ON DYNAMICAL MODELS OF THE SECULAR VARIATION OF THE EARTH'S MAGNETIC FIELD

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ABSTRACT

As already envisioned by Hide (1966), the motions in the liquid core of the Earth that have short time scales (1-10 yrs) are strongly constrained by rotation forces. This viewpoint is consistent with the frozen flux assumption: decadal timescales are too short for the electrical currents to grow and as a result magnetic forces are weak compared to rotation forces. I develop some consequences of this hypothesized importance of the rotation forces in the mechanisms of the secular variation (SV). Among the flows strongly constrained by rotation forces within the Earth's core, a special class has been well studied: the torsional oscillations. Geostrophic motions change indeed with time according to an Alfvèn wave equation. The propagation of the torsional oscillations can be modelled numerically. Finally, analog experiments, such as the DTS experiment in Grenoble, complement well theoretical predictions and numerical models to discuss the competition between magnetic and rotation forces within the fluid outer core on 1yr - 100yrs time scales.

Key words: Earth's core; Earth's magnetic field; Secular Variation.

1. INTRODUCTION

The description of the Earth's magnetic field has recently improved a lot. Using data from the German satellite CHAMP, Olsen et al. (2006) have been able to map the first time derivative of the main magnetic field up to degree 15. The scales inferred for the secular variation are thus smaller than for the core field itself (limited to degree 13 because of the crustal field).

Olsen et al. (2006) estimated the reorganisation time as 20 years for structures of degree 16. Thus, magnetic records from satellites only give us a glimpse of the working of the dynamo acting in the Earth's fluid core. Historical data are longer but less accurate and the lifetime of such an important feature as the patch of reverse magnetic field at the CMB (Core-Mantle Boundary) below South Africa is still discussed (see the two presentations by Gillet and Jackson during this meeting). That may explain why theoretical progress in the understanding of the secular variation in the past few years have not yet matched the dramatic improvement in the description of the field.

In contrast, analog experiments yield very long time series. Also, typical dimensionless numbers for analog experiments are intermediate between what can achieved numerically today and what is geophysically relevant.

In this extended abstract, I argue that we can now await progress from a combination between the much improved monitoring of the Earth's magnetic field from satellites, numerical models of motions in the entire core dedicated to decadal and centennial time scales and laboratory experiments.

2. ROTATION FORCES VERSUS MAGNETIC FORCES

It has been customary to compare the Lorentz force $(\mathbf{j} \wedge \mathbf{B})$ and the Coriolis force $(2\rho(\mathbf{\Omega} \wedge \mathbf{u}))$, where $\mathbf{B}$ is the magnetic field, $\mathbf{j}$ the electrical current density, ρ the density, $\mathbf{\Omega}$ the Earth's rotation rate and $\mathbf{u}$ the velocity field, through the Elsasser number

$$\Lambda = \frac{\sigma B^2}{2\rho\Omega} = \frac{B^2}{2\rho\Omega\mu\eta} \ , \tag{1}$$

where η is the magnetic diffusivity $\eta = 1/\mu\sigma$, μ the magnetic permeability and σ the electrical conductivity of the iron alloy in the fluid outer core. The electrical current density $\mathbf{j}$ is eliminated from the expression of the Lorentz force through Ohm's law $\mathbf{j} = \sigma(\mathbf{E} + \mathbf{u} \wedge \mathbf{B})$, where $\mathbf{E}$ is the electrical field, taking $j \sim \sigma u B$.

It has long been assumed that planetary dynamos are in a magnetostrophic regime, characterized by $\Lambda \sim 1$, which ensures a balance between magnetic and rotation forces. When either the magnetic force or the rotation force is dominant, the motions are strongly constrained. Magnetoconvection studies have supported the idea that both the magnetic and rotation constraints are relaxed when $\Lambda \sim 1$. Taking Λ of order unity allows us to scale the magnetic field intensity with the rotation rate. That gives a reasonable estimate -$1.5\,10^{-3}$T- for the typical intensity of the Earth's magnetic field in the core interior (whereas an intensity of $\sim 3.\,10^{-4}$T at the CMB is directly inferred from main field modelling). I note below, however, that this picture has recently been challenged by a statistical analysis of numerical dynamo models.

In the context of the study of the Secular Variation, the important point is that Λ involves the magnetic diffusivity whilst the magnetic field changes are analysed using the frozen-flux approximation, i.e. neglecting magnetic diffusion. We can thus wonder whether Λ is appropriate to compare magnetic and rotation forces on diffusionless time scales. On rapid time scales, inertial waves and Alfvèn waves are set off when respectively the Proudman-Taylor constraint $\mathbf{\Omega}.\nabla\mathbf{u} = 0$ or the magnetic constraint $\mathbf{B}.\nabla\mathbf{u} = 0$ are violated. Because the frequency $O(\Omega)$ of inertial waves in the Earth's core, measured in days^{-1}, is much higner than the frequency $O(B/a\sqrt{\rho\mu})$ of Alfvèn waves, measured in years^{-1} (a is the the outer radius of the fluid core) we expect the rotation constraint to be much more efficient on rapid, diffusionless, time scales than the magnetic constraint.

I illustrate this idea with a numerical model of axisymmetrical motions in a spherical shell with same radius ratio as the Earth's core. The model has originally been developed to simulate the DTS experiment (see below). In this experiment, a permanent magnet is embedded in the solid inner core and imposes a dipole field decreasing as r^{-3} in the fluid shell. The directions of respectively the magnetic field and the rotation vector much differ. The model can be characterized by four dimensionless numbers, the kinetic Reynolds number *Re*, the Hartmann number *M*, the magnetic Prandtl number P_m and the magnetic Ekman number E_m:

$$P_m = \frac{\nu}{\eta}, \quad M^2 = \frac{a^2 B^2_{r=a}}{\rho\mu\eta\nu}, \quad E_m = \frac{\eta}{\Omega a^2}\ , \qquad (2)$$

where ν is the kinematic viscosity. The numerical illustration below is for $M = 420$, $P_m = 0.75$, $E_m = 2.10^{-4}$ and a small kinetic Reynolds number. Note that the magnetic diffusion time is on the order of $E_m^{-1}\Omega^{-1}$. An initial impulse is imposed to the rotation of the solid inner body and the subsequent evolution of the zonal motions in the fluid shell is monitored. After a transient (not shown here), we do check that the zonal motions have weak gradients in the direction parallel to the rotation axis (see figure 1). These motions propagate away from the inner core at a speed that depends on the magnetic field intensity. As this intensity decreases rapidly away from the inner core, the propagation slows down as larger radii are reached (compare figures 1 and 2).

On time scales short compared to the magnetic diffusion time of the core 10^4 years, the ratio λ between the periods of respectively the inertial waves and the Alfvèn waves appears the appropriate measure of the relative strength of the magnetic and rotation forces:

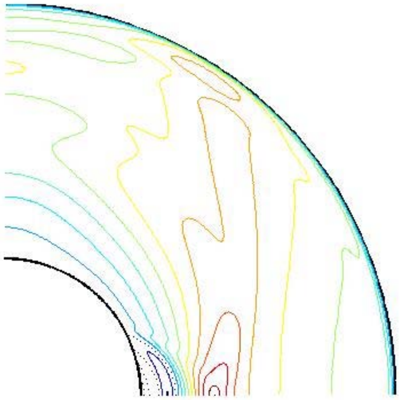

Figure 1. Distribution of angular velocity in the fluid shell after $7./2\pi$ revolutions. Warm and cool contours delineate respectively prograde -as the initial impulse- and retrograde angular velocities.

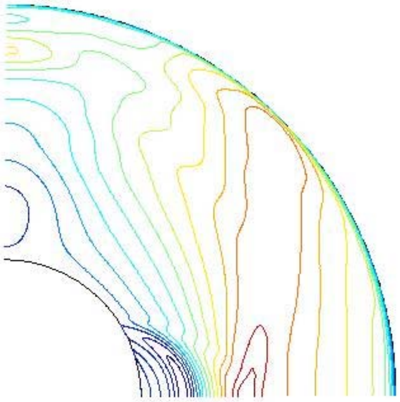

Figure 2. Distribution of angular velocity in the fluid shell after $14./2\pi$ revolutions.

$$\lambda = \frac{B}{a\sqrt{\rho\mu}\Omega} \qquad (3)$$

For the numerical illustration above, we have $\lambda = M\, E_m\, P_m^{0.5} \sim 7.\, 10^{-2}$ (and $\Lambda = \lambda^2\, E_m^{-1} \sim 25$). As we can estimate that $\lambda = O(10^{-4},\, 10^{-3})$ in the Earth's core, we can argue that the motions that take part to the secular variation of the Earth's magnetic field almost obey the Proudman-Taylor theorem and are only weakly sheared in the direction parallel to the rotation axis. In agreement with the reasoning developed here, Dumberry & Bloxham (2006) note that this is not necessarily the case on the longer archeomagnetic time scales.

The importance of the parameter λ has also been recently stressed by Christensen & Aubert (2006) in their study of the scaling laws for dynamos in rotating spherical shells. They find that the various diffusivities do not enter these laws. As a direct result, they suggest that the intensity of the magnetic field is better measured by the parameter λ, which does not involve diffusivities than by the Elsasser number Λ, which depends on the magnetic diffusivity.

In the following, I accept that motions, within the fluid outer core of the Earth, that are slow compared to the rotation rate of the Earth but rapid against the magnetic diffusion time (10^4 years) are strongly constrained by the Coriolis force. This result is well known for axisymmetric motions.

3. GEOSTROPHIC MOTIONS, PROPAGATION OF TORSIONAL ALFVÈN WAVES

Axisymmetric toroidal (zonal) motions that do not vary in the direction of $\mathbf{\Omega}$ are geostrophic. For these motions, which consist of rigid rotations of cylinders centered on the rotation axis, the Coriolis force is exactly equilibrated by the pressure force. The role of geostrophic motions within the Earth's fluid core has long been recognized. Indeed, as the different geostrophic cylinders are coupled together by the magnetic field permeating the Earth's core, the geostrophic motions evolve as torsional Alfvèn waves. These waves have decadal periods and take part to the motions at the core surface. Also, fluctuations in the rate of rotation of the solid mantle arise from the coupling between the torsional waves and the rotational motions of the mantle and of the solid inner core.

The propagation of torsional Alfvèn waves can be described through a slightly simplified equation:

$$s^3 z_T \frac{\partial^2 \left(\omega_g + \Omega\right)}{\partial t^2} = \frac{1}{\rho\mu}\frac{\partial}{\partial s}\left(z_T s^3 \frac{\partial \omega_g}{\partial s} \left\{B_s^2\right\}\right), \quad (4)$$

where $\omega_g(s)$ is the geostrophic angular velocity, z_T denotes the half-height of geostrophic cylinders and $\left\{B_s^2\right\}$ is the average taken on the geostrophic cylinders of B_s^2. It is illuminating to note that $(z_T s^3 \omega_g)$ gives a measure of the angular momentum density per unit of cylindrical radius. Thus, the torsional waves carry angular momentum and near the boundaries $(s = 0)$, $(s = a)$, the amplitude of the waves have to augment to compensate for the lower inertia $(z_T s^3)$. From velocity models at the core surface, Hide & Boggs & Dickey (2000) argue that, in the Earth's core torsional waves propagate inwards.

On their arrival at the tangent cylinder, an imaginary cylinder circumscribing the solid inner and aligned parallel to the Earth's spin axis, torsional waves split. The propagation continues separately in the two regions North and South of the solid inner core. The propagation speed can differ as do the averages $\left\{B_s^2\right\}$ taken on geostrophic cylinders respectively in the Northern and Southern hemispheres. The detection of geostrophic motions inside the cylinder tangent to the inner core from models of Secular Variation is an interesting challenge. Indeed, using magnetic field models that take strong values (up to $3.5\, 10^{-3}$ T.) at the inner core boundary, Mound & Buffett (2005) and Buffett & Mound (2005) give examples of torsional waves reflected back as they hit the tangent cylinder from the exterior, whilst I find that propagation well inside the cylinder tangent to the inner core is possible if the radial magnetic field at the inner core boundary is not much stronger than the magnetic field at the outer core boundary. The coupling between geostrophic motions inside the tangent cylinder and spin of the solid inner core becomes more efficient with increasing strength of the magnetic field at the inner core boundary and for decreasing difference between the electrical conductivities of the solid and fluid cores. On the other hand, a conducting layer at the bottom of the mantle leads to damping of all zonal motions that can be more efficient than the coupling with the inner core, even for moderate values of the integrated conductivity (conductance $\sim 10^8$ S.).

I have briefly summarized recent results on the propagation of torsional Alfvèn waves. Their excitation mechanism remains enigmatic.

4. TORSIONAL ALFVÈN WAVES EXCITED INSIDE THE EARTH'S CORE BECAUSE OF MODULATIONS BY THE SOLAR CYCLE OF THE ELECTRICAL CURRENTS FLOWING IN THE MAGNETOSPHERE

Depending on the electrical conductivity of the Earth's mantle, more or less intense electrical currents are induced in a thin layer at the top of the Earth's core by variations, over the solar cycle, of the magnetic field of external origin. For an eleven years cycle, the skin-depth of the diffusive layer attached at the CMB is of the order of 10 km.

If the distribution of electrical currents in the magnetosphere is modelled by a circular loop, time variation of the currents intensity generates an uniform magnetic field around the core. The electrical currents induced in the core by an axial field are zonal and do not contribute to the ϕ-component of the magnetic force $(\mathbf{j} \wedge \mathbf{B})_\phi$ that can accelerate geostrophic motions ((r, θ, ϕ) spherical coordinates, $\theta = 0$ defines the rotation axis). We can assume, however, that the electrical loop modelling currents in the magnetosphere is symmetrical about the Earth's dipole axis. Then the uniform time-varying field of external origin has an equatorial component that induces non azimuthal electrical currents in the core. They interact with the $m = 1$ part of the main field at the core equator. It can be checked that oscillations of the geostrophic cylinders near the equator of the Earth's core are readily excited by these time varying electrical currents. That sets off torsional Alfvèn waves, which propagate inwards. These waves take part to the fluid motions at the core surface where they interact with the main magnetic field. As a result, there is an interior magnetic field associated to any time varying external magnetic field with a long enough period.

The determination of the intensity of this induced field is difficult as it depends on the geometry and strength of the ambiant magnetic field in the core. It requires a numerical modelling. We can however be guided by a classical model where Alfvèn waves are generated by oscillations of an external source in a semi-infinite fluid region permeated by an uniform field transverse to the fluid solid-interface. For this model, there is equipartition between magnetic and kinetic energy. Thus, a magnetic perturbation of the order of $4.\,10^{-9}\ T.$ would correspond to a velocity perturbation of the order of only $4.\,10^{-8}\ m.s^{-1}$. Numerical simulations in spherical geometry give estimates of the velocity perturbations that are larger but would remain unimportant from a geophysical viewpoint.

This study was first motivated by reports of variations of the field of internal origin with 11yr periodicity. I think that it is important to assess the existence of such variations even if our core models are not able to reproduce their amplitude. I note indeed that solar activity signals in meteorological observations are also underestimated by Global Climate Models by up to a factor of three (Haigh, 2003) while the dynamics of the atmosphere is much better known than the dynamics of the core.

5. ANALOG EXPERIMENTS

The exploitation of dynamo experiments to shed light on the physics of the Earth's core has barely begun.

Analysing magnetic field measurements obtained in the Madison Dynamo Experiment, Spence et al. (2006) have shown that a dipole moment cannot be induced by an axisymmetric velocity field exposed to an axisymmetric magnetic field. This result stems from the calculation of the dipole moment as a function of electrical currents and sets apart once again the axial dipole field.

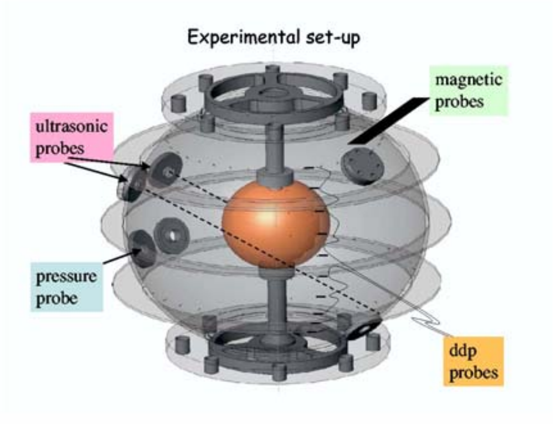

Figure 3. Overview of the DTS experiment showing the variety of possible measurements

An experiment (Nataf et al., 2006), partially devised in the image of the Earth's core, has just been built in Grenoble (figure 3). The rotations of the strongly magnetized inner body and of the outer sphere can be set independently. The outer sphere rotates beneath a magnetometer that is fixed. The electric potential is measured between electrodes set in the outer sphere made of stainless steel. A piezometer affixed also to the outer boundary monitors high frequency pressure changes. Using ultrasounds emitted from a probe inserted in the stainless steel wall and reflected back by particles dispersed in the liquid metal (sodium), the velocity in the fluid interior can be measured along two lines. The first study (Nataf et al., 2006) was devoted to the average state. Investigation of the time changes now begins. Correlations between differences of electrical potential at various locations have enabled to detect short-lived electrical currents that depend on ϕ as $e^{i2\phi}$, $e^{i3\phi}$. They can be characterized from only 1s of data while the experimental runs can last hours. The reorganization time of the distribution of electrical currents is thus much shorter than the duration of experimental runs, allowing us to gain an insight into the dynamics of liquid metals in presence of rotation and magnetic field that cannot be obtained from geomagnetic data. Inferences based on data collected at the surface will also be confronted to velocity measurements in the fluid. Thus, we may eventually test techniques developed to analyse geomagnetic records. In spite of its small size (42cm-diameter), the experiment cannot be simulated numerically. As an example, it is possible to calculate models, for short spans of time, with all the parameters appropriate to the experiment but the rotation rates of the two spheres, by a factor 10^2. Parameters of laboratory experiments are intermediate between those respectively of the Earth's core and of direct numerical simulations.

6. CONCLUSION

Assuming that the motions responsible for the secular variation of the Earth magnetic field are strongly constrained by the rotation of the planet opens new questions. We can wonder whether these motions have a preferred scale as it is usual in geophysical flows. Also, can we assume that the effect of the rotation forces is to favour motions in the azimuthal direction? Finally, can we rely on studies of geophysical turbulence to make a priori assumptions on the spectrum of the motions?

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