

# Extending the historical field model *gufm1* from 1990 to 2005

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**GEO SPACE**

## Summary

We present an updated version of the time-dependent, internal geomagnetic field model *gufm1* (Jackson et al. 2000). The new model spans the interval from 1590 to 2005. It includes updated survey and observatory records from 1990 to 2005 as well as both scalar and vector data from the Orsted and CHAMP satellites and scalar data from the SAC-C satellite. The external field estimated from this satellite data by Olsen et al. (2006) in their CHAOS model has been subtracted to facilitate an accurate determination of the internal field. The vector satellite data is also treated using the anisotropic attitude error formalism of Holme and Bloxham (1996).

The new model (*gufm-2005*) is constructed with the same spatial and temporal damping and 3 $\sigma$  data rejection criteria as *gufm1*. Despite being derived from over 120,000 more observations, *gufm1-2005* has similar spatial and temporal norms and global misfit as *gufm1*. It fits both the satellite data and the observatory data well giving reasonable residual distributions and confirms the presence of a number of sharp changes in SV in Europe over the past decade.

In the future we plan to develop this model further by improving the statistical characterisation of non-core fields, by applying maximum entropy regularisation techniques and by exploiting new data sources including eventually SWARM.

## 2. Modelling Method

We assume the mantle is to a good approximation an insulator and use a potential representation of the field,

$$\mathbf{B} = -\nabla V \quad \text{where} \quad \nabla^2 V = 0 \quad \text{and} \quad V \rightarrow \frac{1}{r^2} \quad \text{as} \quad r \rightarrow \infty$$

Considering only internal (core) sources, we expand the potential as a sum of spherical harmonics,

$$V(t) = a \sum_{l=1}^{\infty} \sum_{m=0}^l \left(\frac{a}{r}\right)^{l+1} \{g_l^m(t) \cos(m\phi) + h_l^m(t) \sin(m\phi)\} P_l^m(\cos \theta)$$

Since we wish to model the time-dependent field we expand the coefficients in a cubic B-spline basis, for example,

$$g_l^m(t) = \sum_n g_l^{mn} M_n(t) \quad \text{where} \quad M_n(t) \text{ are the localised cubic B-splines.}$$

We invert to find the model parameters that minimise the regularised least squares objective function,

$$\Theta(\mathbf{m}) = [\mathbf{d} - \mathbf{f}(\mathbf{m})]^T \mathbf{C}_e^{-1} [\mathbf{d} - \mathbf{f}(\mathbf{m})] + \mathbf{m}^T \mathbf{C}_m^{-1} \mathbf{m} \quad \text{where} \quad \mathbf{C}_m^{-1} = (\lambda_S \mathbf{S}^{-1} + \lambda_T \mathbf{T}^{-1})$$

and  $\mathbf{m}^T \mathbf{S}^{-1} \mathbf{m}$  is the Ohmic heating norm while  $\mathbf{m}^T \mathbf{T}^{-1} \mathbf{m}$  is  $(\partial_t^2 B_r)^2$ , integrated over time and space.

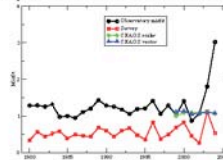
Since some of the data is nonlinearly related to the model parameters, the preferred model is found iteratively,

$$\mathbf{m}_{i+1} = \mathbf{m}_i + (\mathbf{A}^T \mathbf{C}_e^{-1} \mathbf{A} + \mathbf{C}_m^{-1})^{-1} [\mathbf{A}^T \mathbf{C}_e^{-1} (\mathbf{d} - \mathbf{f}(\mathbf{m}_i)) - \mathbf{C}_m^{-1} \mathbf{m}_i]$$

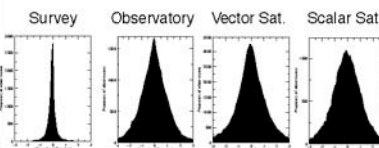
Damping parameters are chosen here to be identical to *gufm1* to aid comparisons.

## 3. Results

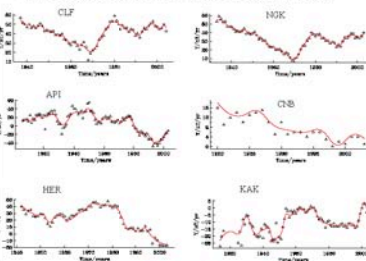
Time-dependent misfit for new datasets



Residual distributions



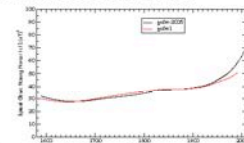
Fit to observatory annual means (dY/dt)



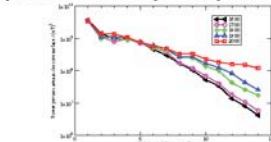
Comparison of model Statistics

Property	<i>gufm-2005</i>	<i>gufm1</i>
No. accepted data	485 613	365 694
Misfit	1.14	1.16
Av. Spatial Norm	$3.6 \times 10^3 (\text{nT})^2$	$3.5 \times 10^3 (\text{nT})^2$
Av. Temporal Norm	$7.44 \times 10^3 (\text{nT})^2 \text{yr}^{-2}$	$6.8 \times 10^3 (\text{nT})^2 \text{yr}^{-2}$
rms SV at core surf.	1908 nT/yr	1817 nT/yr

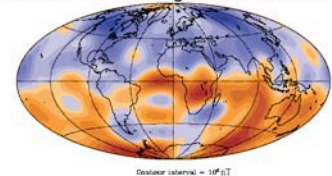
Evolution of spatial norm



Spherical harmonic power spectra evolution

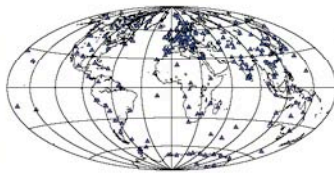


Core surface radial magnetic field in 2005



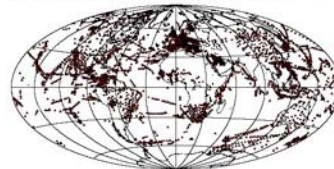
## 1. New Data Sources

(i) Observatory Annual Means 1990-2005: 8066 new data, total misfit=1.21



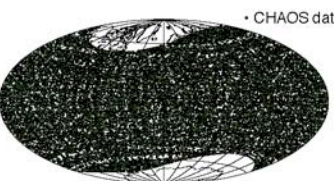
Error estimates from generalised cross validation on time series of each component ( $dX/dt, dY/dt, dZ/dt$ ) at each observatory. e.g. NGK (3.3, 1.0, 4.9) nT/yr ALE (6.3, 6.0, 18.5) nT/yr

(ii) Marine and continental surveys 1980-2005: 11846 data, misfit=0.69



Includes repeat station data  
 Tesseral culling ( $0.1^\circ$ ) to limit effects of dense sampling  
 Isotropic errors of 100nT assumed

(iii) Vector satellite data (Orsted and CHAMP) 1999-2005: 72660 data, misfit=1.09

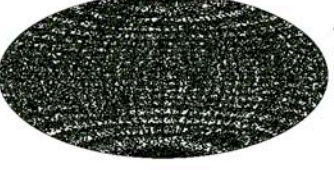


CHAOS dataset (Olsen et al. 2006) used:

non-polar latitudes only  
 $K_p < 20$  ( $< 7$  nT peak to peak)  
 $d\text{Dst}/dt < 2$  nT/hr  
 Original sampling 60s  
 Resampled; tesserae  $10^\circ$  in longitude,  $\cos\theta/18$  in latitude  
 Tesserae reset every 0.1yr for good temporal sampling

Anisotropic attitude errors accounted for using CHAOS Euler angles and error estimates of  $\sigma=2.6$  nT,  $\chi=60$  arcsec,  $\psi=10$  arcsec

(iv) Scalar satellite data (Orsted, CHAMP and SAC-C) 1999-2005: 27615 data misfit=1.08



CHAOS dataset again used  
 All latitudes now considered  
 Merging electric field  $< 0.8$  V/m  
 Same sampling strategy  
 Estimated errors of 2.6nT

## 4. Discussion: Suggestions for future improvements

- Need to exploit large numbers of densely sampled satellite data better e.g. by explicitly including correlated errors due to crustal field at satellite altitude via a statistical model. Or perhaps by developing a deterministic model of errors due to crustal field (if this is now feasible?).
- Application of more sophisticated treatments of dense survey data rather than simple tesseral culling.
- Accounting for correlated errors (due to external fields) between different components at individual observatories.
- Treating all 20th century repeat station data as pseudo-observatory data to improve SV constraints.
- Use of archaeomagnetic intensity data to constrain axial dipole prior to 1840 and to augment sparse early data.

## 5. Conclusions

- The historical field model *gufm1* has been successfully extended to 2005 and now includes recent satellite and observatory data which are satisfactorily fit by the new model.
- The global model reproduces several sharp changes in secular variation observed in the past 10 years at European observatories indicating that these are robust features.

## 6. References

- Jackson, Jonkers and Walker (2000) Four centuries of geomagnetic secular variation from historical records. *Phil. Trans. Roy. Soc. Lond.* **358**, pp.957-990  
 Holme and Bloxham (1996) The treatment of attitude errors in geomagnetic data. *Phys. Earth Planet. Int.* **98** 221-233  
 Olsen, Lühr, Sabaka, Manda, Rother, Toffner-Clausen and Choi (2006) CHAOS- A model of Earth's magnetic field derived from CHAMP, Orsted, and SAC-C magnetic satellite data. *Geophys. J. Int.* (in press)