Reducing the Backus effect using Backus' constraints

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Determining a geomagnetic field model from intensity measurements only is non-unique. This non-uniqueness leads to strong errors on the field direction predicted by the model, an effect known as the "Backus effect", and hampers optimal use of field intensity satellite data for years when no vector data are available (e.g. POGO satellite scalar data taken between 1965 and 1971). Assuming that the field is well resolved during years where vector satellite data are available, we investigate the idea of using physical constraints from core dynamics to alleviate the Backus effect during years where only scalar data are available. On decadal time scales, magnetic diffusion is generally believed to be negligible in the core and the magnetic flux is said to be frozen. Under this assumption, the core field satisfies a set of mathematical constraints known as the "Backus' constraints". We show that constraining geomagnetic models with the Backus' constraints significantly reduces the Backus effect. Although constrained models from scalar data only are not as good as models from vector data, this provides a way to build more realistic field models at times where no vector data is available.

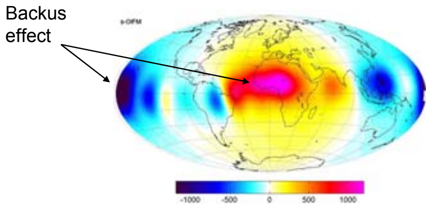
1 Introduction

- **Backus effect**

The so-called "Backus effect" arises when one attempts to model the geomagnetic field globally from scalar-only data. It consists of large errors, especially in the mid- and low-latitude regions. It is related to a formal non-uniqueness of the inversion solution, as was shown by Backus (1970).

The Backus effect was first observed in field models derived from POGO scalar-only data. MAGSAT, and later Ørsted and CHAMP, made it possible to compare field models derived from a set of vector data and from the corresponding set of intensities.

Differences between the OIFM model (Olsen *et al*. 2000) and an OIFM-like model computed from the same scalar data and the intensities of vector data, vertical component:



- **Backus' constraints**

The frozen-flux (FF) assumption consists of neglecting the diffusion term in the induction equation in the core (Roberts and Scott, 1965). This approximation holds for variations of the large scale magnetic field (L ~ 1000 km) of time scale T << 30000 yrs.

Under the FF assumption, the radial component of the magnetic field at the CMB (which, unlike the other components, is continuous through the CMB) is related to the flow at the core surface by

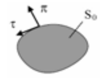
$$\boxed{\frac{\partial B_r}{\partial t} = -\nabla_H \cdot (\mathbf{u} B_r)}$$

To be compatible with the FF assumption, the field must satisfy:

$$\boxed{\begin{gathered}\int_{S_0} \frac{\partial B_r}{\partial t} dS = 0 \\ \left(\frac{\partial B_r}{\partial t}\right)_C = 0\end{gathered}}$$

S_0: null-flux curve $B_r = 0$

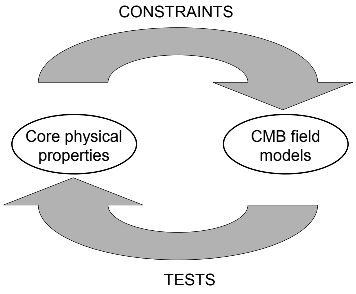
C: critical point (intersection of 2 nfc)



These constraints are necessary and sufficient (Backus, 1968) and are referred to as the "Backus' constraints".

- **CMB field models and core physical properties**

Using Backus' constraints, the FF assumption can be *tested* against CMB field models computed from magnetic data. Such a test is positive over the the time interval 1980-2000 (see the presentation "Magnetic diffusion patches at the top of the Earth's core", by Chulliat, Olsen & Sabaka). Therefore the FF assumption can also be used to *constrain* main field models.



CONSTRAINTS

Core physical properties

CMB field models

TESTS

2 Data

Ørsted scalar and vector data.

Same selection as in OIFM (Olsen *et al*. 2000):

- nightside data (local time about 22:00)
- quiet conditions (Kp < 1+, |Dst| < 10 nT, |d(Dst)/dt| < 3 nT/h)
- no scalar data at high latitudes

3 Analysis

- **Constructing constrained core field models**

Method = Iteratively Reweighted Least-Squares.

Minimizing the function

$$\Phi(\mathbf{m}) = [\gamma - \mathbf{f}(\mathbf{m})]^T \mathbf{C}_e^{-1} [\gamma - \mathbf{f}(\mathbf{m})] + \lambda\, \mathbf{m}^T \mathbf{C}_m^{-1} \mathbf{m} + \mu\, |\mathbf{L}^T(\mathbf{m})\mathbf{m} - \mathbf{F}_0|^2$$

λ = damping parameter
μ = constraint parameter
$\mathbf{L}^T(\mathbf{m})$ = constraint matrix
$\mathbf{F}_0$ = fluxes of reference model (CM4 in 1980)

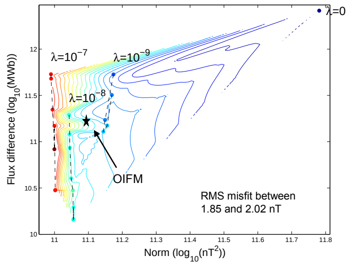
- **Exploring the space of the (λ, μ) parameters**

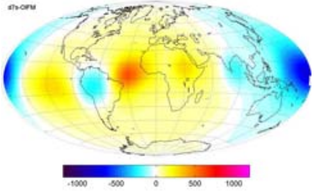
More details:

- Data corrected for CM4 crustal field
- MF damped with minimum energy norm at the core
- Core field = low-pass filtered MF
- Starting MF model = reference model
- SV model = OSVM
- Iterations stop once convergence test is satisfied.
- Fixed Huber weights and outliers

4 OIFM-like, FF-constrained models from scalar-only data

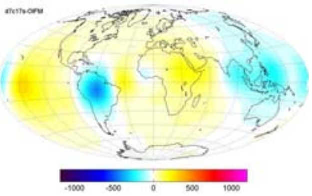
Misfits of OIFM-like models from scalar-only data:





Effect of damping at Earth's surface

($\lambda=10^{-7}$)

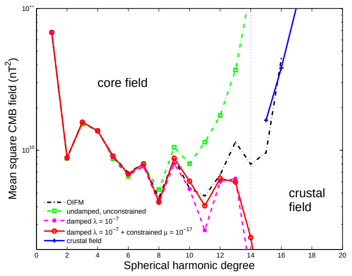


Effect of damping + FF-constraints at Earth's surface

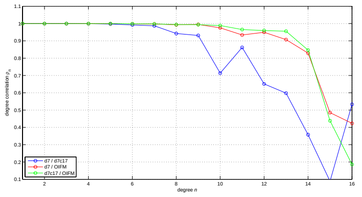
($\lambda=10^{-7}$, $\mu=10^{-16}$)

5 Spectra and degree correlations

CMB power spectra of OIFM-like models from scalar-only data:



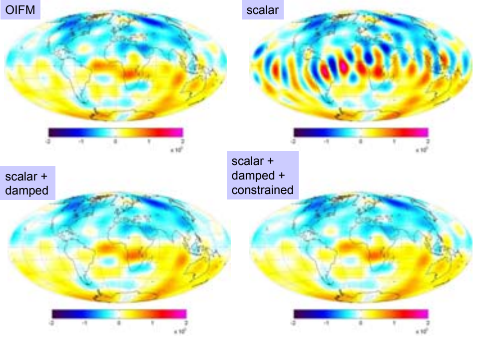
Degree correlations of coefficients of OIFM-like models from scalar-only data:



=> The FF-constrained model is closer to OIFM than the damped model.

6 How does it work?

Vertical component at CMB:



7 Conclusions

1. The Backus effect at the Earth's surface can be reduced by about 50 % using damping of the model built from scalar-only data.
2. It can further be reduced by 50 % using Backus' constraints associated with the frozen-flux assumption at the top of the Earth's core.
3. The coefficients of the frozen-flux constrained scalar models are closer to the vector model than those of the damped scalar model.

References:

Backus, G. E., "Kinematics of geomagnetic secular variation in a perfectly conducting core", *Phil. Trans. R. Soc. Lond. A*, **263**, 239-266.

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