf{B}$$

$$\frac{\partial\Theta}{\partial t}+\mathbf{v}\cdot\nabla\Theta=-\mathbf{v}\cdot\nabla\Theta+q_\kappa\nabla^2\Theta$$

$\mathbf{B}=(B_P,B_T)$: magnetic field,
$\mathbf{v}=(v_P,v_T)$: fluid velocity,
Θ: temperature perturbation,
5 independent variables (fields)

Parameters in our study

$R_0 = E = 1.25\times10^{-6}$, $q_\kappa = 1$

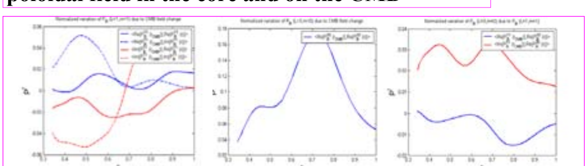
Error Covariances

- P^f is the ensemble generated error covariance for the geodynamo model. It represents covariances between different radial locations in the core, and between different state variables. This information is used to determine how magnetic field observations can be used to correct the state variables within the core in a dynamically consistent manner.
- Ensemble size = 50 with Gaussian noise perturbations.
- Error covariances is given by:

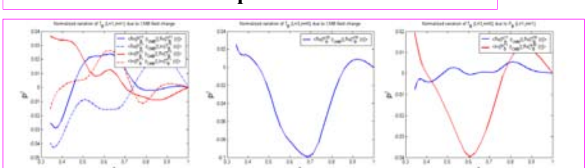
$$\mathbf{P}^f(L_1,m_1;L_2,m_2)$$
$$=<(\mathbf{X}^{L_1,m_1}-\boldsymbol{\mu}^{L_1,m_1})/\boldsymbol{\sigma}^{L_1,m_1},(\mathbf{X}^{L_2,m_2}-\boldsymbol{\mu}^{L_2,m_2})/\boldsymbol{\sigma}^{L_2,m_2}>$$
$$=\frac{1}{N_{ens}-1}\sum_{i=1}^{N_{ens}}\begin{bmatrix}(\tilde{\mathbf{P}}_{\mathbf{V}}^{L_1,m_1}{}_i-\tilde{\boldsymbol{\mu}}_{\mathbf{P}_{\mathbf{V}}^{L_1,m_1}})/\sigma_{\mathbf{P}_{\mathbf{V}}^{L_1,m_1}}\\(\tilde{\mathbf{T}}_{\mathbf{V}}^{L_1,m_1}{}_i-\tilde{\boldsymbol{\mu}}_{\mathbf{T}_{\mathbf{V}}^{L_1,m_1}})/\sigma_{\mathbf{T}_{\mathbf{V}}^{L_1,m_1}}\\(\tilde{\mathbf{P}}_{\mathbf{B}}^{L_1,m_1}{}_i-\tilde{\boldsymbol{\mu}}_{\mathbf{P}_{\mathbf{B}}^{L_1,m_1}})/\sigma_{\mathbf{P}_{\mathbf{B}}^{L_1,m_1}}\\(\tilde{\mathbf{T}}_{\mathbf{B}}^{L_1,m_1}{}_i-\tilde{\boldsymbol{\mu}}_{\mathbf{T}_{\mathbf{B}}^{L_1,m_1}})/\sigma_{\mathbf{T}_{\mathbf{B}}^{L_1,m_1}}\\(\tilde{\boldsymbol{\Theta}}^{L_1,m_1}{}_i-\tilde{\boldsymbol{\mu}}_{\boldsymbol{\Theta}^{L_1,m_1}})/\sigma_{\boldsymbol{\Theta}^{L_1,m_1}}\end{bmatrix}\begin{bmatrix}(\tilde{\mathbf{P}}_{\mathbf{V}}^{L_2,m_2}{}_i-\tilde{\boldsymbol{\mu}}_{\mathbf{P}_{\mathbf{V}}^{L_2,m_2}})/\sigma_{\mathbf{P}_{\mathbf{V}}^{L_2,m_2}}\\(\tilde{\mathbf{T}}_{\mathbf{V}}^{L_2,m_2}{}_i-\tilde{\boldsymbol{\mu}}_{\mathbf{T}_{\mathbf{V}}^{L_2,m_2}})/\sigma_{\mathbf{T}_{\mathbf{V}}^{L_2,m_2}}\\(\tilde{\mathbf{P}}_{\mathbf{B}}^{L_2,m_2}{}_i-\tilde{\boldsymbol{\mu}}_{\mathbf{P}_{\mathbf{B}}^{L_2,m_2}})/\sigma_{\mathbf{P}_{\mathbf{B}}^{L_2,m_2}}\\(\tilde{\mathbf{T}}_{\mathbf{B}}^{L_2,m_2}{}_i-\tilde{\boldsymbol{\mu}}_{\mathbf{T}_{\mathbf{B}}^{L_2,m_2}})/\sigma_{\mathbf{T}_{\mathbf{B}}^{L_2,m_2}}\\(\tilde{\boldsymbol{\Theta}}^{L_2,m_2}{}_i-\tilde{\boldsymbol{\mu}}_{\boldsymbol{\Theta}^{L_2,m_2}})/\sigma_{\boldsymbol{\Theta}^{L_2,m_2}}\end{bmatrix}^T$$

(P_V, T_V): poloidal and toroidal velocity scalars
(P_B, T_B): poloidal and toroidal magnetic field scalars
Θ: density perturbation
μ= mean, σ= standard deviation.

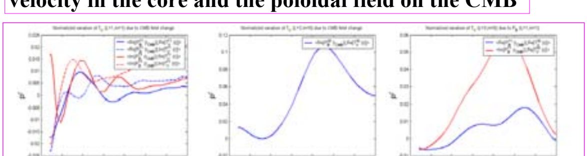
Examples of error covariance between the poloidal field in the core and on the CMB



Examples of the covariance between the toroidal field in the core and the poloidal field at the CMB



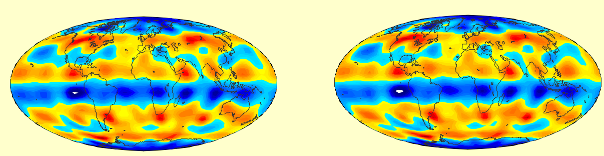
Examples of error covariances between the toroidal velocity in the core and the poloidal field on the CMB



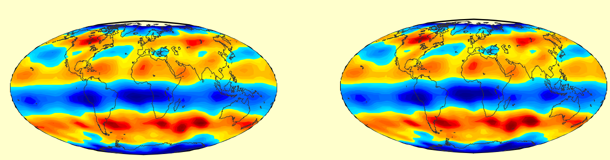
Application to Surface Geomagnetic Data

- **The covariances have been tested with the surface geomagnetic data from 1900 to 2000.**
- **Analysis is made every 20 years, with the forecast and the observation at the given time. It is then used as an initial state for forecast over the next 20 years.**
- **Only corrections to the poloidal field in the core are carried out, no cross correlation between different state variables are considered.**
- **The results for the radial component of the magnetic field B_r (excluding the dipole field) on the CMB.**

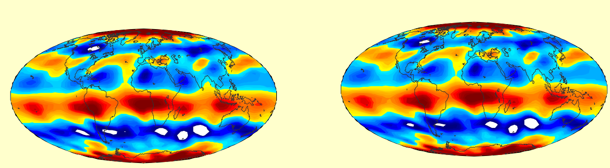
Forecast | **Analysis**



Year 1960



Year 1980



Year 2000

Conclusion

1. We have calculated the model error covariances for multivariate geomagnetic data assimilation. These covariances provide knowledge on corrections of state variables in the core to the poloidal field on the CMB continued downward from surface observations.
2. Correlations of the spectral coefficients with the same degree and order are dominant.
3. Cross-correlations between different state variables are significant.
4. We shall continue to work on implementing the error covraiances to our first generation geomagnetic data assimilation system MoSST_DAS.