Constraining Numerical Geodynamo Model with Surface Observations:
Geomagnetic data assimilation

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1. Introduction

   Motivation, Challenge, Approach

2. Assimilation with 100 year surface geomagnetic observations

   First taste of geomagnetic data assimilation

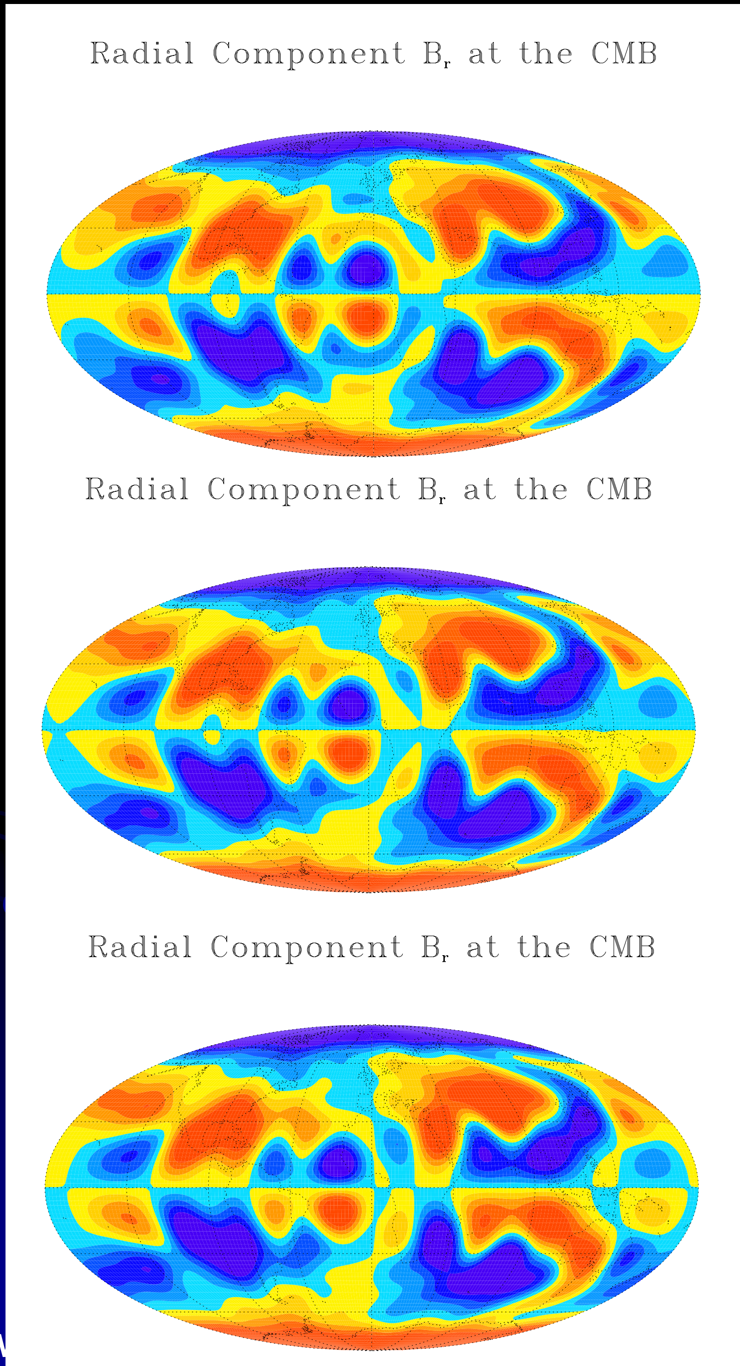
3. Assimilation with synthetic data from numerical model

   Response of the system to surface observation

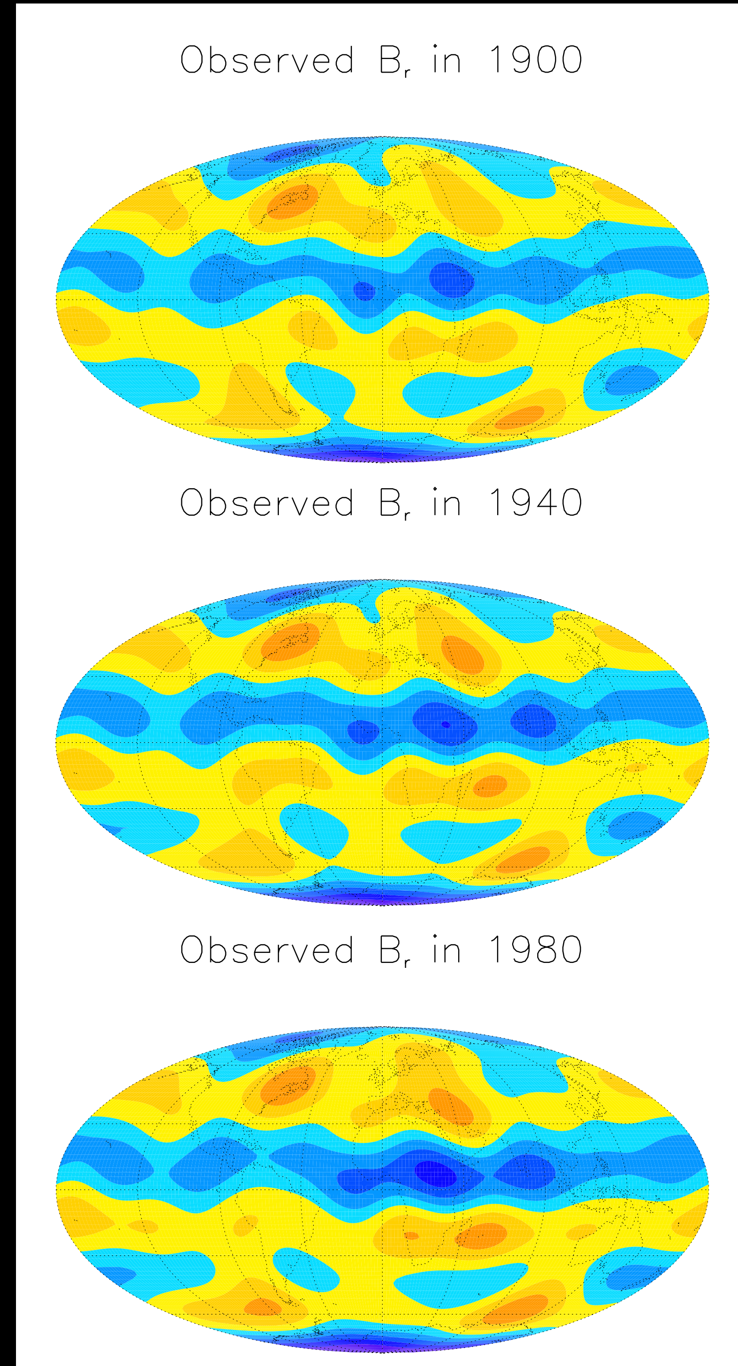
4. Error covariance studies

   Cross-correlation in space and between state variables (fields)

5. Discussion



Dipole-less Br
(Model results from MoSST)



Dipole-less Br
(Data provided by Sabaka)

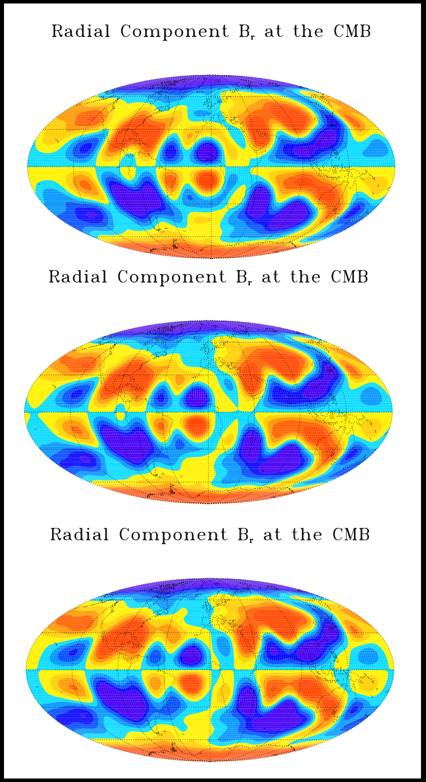
SW

Introduction: Motivation

Can we use surface geomagnetic observation to constrain numerical model?

Assimilating surface observation with numerical model output:

- Improve numerical model
- Improve knowledge of "true state" in the core
- Enable other geophysical applications
  - Time-variable gravity
  - Core-mantle interactions
  - Surface deformation
- Forecast geomagnetic secular variation



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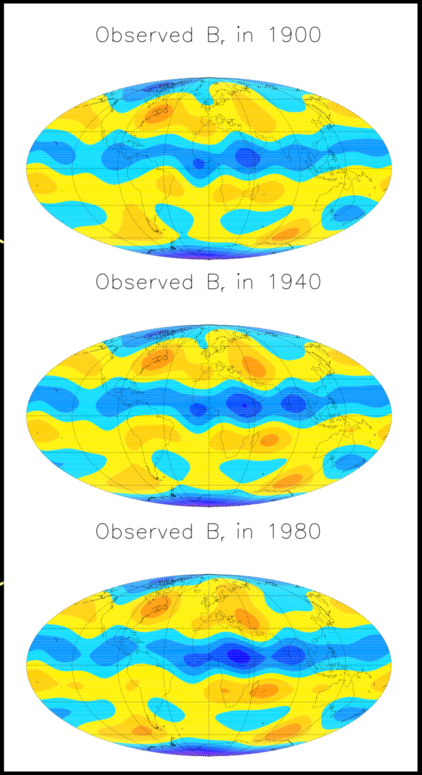


Dipole-less Br
(Data provided by Sabaka)

Sequential data assimilation algorithm for our model

Introduction: Sequential Assimilation

Assimilation Procedure

- Denote by **x** the state variable vector
- At time t_k, an observation $\mathbf{x}^o$ and a model forecast $\mathbf{x}^f$ are made
- Assimilation is carried out at the time to create an analysis $\mathbf{x}^a$.
- The analysis will be used as the initial condition for numerical modeling.
- The process will be repeated at a later time t_{k+1}.

$$\mathbf{x}^a = \mathbf{x}^f + \mathbf{K}\left(\mathbf{x}^o - \mathbf{H}\mathbf{x}^f\right)$$

$\mathbf{x}^o$: observation

$\mathbf{x}^f$: forecast

$\mathbf{x}^a$: analysis

K: gain matrix

H: observation operator

Two algorithms for **K**
- Optimal interpolation (OI)
- Covariance analysis

Introduction: Challenges and Approaches

1. Problems with numerical models

- **Field strength dependence**

$$\left|\mathbf{B}\right| \propto \sqrt{R_{th}}$$

- **System very sensitive**

$$\frac{\partial}{\partial t}\overline{V}_{\phi} = \frac{1}{R_0}\int_{\Sigma}\left(\mathbf{J}\times\mathbf{B}\right)_{\phi} d\Sigma$$

- **Different time scales**

$$\text{Time scale} \propto \sqrt{R_o}$$

Parameters	Outer core	Numerical Model
Rayleigh #: R_{th} (Buoyancy force)	Not well known	Not very supercritical
Rossby #: R_o (Fluid inertia)	10^{-9}	10^{-6}

1. Assimilate scaled observed field $\alpha(t)\mathbf{B}^{obs}$
2. Limit spatial correlation length scale
3. Optimize time scale with error growth

Introduction: Challenges and Approaches

2. Limited surface observations

- Impact on the physical sate inside the core
- Assessment of the assimilation effect
- Assimilation spin-up time

Quality of the observed poloidal field back in time!

Back in Time	Degree L_{obs}
~ 10 year	12 ~ 14
~ 10^2 year	8 ~ 10
~ 10^3 year	4 ~ 6

1. Modify "hidden" physical variables from surface observations (e.g. co-variance)
2. Synthetic "observations" for sufficient model solution response time

Optimal Interpolation:

Specification:

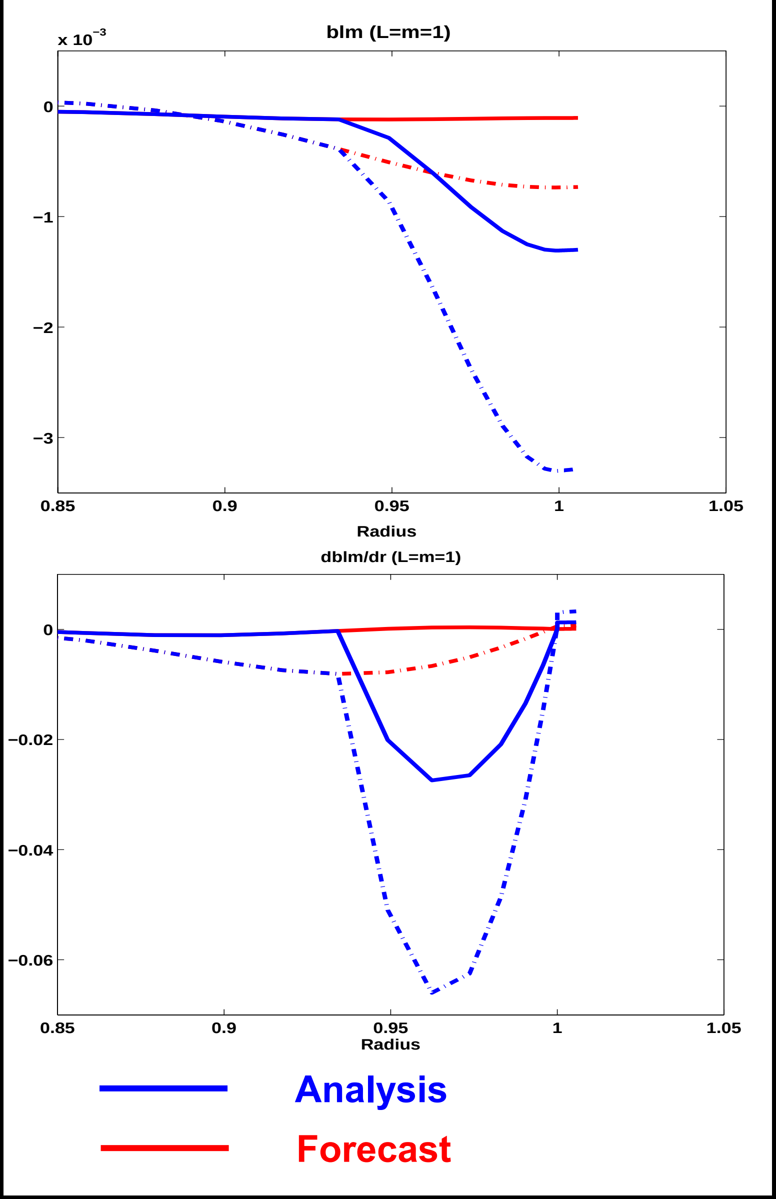
- **Field Scaling factor:** ***Dipole coefficient***
- **Time scaling:** ***magnetic free decay time***
- **Interpolation domain:** ***200 km deep into the core.***
- **Geomagnetic data:** **1900 - 2000**

Advantage:

Simple, less computing, independent of the model details

Disadvantage:

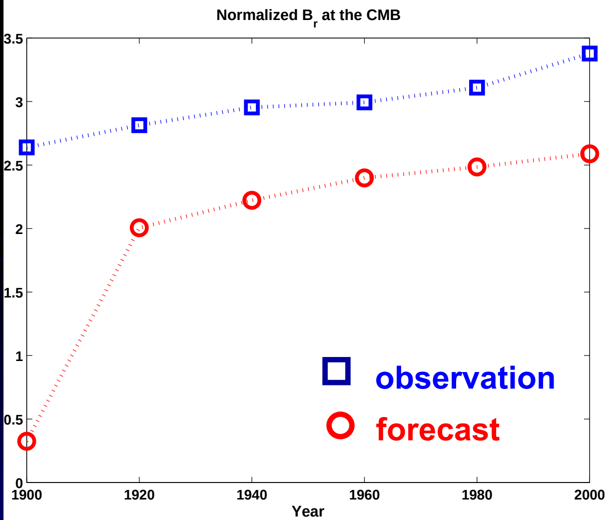
Not “optimal”

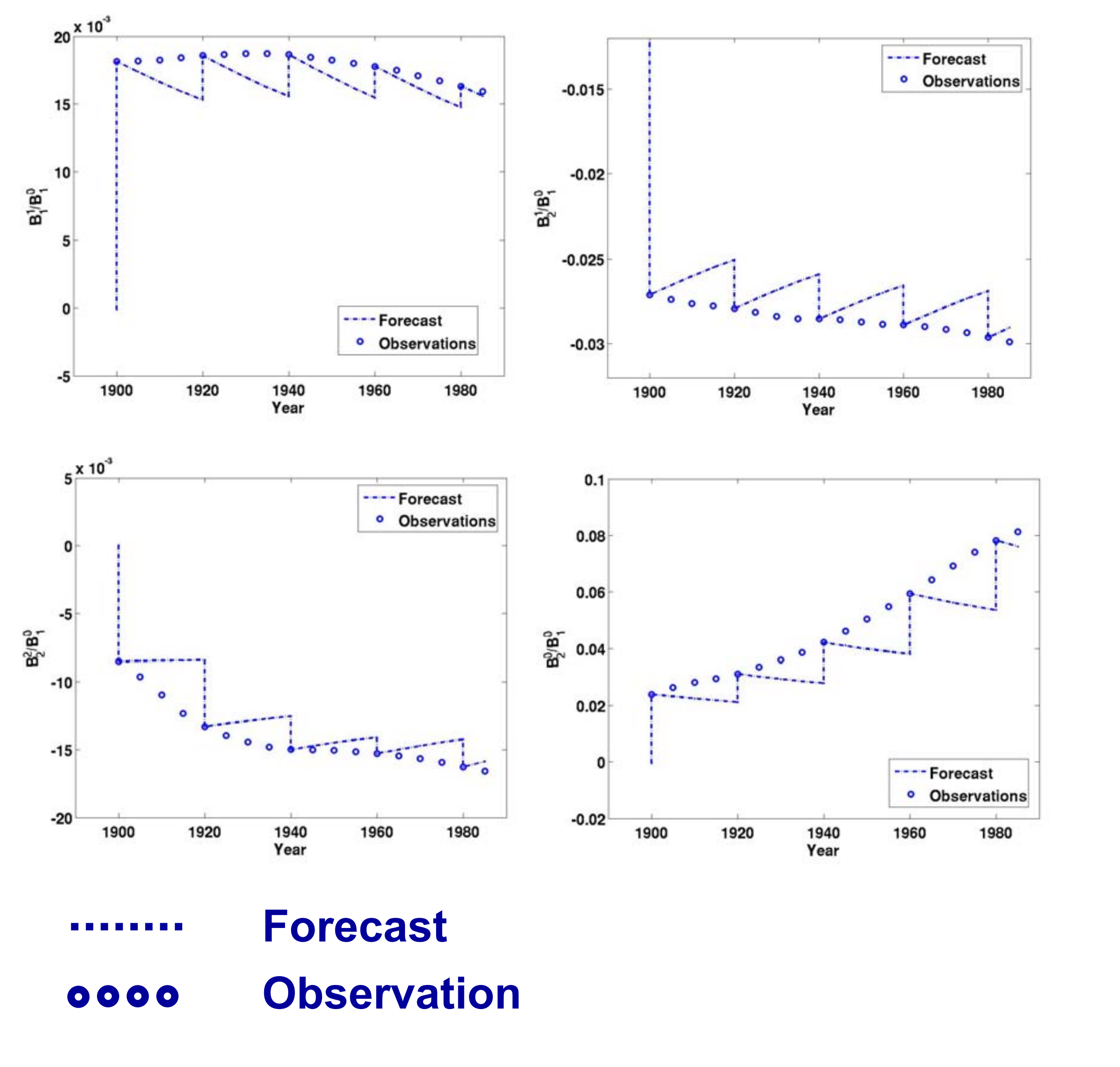


Optimal Interpolation: Results

Fall AGU, San Francisco, 12/2005

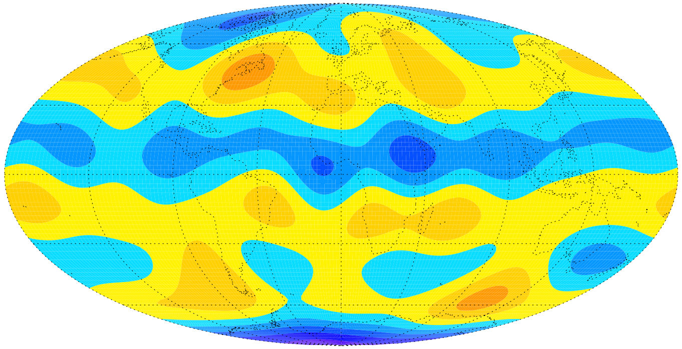
Difference between the observed field and the forecast (errors) decreases with "spin-up" time



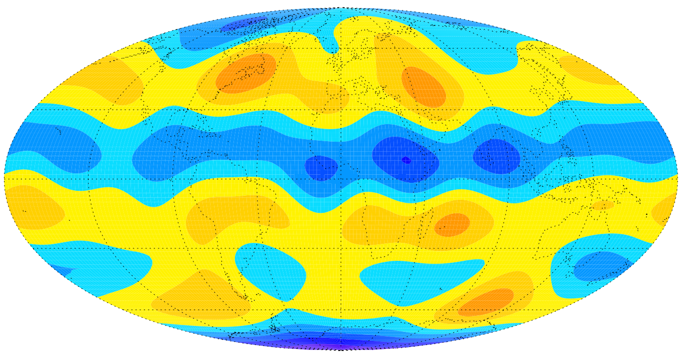


SWARM Science Workshop, Nantes, France, 2006

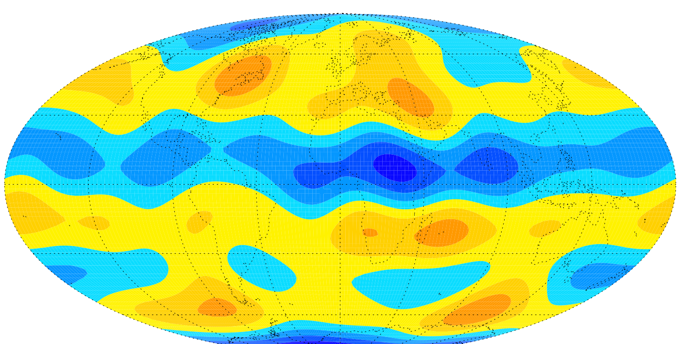
Observed B_r in 1900



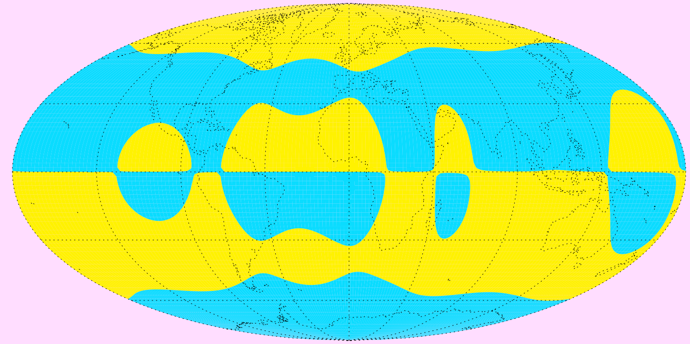
Observed B_r in 1940



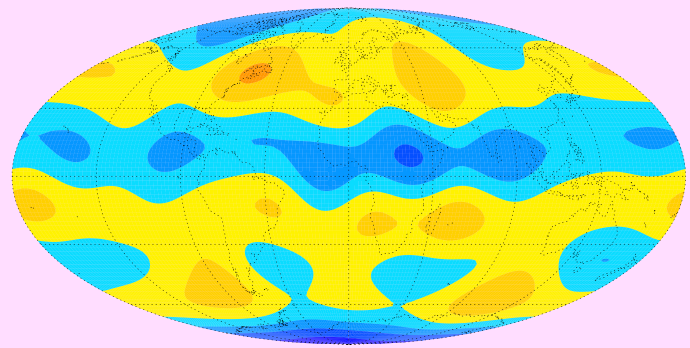
Observed B_r in 1980



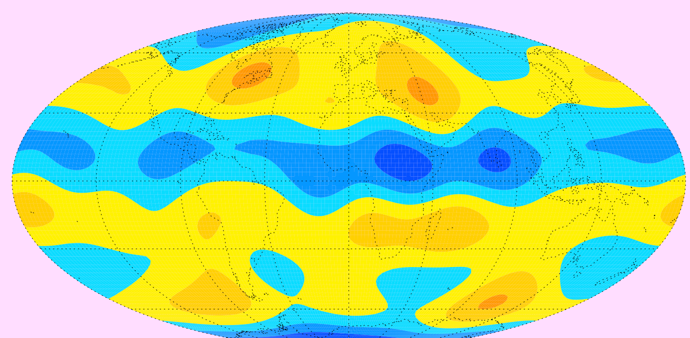
Predicted B_r in 1900



Predicted B_r in 1940



Predicted B_r in 1980



SWARM

Synthetic Geomagnetic Assimilation:

Specification

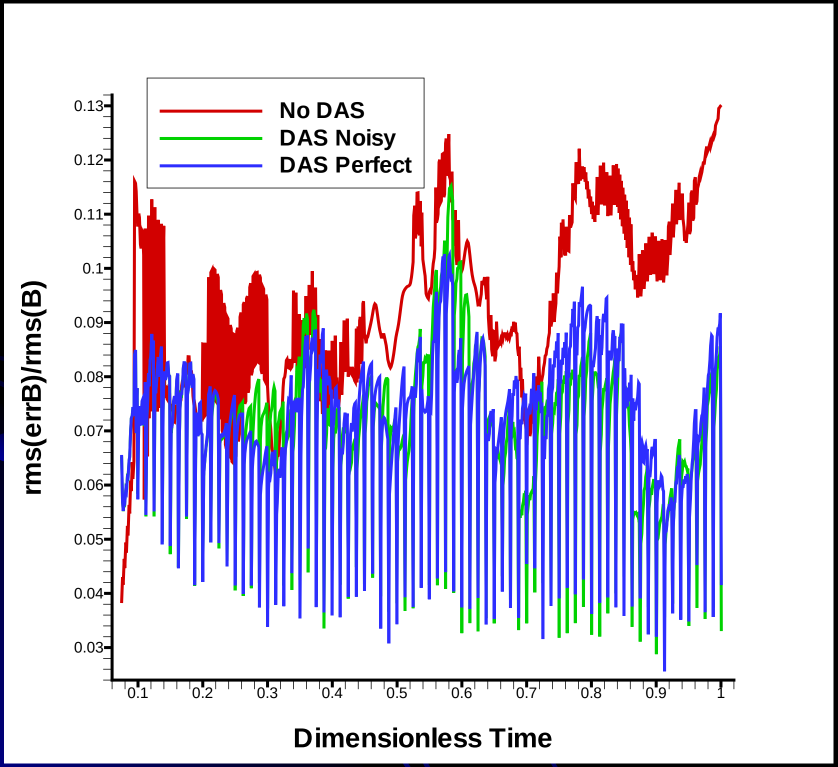
1. Model: $R_1 = 14500$ (forecast)
2. Truth: $R_2 = 15000$ (observation)
3. Assimilation time:
   $t = \tau_\eta$ (20000 years for the core)
4. Assimilation interval:
   $\Delta t = 0.01\ \tau_\eta$
5. Assimilation domain:
   200km (below the CMB)

Goals

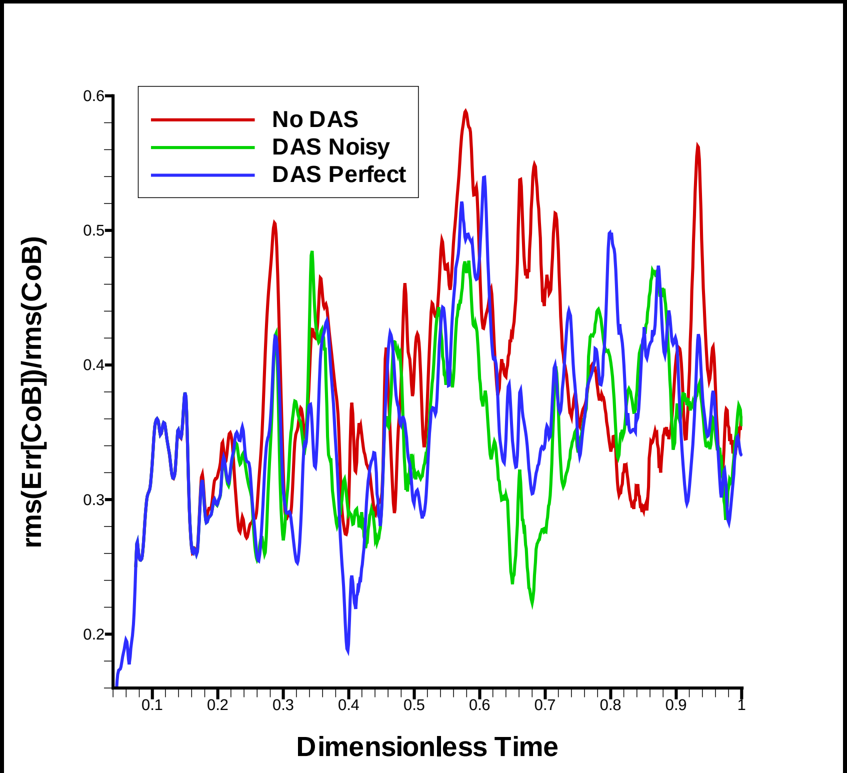
1. Understand the error evolution on very long time scales
2. Understand the changes in other physical fields due to poloidal magnetic field assimilation
3. Understand the improvement of the model forecast capability over time.

Synthetic Geomagnetic Assimilation: Results

1. The error growth rate for the poloidal magnetic field in the region near the CMB decreases over time.

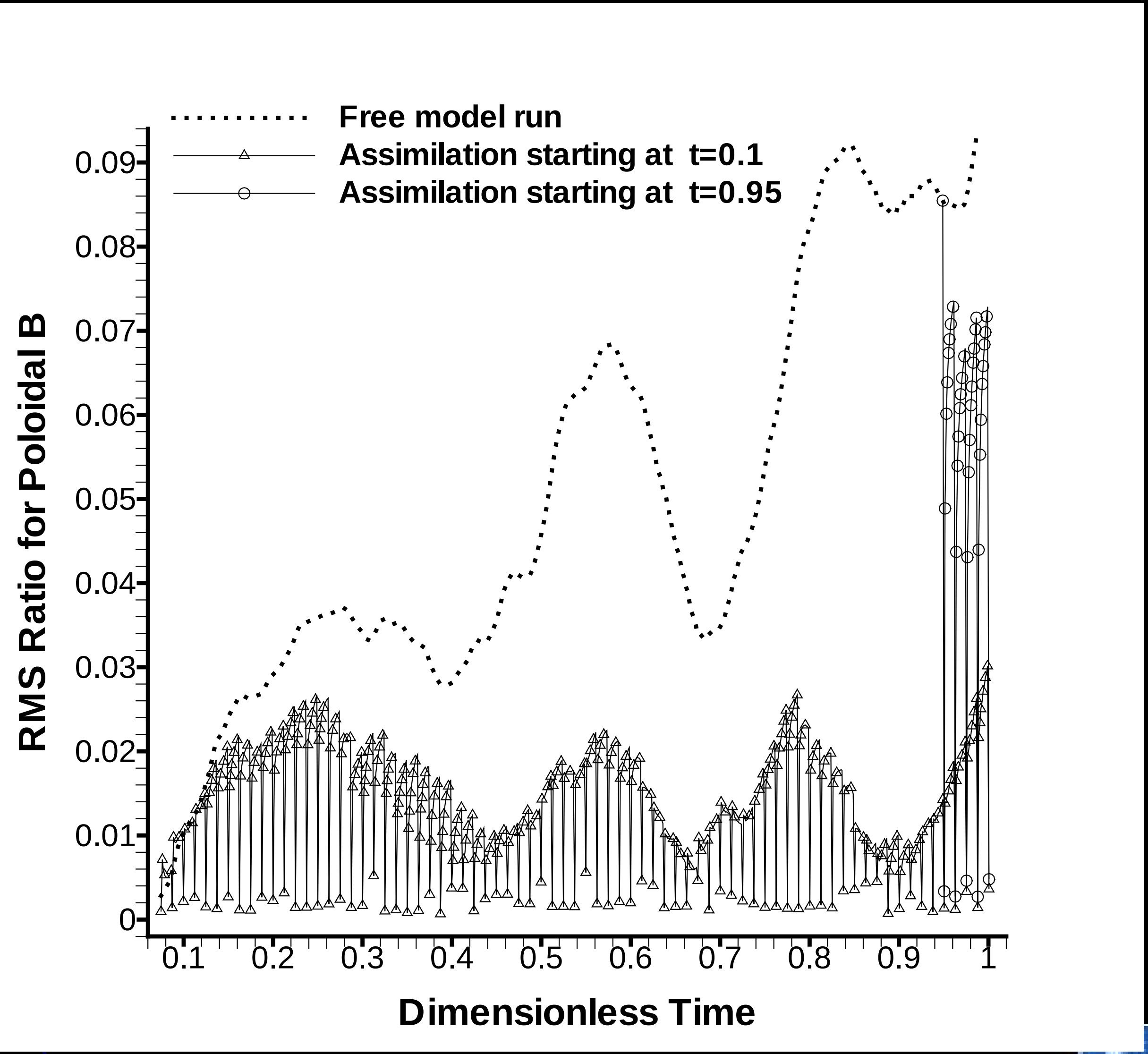


2. The error of the poloidal field over the entire outer core is bounded.



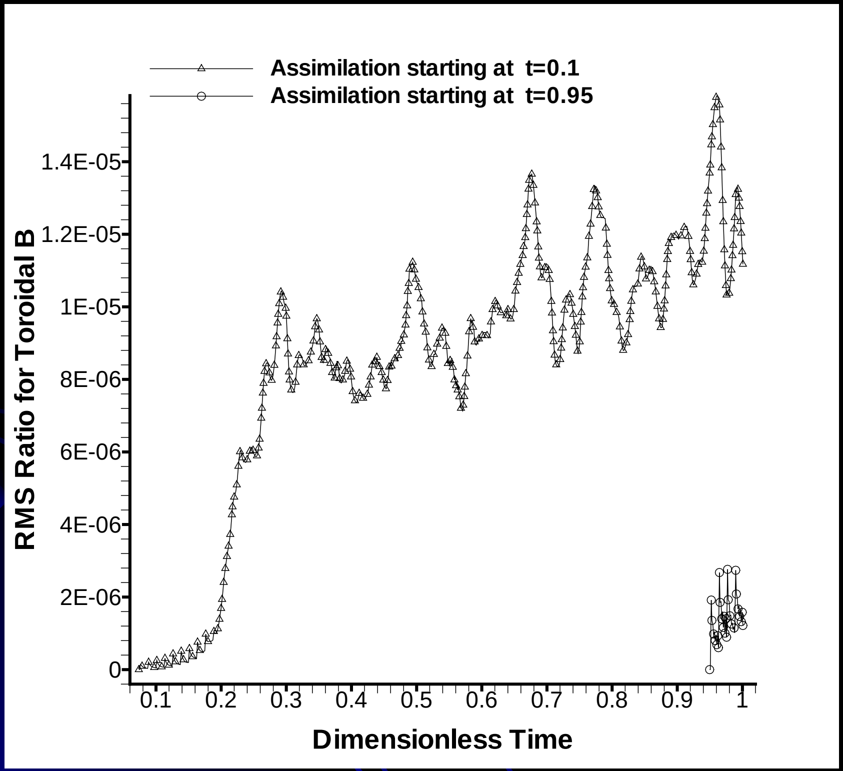
Synthetic Geomagnetic Assimilation: Results

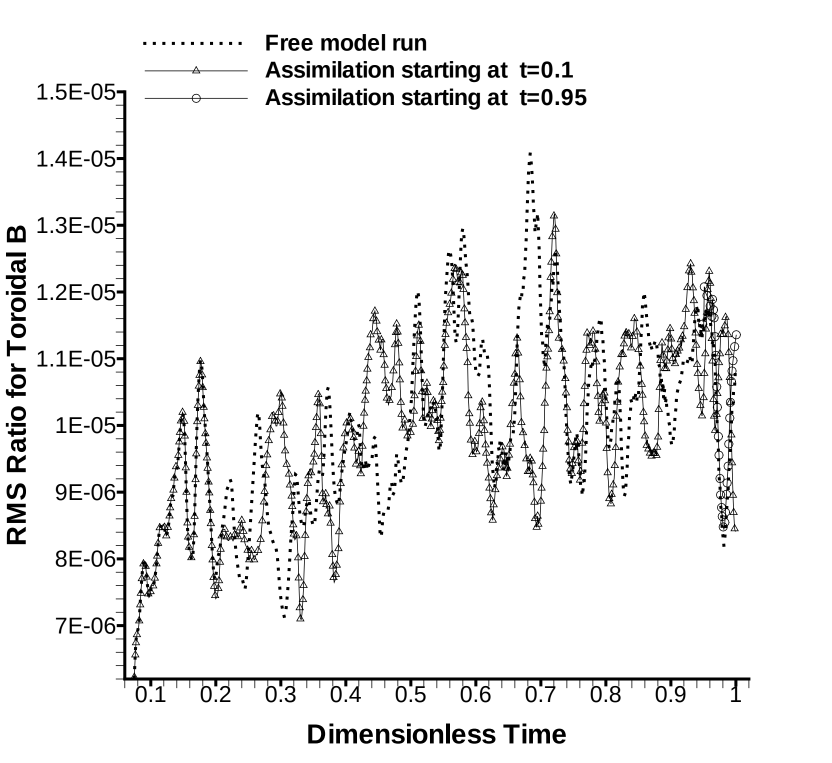
3. The longer the spin-up time, the closer the model output to the “truth”



Synthetic Geomagnetic Assimilation: **Results**

4. **Other state variables respond to the observation constraint**





Ensemble Calculation of Covariance

1. **Evaluate covariance matrix with an ensemble of calculations of perturbation to the model.**
2. **Updating "hidden" physical variables with the surface observation via covariance matrix, Poloidal field inside the core Toroidal field, velocity field, density perturbation in the core**
3. **For details, visit our poster (Sun *et al*)**

Advantage:

Optimal for given model

Disadvantage:

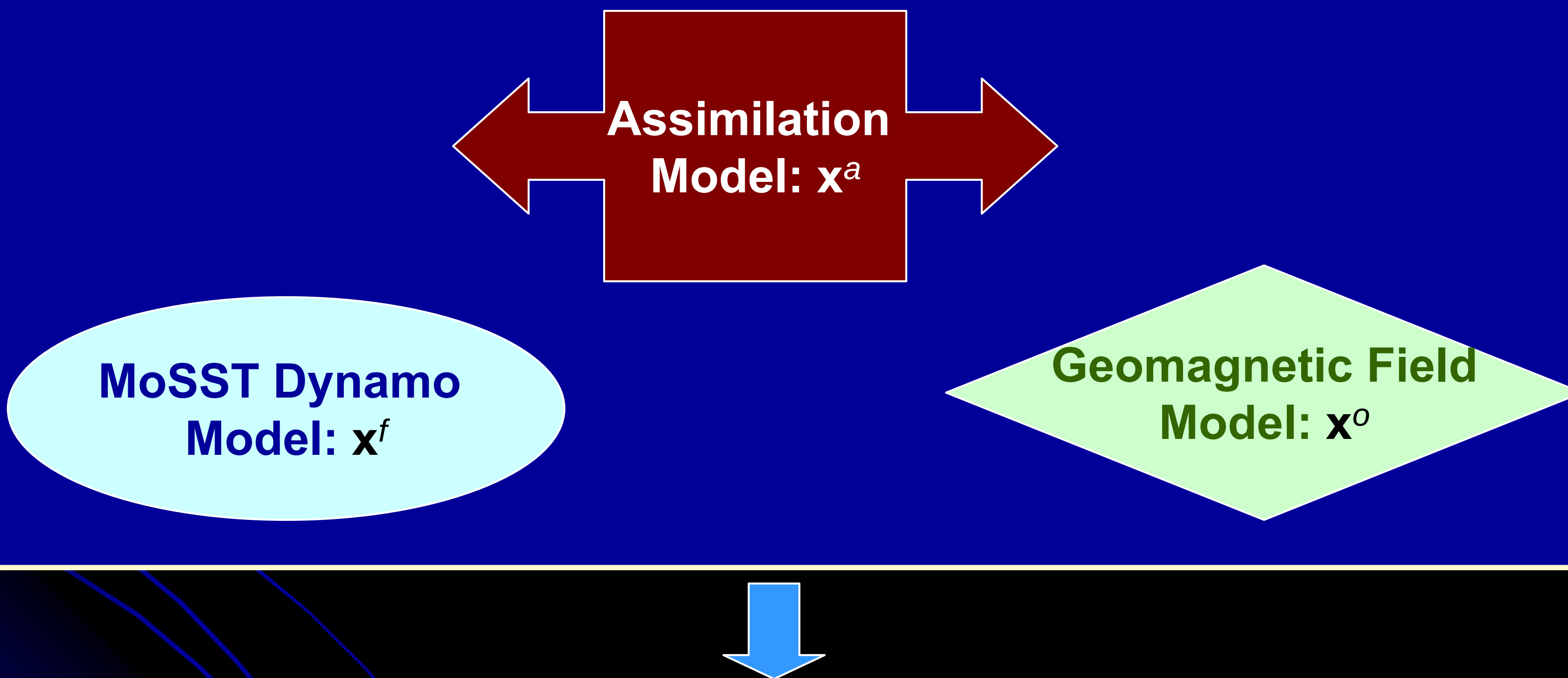
Very expensive, 2 orders of magnitude increase in CPU requirement

Discussion

1. We have discussed the challenges in geomagnetic data assimilation, and our approaches and first results.
2. Our test with 100 year geomagnetic data assimilation demonstrates that errors between observation and forecast decrease in time.
3. Our initial studies with synthetic data show that errors between “truth” and “model” (for poloidal field) are bounded over much longer time scales.
4. Other fields are also affected by surface observation. Further assessment is necessary.
5. Covariance studies can improve OI algorithm by introducing optimal cross-correlation between variables.

Our Goal

Geomagnetic Data Assimilation Framework: MoSST_DAS



Dynamo state in the core consistent with (regulated by) the surface observations