

**MIRACLE and CHAMP:  
some results;  
MIRACLE and SWARM:  
some opportunities**

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– Main topics of this presentation:

- How can we derive ionospheric electrodynamic parameters (currents, conductances, fields) from multi-satellite data
- How can ground-based data be used together with multi-satellite data for studies of ionospheric electrodynamics

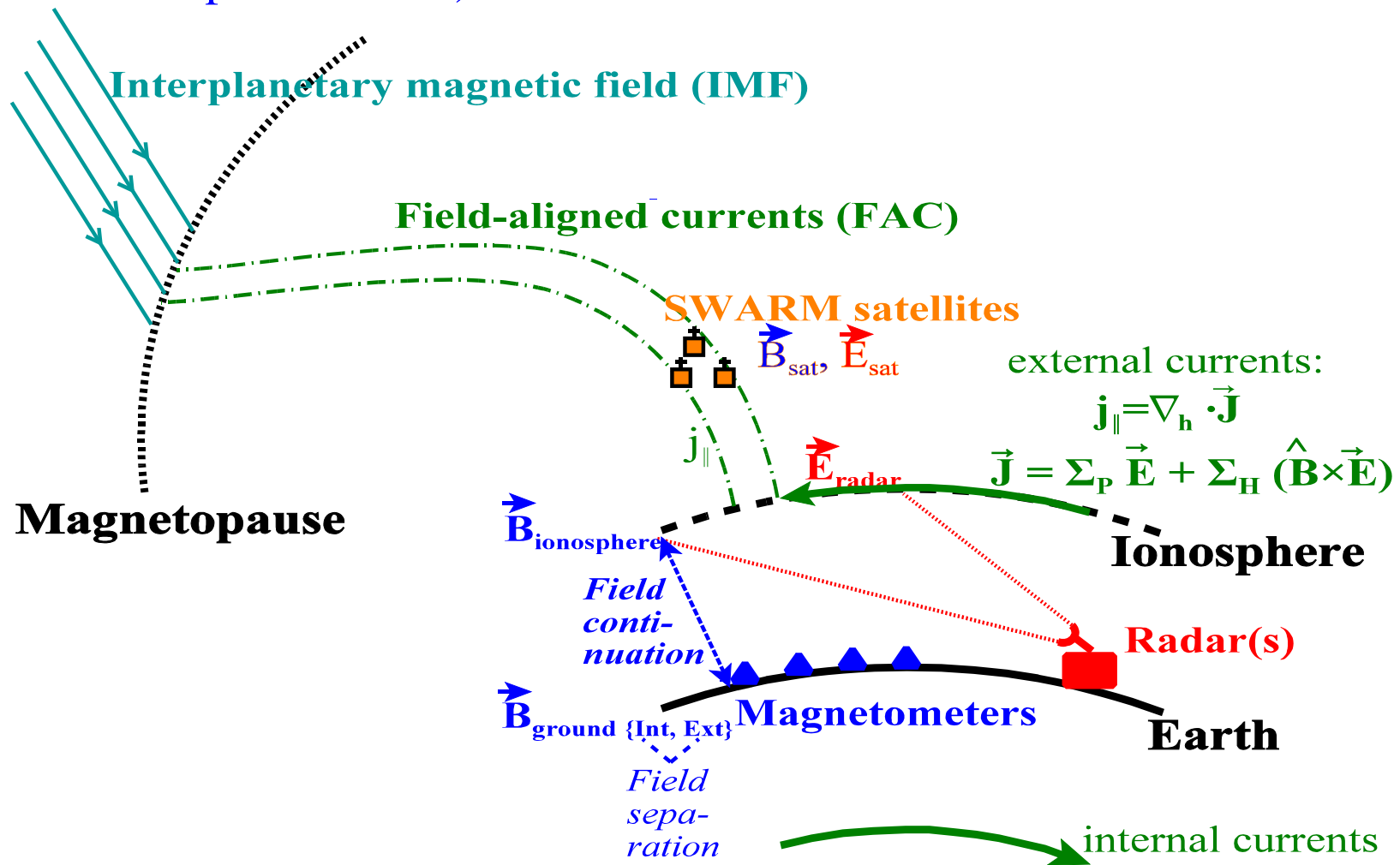
– Structure of this presentation:

- 1.) Introduction and basics
- 2.) Some CHAMP results using the 1D Spherical Elementary Current System (SECS) technique
- 3.) Some new opportunities with SWARM

- **Note:** In this presentation, **all magnetic fields mentioned are disturbance magnetic fields caused by ionospheric current systems**

# 1.) INTRODUCTION AND BASICS:

- Relevant current systems for ionospheric electrodynamics (schematic, thin sheet ionosphere model):

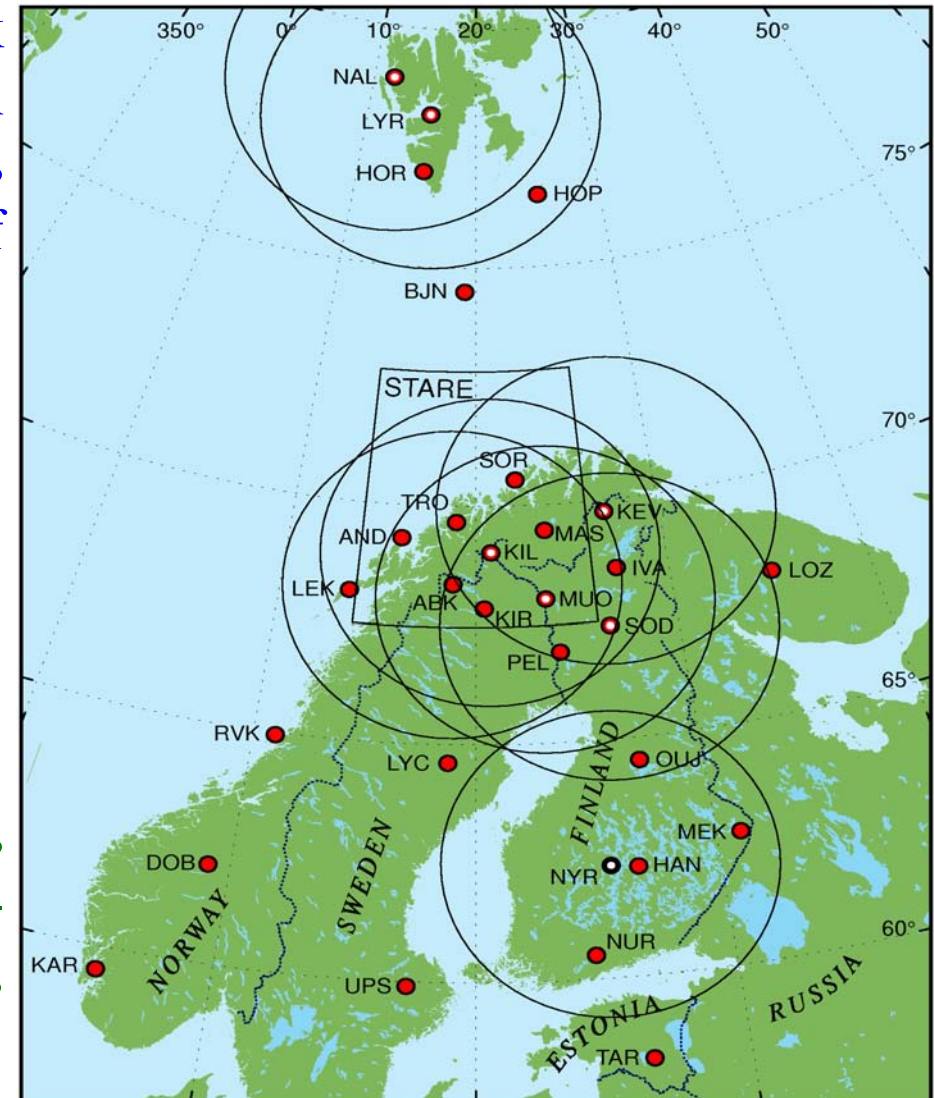


- Ground-based instrument network to perform integrated studies on ionospheric electrodynamics together with/ in support of SWARM:

**MIRACLE network**, consisting of:

- IMAGE magnetometers
- STARE radar (until March 2005)
- all-sky cameras

+ other ground-based instruments (EISCAT, SuperDARN, Fabry-Perot interferometers, photometers, riometers, etc.)



- Magnetometer
- All-sky camera
- All-sky camera and Magnetometer

October 2004

– Some basics:

a) *Curl-free and divergence-free 2D spherical elementary current systems (SECS):*

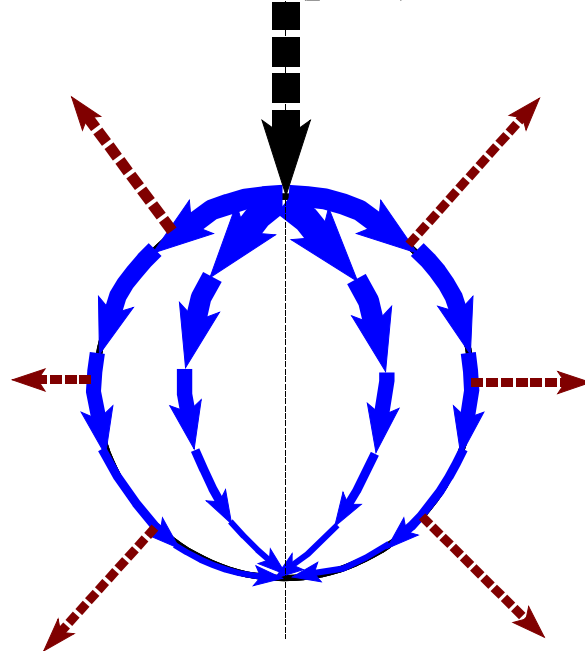
- written in spherical coordinate system  $(r', \vartheta', \varphi')$  with  $\vartheta' = 0$  at their “pole”  $\vec{r}_0$

$$\vec{J}_{cf,el,sph}(\vec{r}') = \frac{I_{0,cf}}{4\pi R_I} \cot(\vartheta'/2) \underline{e}_{\vartheta'}$$

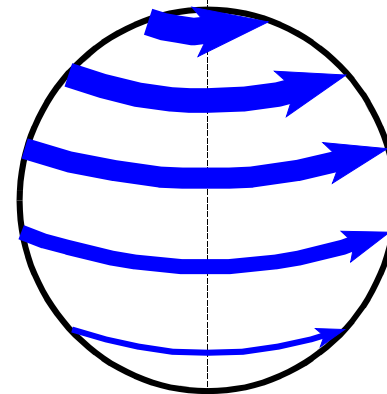
$$\vec{J}_{df,el,sph}(\vec{r}') = \frac{I_{0,df}}{4\pi R_I} \cot(\vartheta'/2) \underline{e}_{\varphi'}$$

$\vartheta' = 0$  (pole)

$\vartheta' = 0$  (pole)



**Curl-free elementary system  
(with associated FACs)**



**Divergence-free elementary system**

- Expansion of any horizontal current system  $\vec{J}$  in terms of SECS:

$$\vec{J}(\vec{r}) = \sum_{\text{Ionosph.}} \left( \frac{I_{0,df}(\vec{r}')}{4\pi R_I} \cot(\tilde{\vartheta}/2) \underline{e}_{\tilde{\varphi}} + \frac{I_{0,cf}(\vec{r}')}{4\pi R_I} \cot(\tilde{\vartheta}/2) \underline{e}_{\tilde{\vartheta}} \right)$$

$I_{0,df}(\vec{r}')$ ,  $I_{0,cf}(\vec{r}')$ : scaling factor of elementary current system with pole at  $\vec{r}'$ ;  
 $\tilde{\vartheta}$  and  $\tilde{\varphi}$ : coordinates of  $\vec{r}$  in spherical coordinate system with pole at  $\vec{r}'$ )

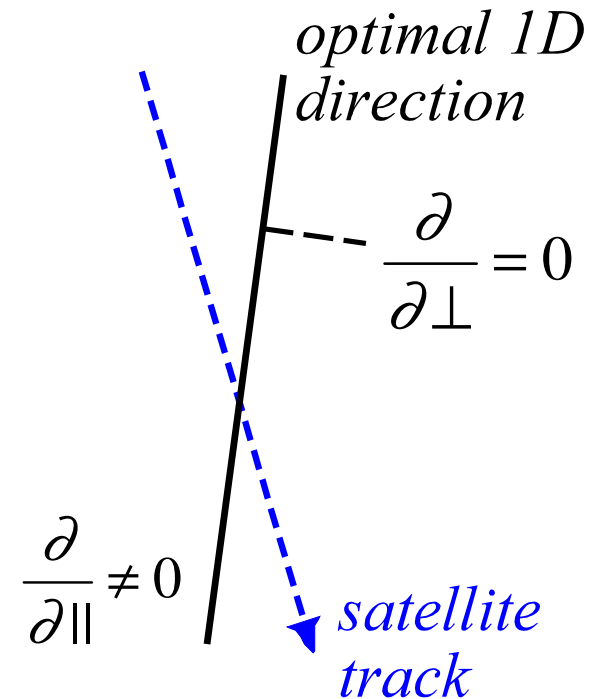
- **Note:** If the physical field to be expanded is known to be either divergence-free or curl-free
  - ▮ only one type of elementary systems is needed, expansion will by definition produce exactly divergence-/ curl-free system!
- **Example:** Ground equivalent currents  $\vec{J}_{eq} = \frac{2}{\mu_0} \hat{z} \times \vec{B}_h$  are divergence-free

– **For data of single satellites:**

- Only data along a single line available
  - 2D modelling not possible
  - ⇒ use 1D assumption, i.e., derivatives vanish along one horizontal direction
  - ⇒ use 1D SECS

- Notes:

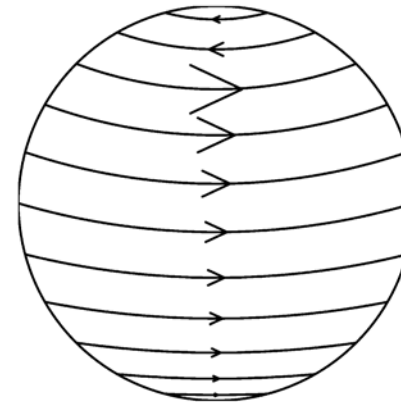
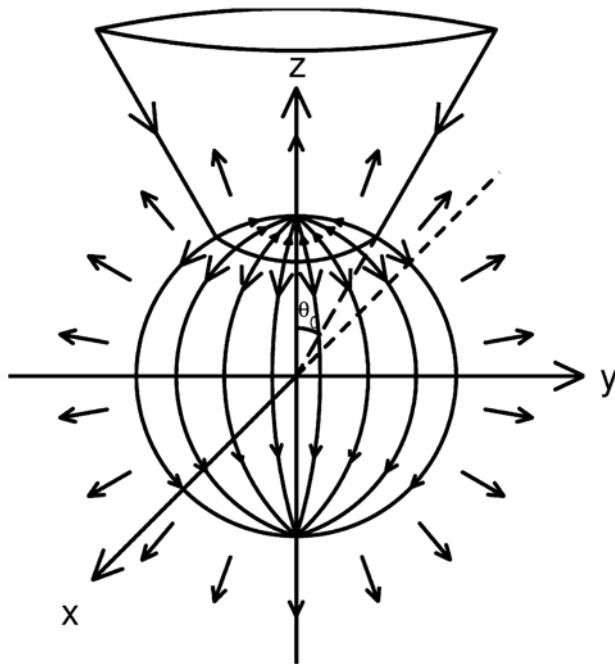
- ☞ We can check from the magnetic data itself whether 1D assumption applies, or not
- ☞ 1D direction does not need to be perpendicular to the satellite track
- ☞ “Optimal 1D direction” can be inferred from magnetic data



☞ **b) Curl-free and divergence-free 1D spherical elementary current systems (SECS):**

- written in “global” (e.g., geographic) spherical coordinate system  $(r, \vartheta, \varphi)$
- obtained by integration of 2D SECS over all  $\varphi$  at a certain  $\vartheta_0$

$$\vec{J}_{cf,el,sph}(\vartheta, \vartheta_0) = \frac{I_{0,cf}}{2R_I} \mathbf{e}_\vartheta \begin{cases} -\tan(\vartheta/2), \vartheta < \vartheta_0 \\ \cot(\vartheta/2), \vartheta > \vartheta_0 \end{cases} \quad \vec{J}_{df,el,sph}(\vartheta, \vartheta_0) = \frac{I_{0,df}}{2R_I} \mathbf{e}_\varphi \begin{cases} -\tan(\vartheta/2), \vartheta < \vartheta_0 \\ \cot(\vartheta/2), \vartheta > \vartheta_0 \end{cases}$$





### c) How to obtain the currents from the magnetic field measurements

- Mathematically: Solve inversion problem

$$\underline{\underline{T}} \cdot \underline{I} = \underline{Z}$$

where

$$\underline{Z} = \begin{pmatrix} Z_{1,r} \\ Z_{1,\vartheta} \\ Z_{1,\varphi} \\ \vdots \\ Z_{n_{obs},r} \\ Z_{n_{obs},\vartheta} \\ Z_{n_{obs},\varphi} \end{pmatrix} \quad \underline{I} = \begin{pmatrix} I_{0,cf,1} \\ I_{0,cf,2} \\ \vdots \\ I_{0,cf,n_{el}} \\ I_{0,df,1} \\ I_{0,df,2} \\ \vdots \\ I_{0,df,n_{el}} \end{pmatrix} \quad \underline{\underline{T}} = \begin{pmatrix} T_{11,r,cf} & T_{12,r,cf} & \cdots & T_{1n_{el},r,cf} & T_{11,r,df} & T_{12,r,df} & \cdots & T_{1n_{el},r,df} \\ T_{11,\vartheta,cf} & T_{12,\vartheta,cf} & \cdots & T_{1n_{el},\vartheta,cf} & T_{11,\vartheta,df} & T_{12,\vartheta,df} & \cdots & T_{1n_{el},\vartheta,df} \\ T_{11,\varphi,cf} & T_{12,\varphi,cf} & \cdots & T_{1n_{el},\varphi,cf} & T_{11,\varphi,df} & T_{12,\varphi,df} & \cdots & T_{1n_{el},\varphi,df} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ T_{n_{obs}1,\varphi,cf} & T_{n_{obs}2,\varphi,cf} & \cdots & T_{n_{obs}n_{el},\varphi,cf} & T_{n_{obs}1,\varphi,df} & T_{n_{obs}2,\varphi,df} & \cdots & T_{n_{obs}n_{el},\varphi,df} \end{pmatrix}$$

**Z**: vector of observations

**I**: vector of (internal and external) scaling factors

**T**: transfer matrix (describes magnetic field effect at observations points due to a single curl-free or divergence-free SECS)

- Method to solve problem: **Singular value decomposition (SVD)**

– **Properties and advantages of SECS:**

- 1D/2D SECS are complete basis functions for any continuously differentiable 1D/2D vector field on a sphere
- SECS are local basis functions
- Location of elementary system poles can freely be chosen, adjusted to the geometry/ data availability of the specific problem
- No need to select globally smallest resolvable wavelength, like in methods based on harmonic functions (spherical (cap) harmonic analysis, Fourier analysis)
- SECS naturally divide up curl-free and divergence-free parts of vector fields, which often have different physical meaning
- *Particularly for solving current systems from magnetic satellite data:*
  - Local and remote currents can be deduced on the same spatial scale  
⇒ it makes sense to combine them for further calculations
  - Satellite and ground-based magnetic data can be combined in the same inversion, when both data available

### 3.) SOME CHAMP RESULTS USING THE 1D SECS TECHNIQUE:

– Examples here:

- Derivation of full current system ( $j_{\parallel}$ ,  $J_{\vartheta}$ ,  $J_{\varphi}$ ) on same spatial scale, global statistics
- Derivation of the parameter  $\alpha = \Sigma_H/\Sigma_P$ , statistics between  $\alpha$  and  $J_{\varphi}$ , global statistics

• Notes:

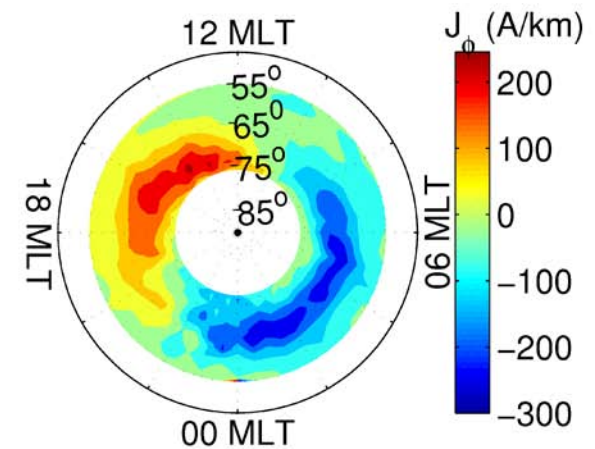
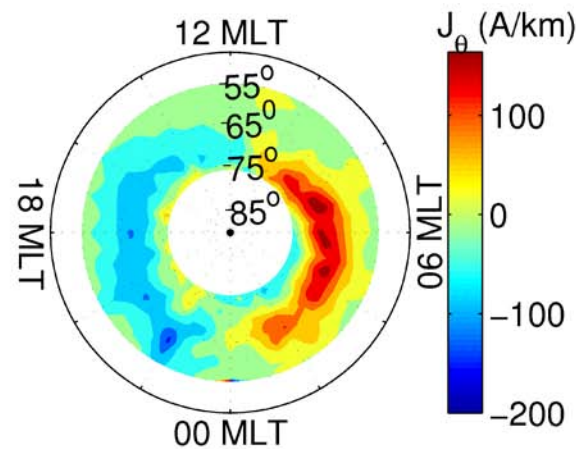
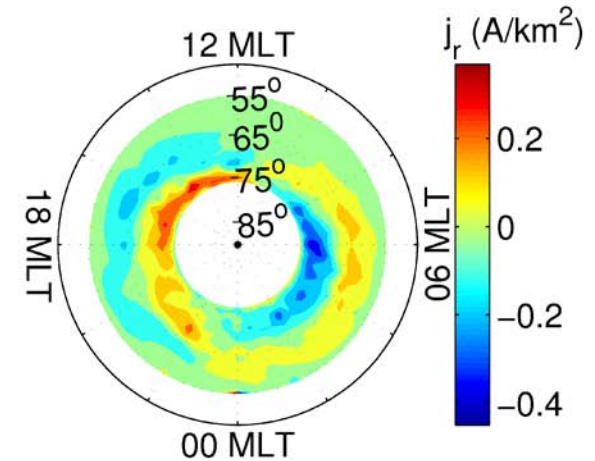
- ☞  $\alpha$  is an important parameter, as it tells about the energy spectrum of precipitating particles, and about the altitude distribution of ionospheric currents
- ☞  $\alpha$  is also needed as an input parameter for the “method of characteristics” that infers  $\Sigma_H$ ,  $\Sigma_P$  from magnetic and electric field data

a) Statistics of  $j_{\parallel}$ ,  $J_{\theta}$ ,  $J_{\phi}$ , based on 6112 passes with sufficiently good 1D condition:

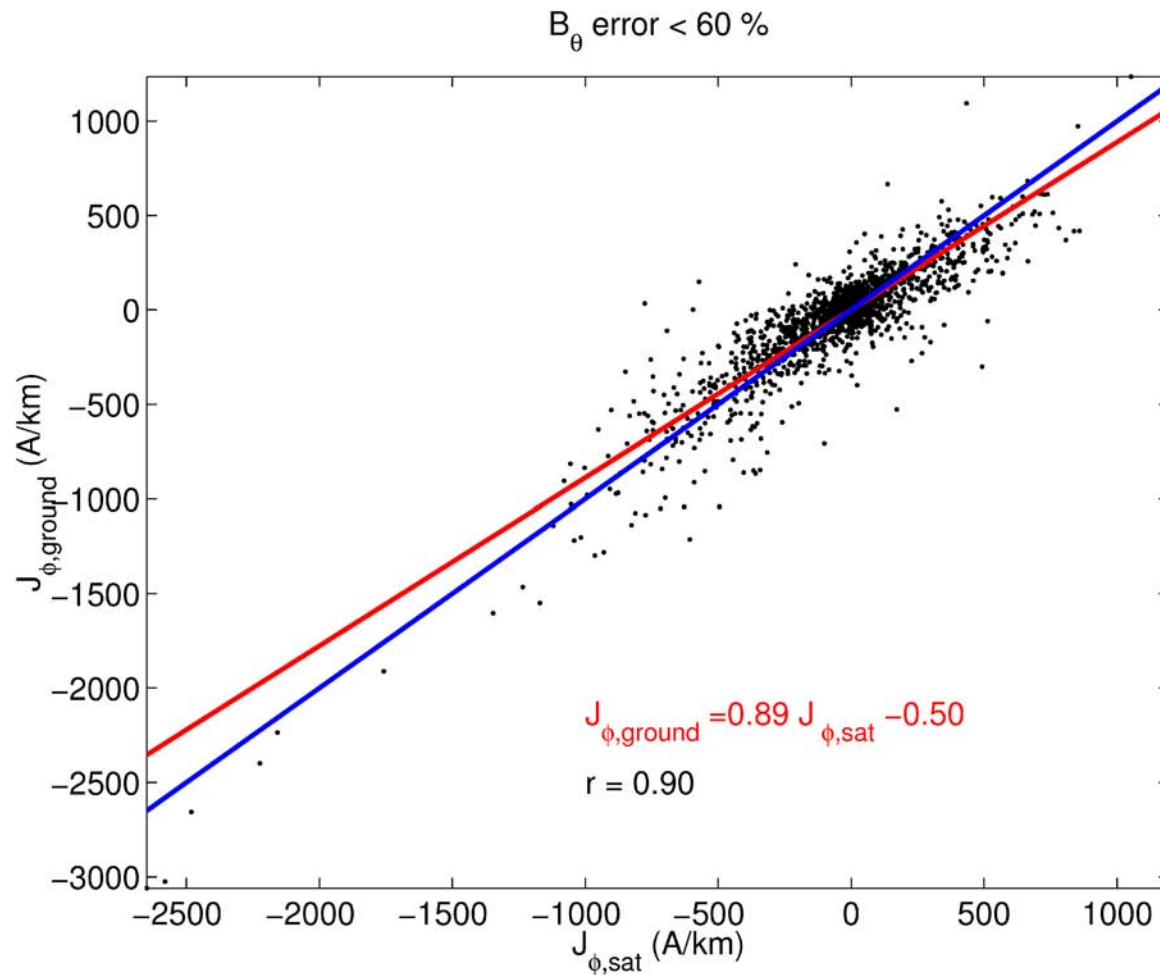
⇒ binning with respect to activity, season, IMF, etc. possible

$0.00 \leq |I_{\phi}| < \text{Inf MA}$

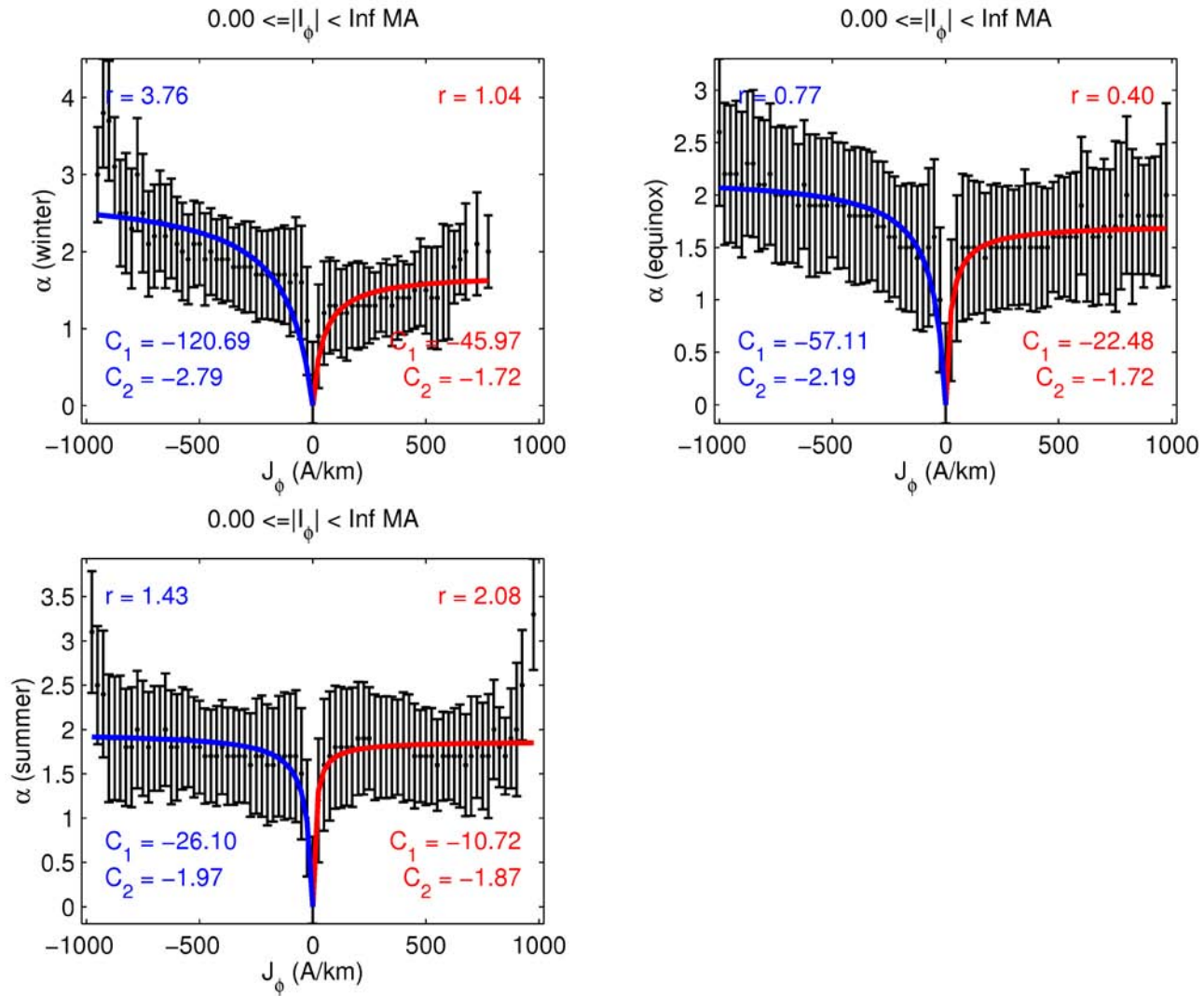
6112 overflights



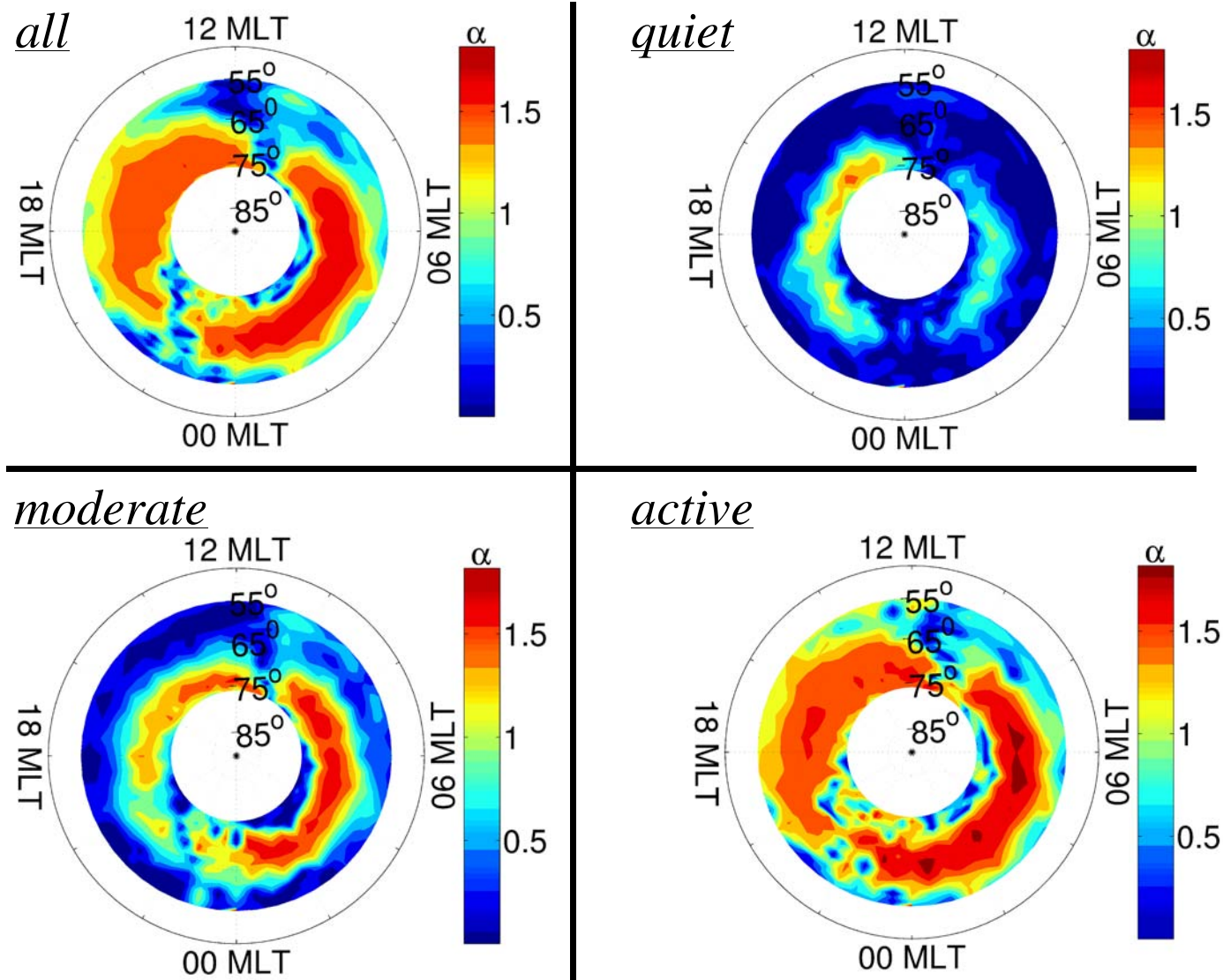
b) Confirmation of 1D SECS satellite results from Champ for  $J_\phi$  with 2D SECS ground-based results from MIRACLE for  $J_{\phi,eq}$ :



c) Statistical derivation of  $\alpha$  as a function of  $J_\phi, \left\{ \begin{matrix} sat \\ ground \end{matrix} \right\}$ :



d) Global maps of  $\alpha$ , total and binned by activity level:



#### 4.) NEW OPPORTUNITIES USING MULTI-SATELLITES:

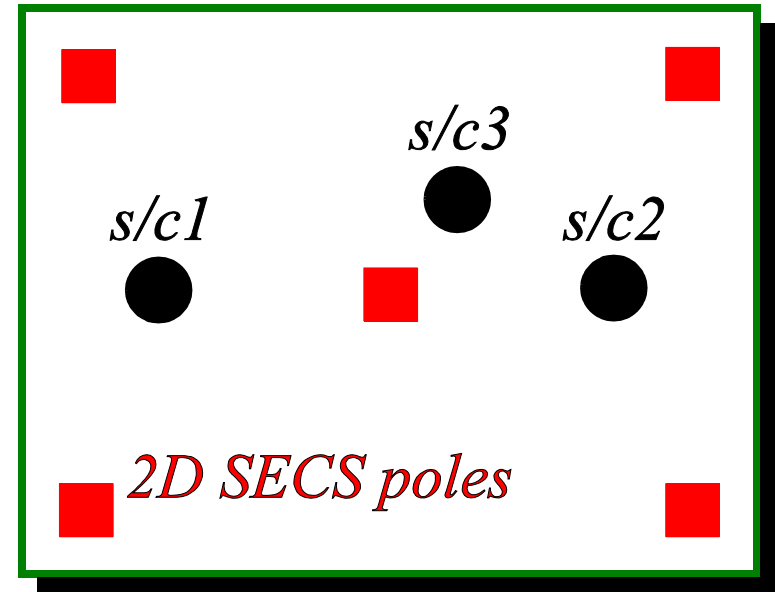
- Major new possibilities with SWARM for ionospheric physics:
    - 2-3 satellites available closeby  $\Rightarrow$  gradients of ionospheric parameters can be measured
    - electric field data available, also with gradients
  - Focus here on:
    - Techniques to derive local *and* remote currents (using SECS), and conductances with  $\vec{E}$ -field
- (Other techniques, not discussed here:
- Derive local currents at the satellites only (by evaluation of  $\nabla \times \vec{B}$ ) )



a) Full 2D SECS technique:

– Properties:

- evaluates both local and remote currents
- assumption of stationary situation optionally
- makes use of known current geometry
- needs (at least) 3 satellites to work



– Would be optimal for more satellites, but here:

- If real situation is close to 1D, this is not well represented by only a few 2D SECS
- However, we cannot put too many 2D SECS, as only 3 satellites are available for input data

⇒ probably better to use a hybrid 1D/2D SECS approach

b) Combined (hybrid) 1D/2D SECS technique:

– Properties like full 2D SECS method, but:

- needs only 2 satellites to work
- needs stationarity assumption for small number of satellites
- basis system is partly linear dependent  $\Rightarrow$  should not matter in practise, but tests needed

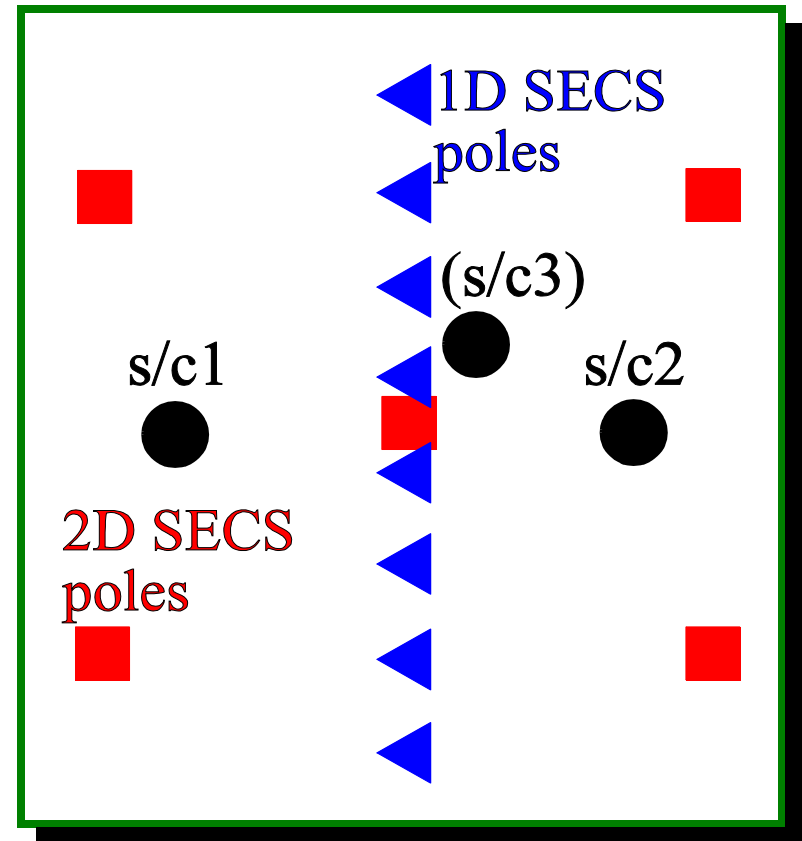
– In both cases c) and d):

- With knowledge of  $\vec{E}$ ,  $\Sigma_H$  and  $\Sigma_P$  can simply be derived from  $\vec{J}$  by inversion of Ohm's law:

$$\Sigma_H = \frac{(\vec{J} \times \vec{E})_r}{|\vec{E}|^2}$$

and

$$\Sigma_P = \frac{\vec{J} \cdot \vec{E}}{|\vec{E}|^2}$$



c) Further collaboration possibilities between SWARM and MIRACLE, *if* a planned new VHF radar for MIRACLE is realized:

– Ground-based data of  $\vec{B}_G$ ,  $\vec{E}_{radar}$ , and estimate of  $\alpha$ , can be used to infer 2D real current system and conductances independently from SWARM data (method of characteristics)  $\Rightarrow$  during MIRACLE overflights, *calibration of SWARM results possible!*

– With  $\vec{E}_{radar}$  and  $\vec{E}_{sat}$ , studies of altitude dependence of the electric field and its gradients become possible

☞ Why the electric field can be altitude-dependent?

- Effect of inductive electric fields
- Effect of space charges in lower ionosphere
- Effect of significant field-aligned conductance in lower ionosphere
- Scale-dependence

☞ coordinated analysis of satellite and radar electric fields important!

5.) GENERAL CONCLUSION:

*A LOT OF EXCITING NEW SCIENCE  
POSSIBILITIES ARE OPENING UP WITH  
SWARM,  
PARTICULARLY ALSO IN CONNECTION WITH  
GROUND-BASED EQUIPMENT!*

– Method references:

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