Magnetic diffusion patches at the top of the Earth's core

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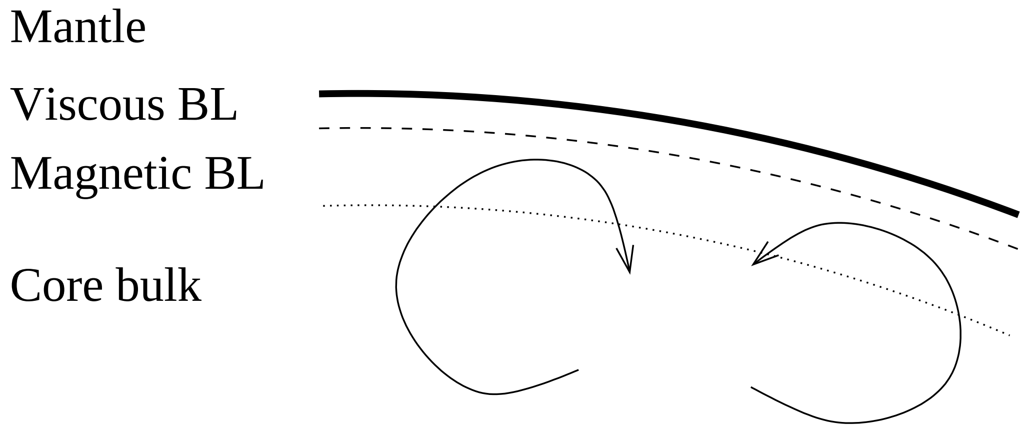
The frozen-flux assumption

Magnetic induction equation:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$



Boundary layer	Thickness	$[B_r]/B$	$[B_H]/B$
viscous	30 cm	10^{-7}	10^{-4}
magnetic	60 km	10^{-2}	1

(Jault & Le Mouël, 1991)

$$\begin{cases} T_B << T_d \approx 30000 \text{ yrs} \\ L_B \approx L_u \approx 1000 \text{ km} \end{cases} \Longrightarrow \frac{\partial B_r}{\partial t} = -\nabla_H \cdot (\mathbf{u} B_r)$$

(Roberts & Scott, 1965)

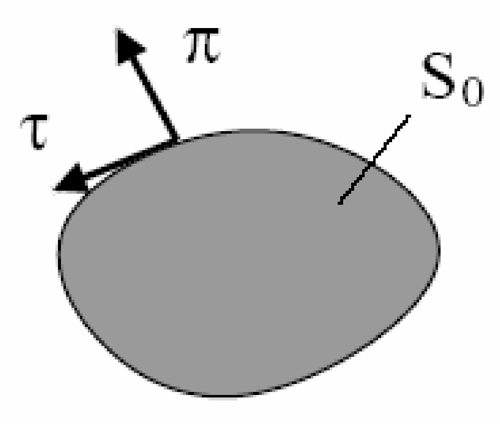
Backus' constraints

Necessary and sufficient conditions for the CMB field to be compatible with the frozen-flux assumption:

$$\int_{S_0} \frac{\partial B_r}{\partial t} dS = 0$$

$$\left(\frac{\partial B_r}{\partial t} \right)_C = 0$$

(Backus, 1968)



S_0: null-flux curve $B_r = 0$

C: critical point (intersection of 2 nfc)

Data

Ørsted scalar and vector data

Same selection as OIFM (Olsen *et al*. 2000):

- night side data (local time about 22:00)
- quiet conditions ($Kp < 1+$, $|Dst| < 10$ nT, $|d(Dst)/dt| < 3$ nT/h)
- scalar-only data at high latitudes

Analysis

- **Constructing constrained core field models**

Method = Iteratively Reweighted Least-Squares.

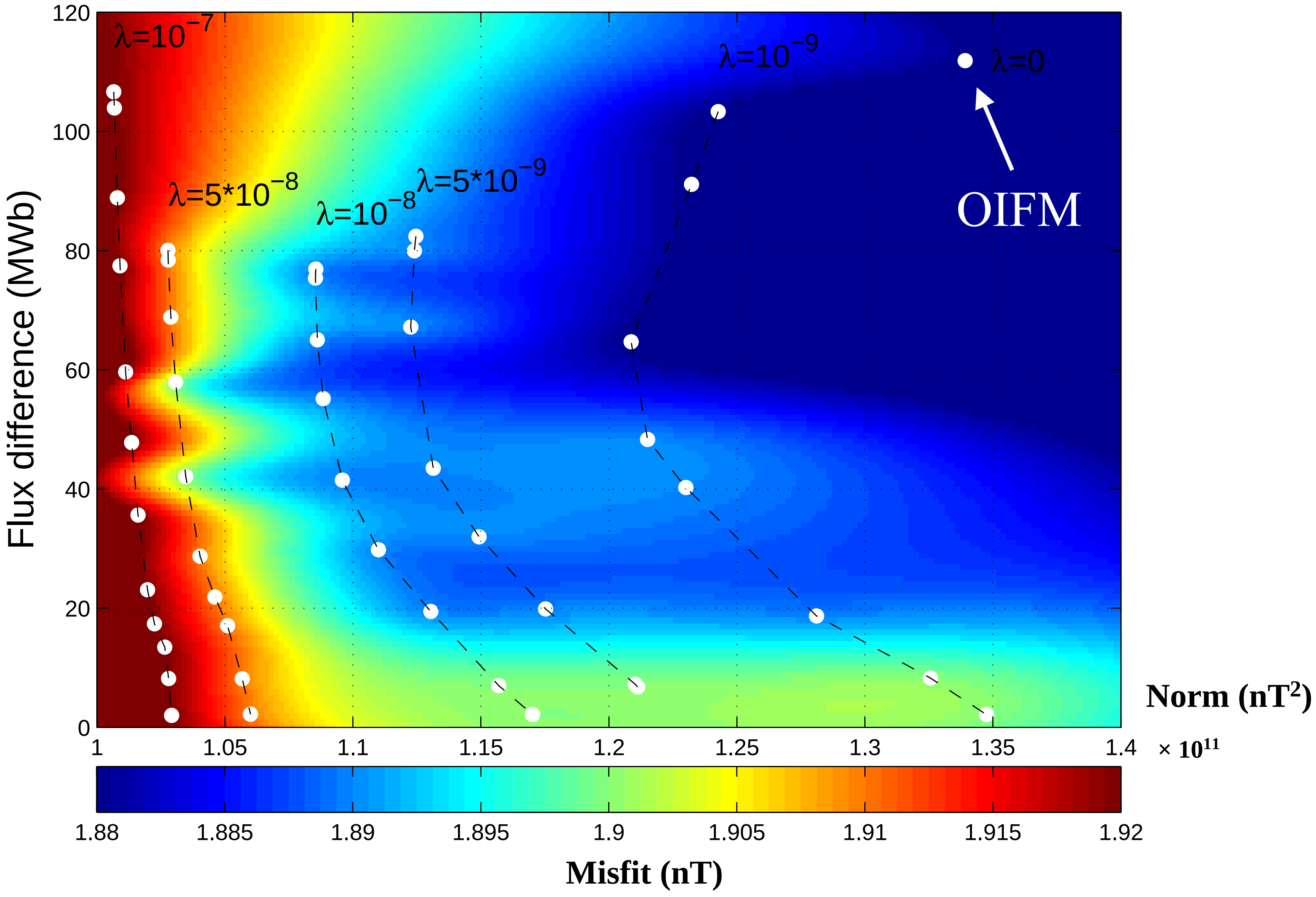
Minimizing the function

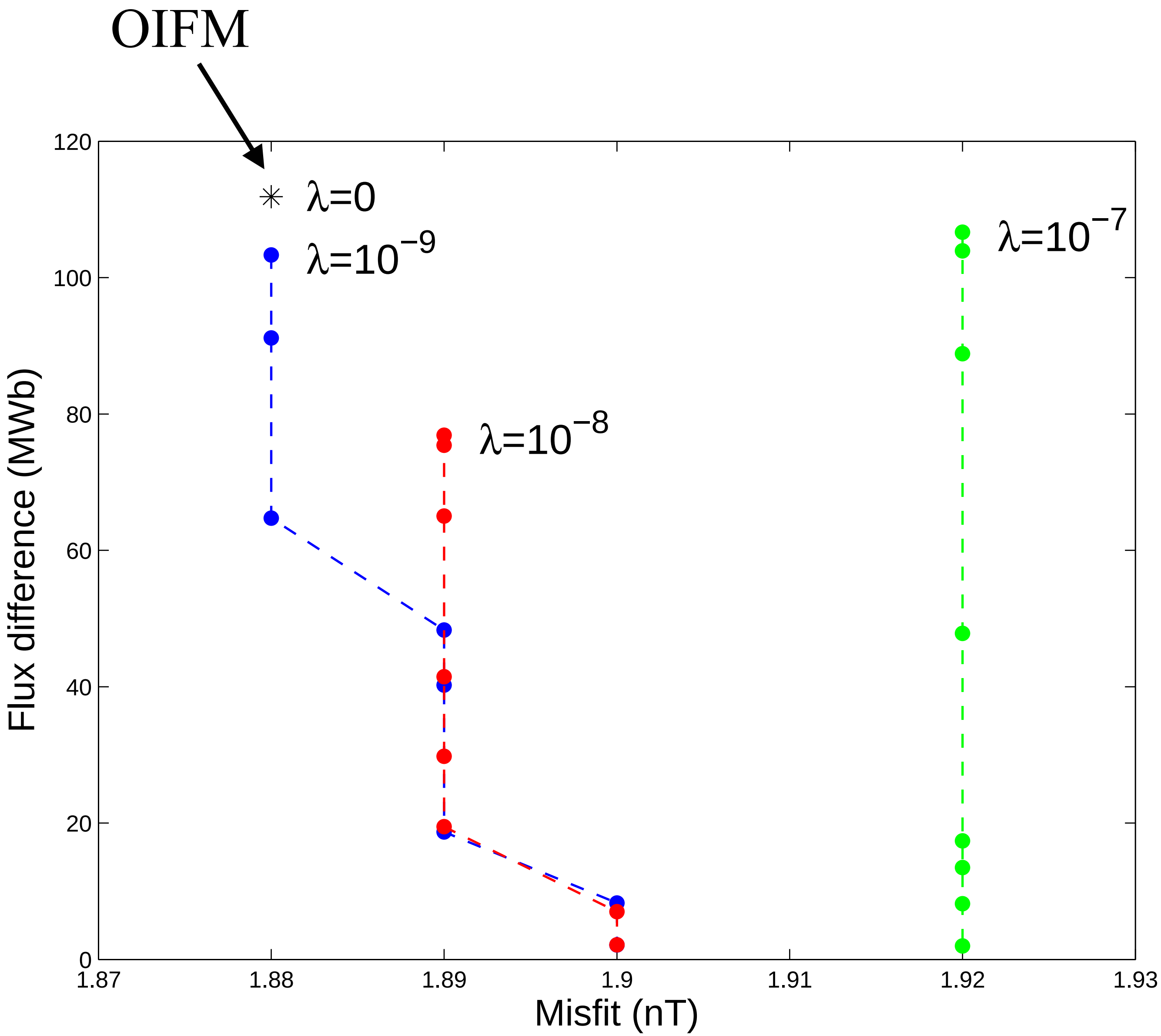
$$\Phi(\mathbf{m}) = [\gamma - \mathbf{f}(\mathbf{m})]^T \mathbf{C}_e^{-1} [\gamma - \mathbf{f}(\mathbf{m})] + \lambda\, \mathbf{m}^T \mathbf{C}_m^{-1} \mathbf{m} + \mu\, |\mathbf{L}^T(\mathbf{m})\mathbf{m} - \mathbf{F}_0|^2$$

λ = damping parameter
μ = constraint parameter
$\mathbf{L^T(m)}$ = constraint matrix
$\mathbf{F_0}$ = fluxes of reference model (CM4 in 1980)

- **Exploring the space of the (λ, μ) parameters**

Misfits of OIFM-like models



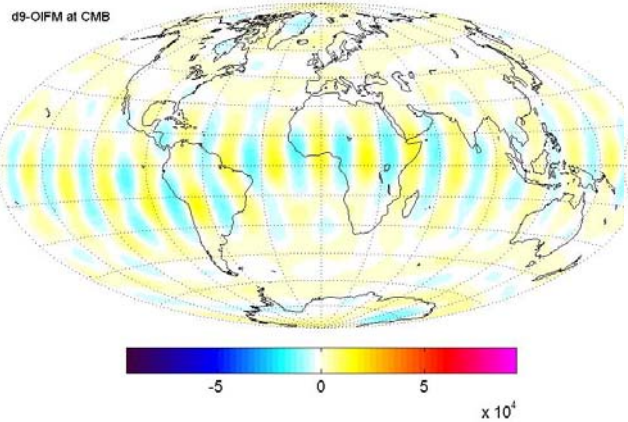


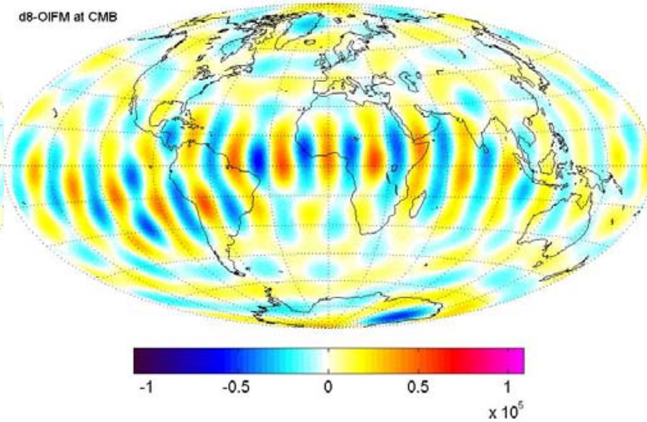
OIFM
λ=0
λ=10^{−9}
λ=10^{−8}
λ=10^{−7}
Flux difference (MWb)
Misfit (nT)
0
20
40
60
80
100
120
1.87
1.88
1.89
1.9
1.91
1.92
1.93

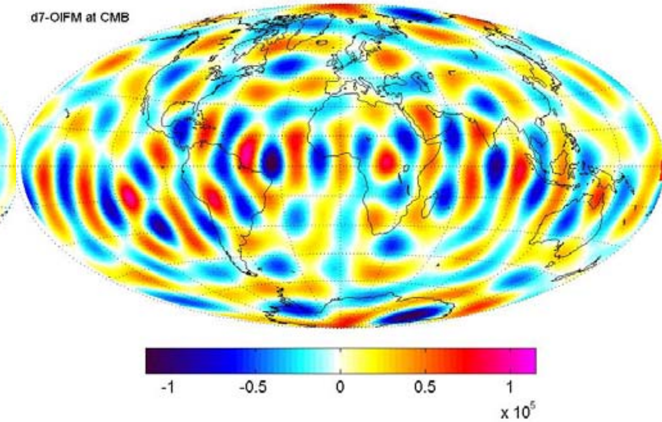
Effect of the damping on B_r

Differences with OIFM at the CMB:

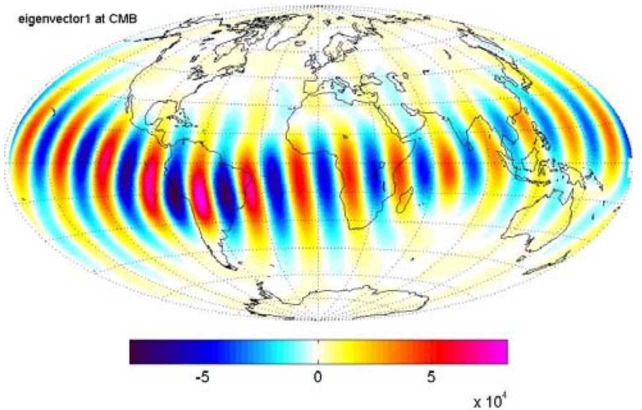
$\lambda = 10^{-9}$ $\lambda = 10^{-8}$ $\lambda = 10^{-7}$





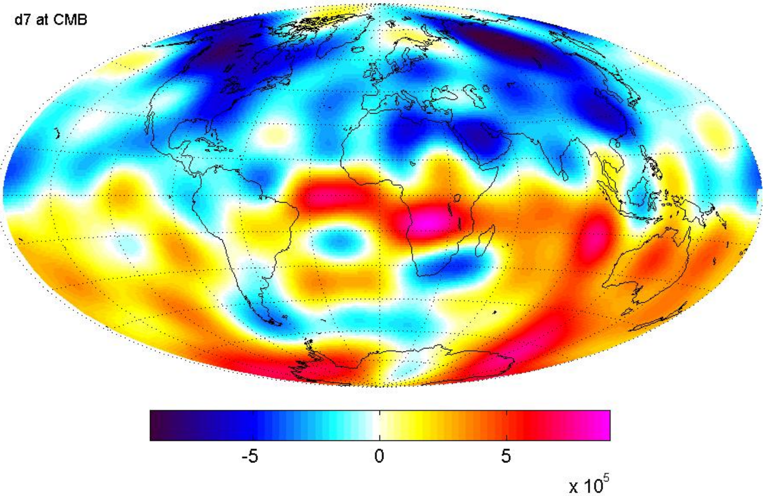


Eigenvector least constrained by the data:
(smallest eigenvalues of G^TG)

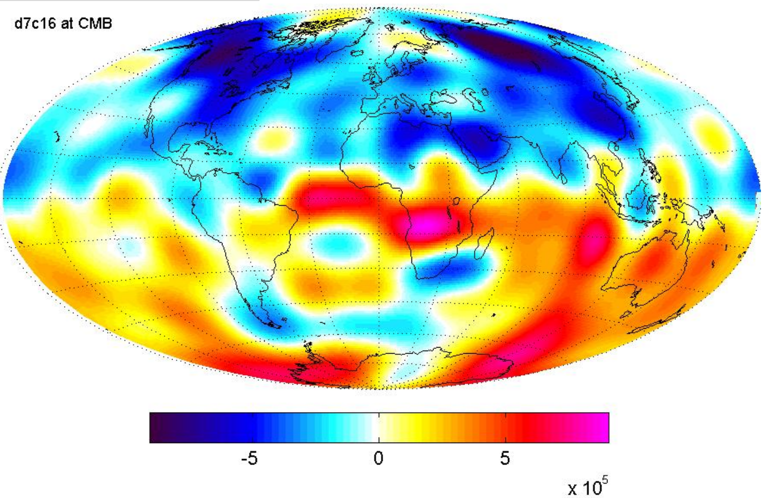


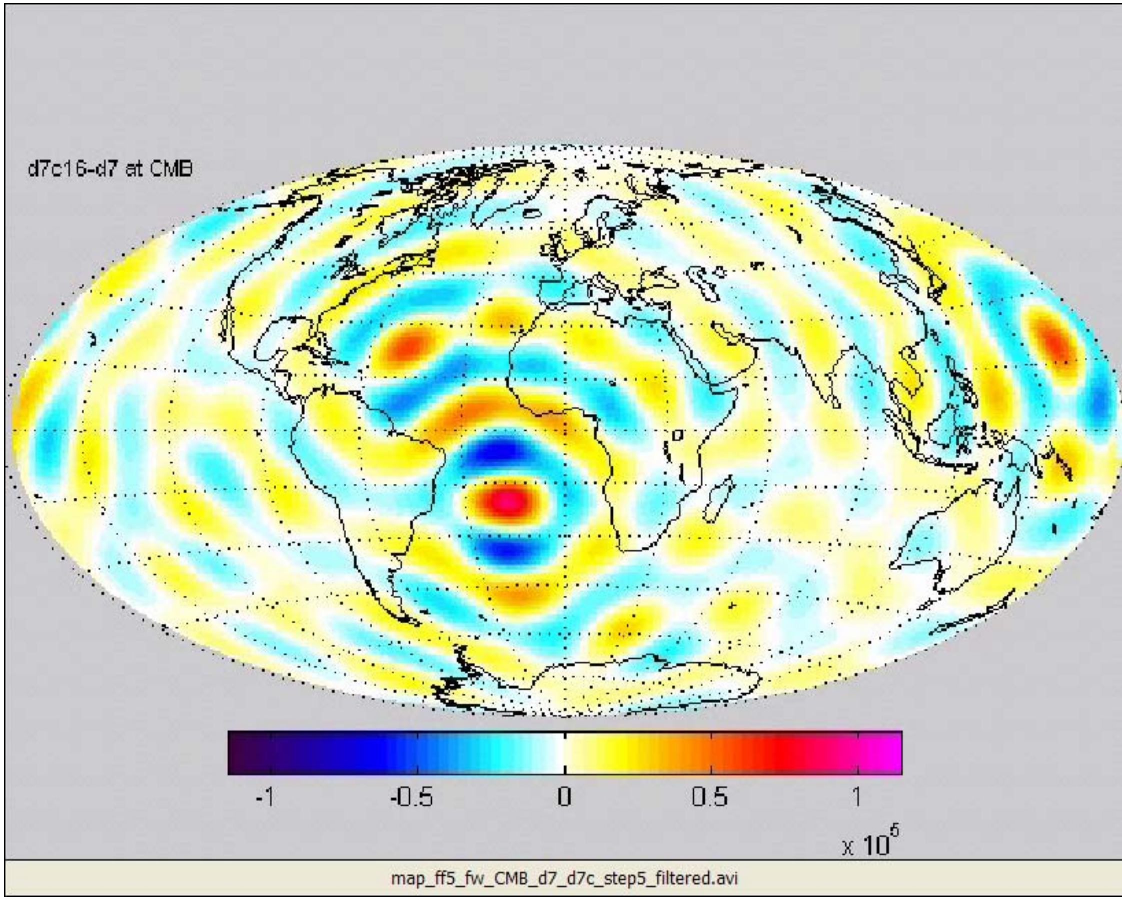
Effect of the FF constraint, $\lambda = 10^{-7}$

$\mu = 0$

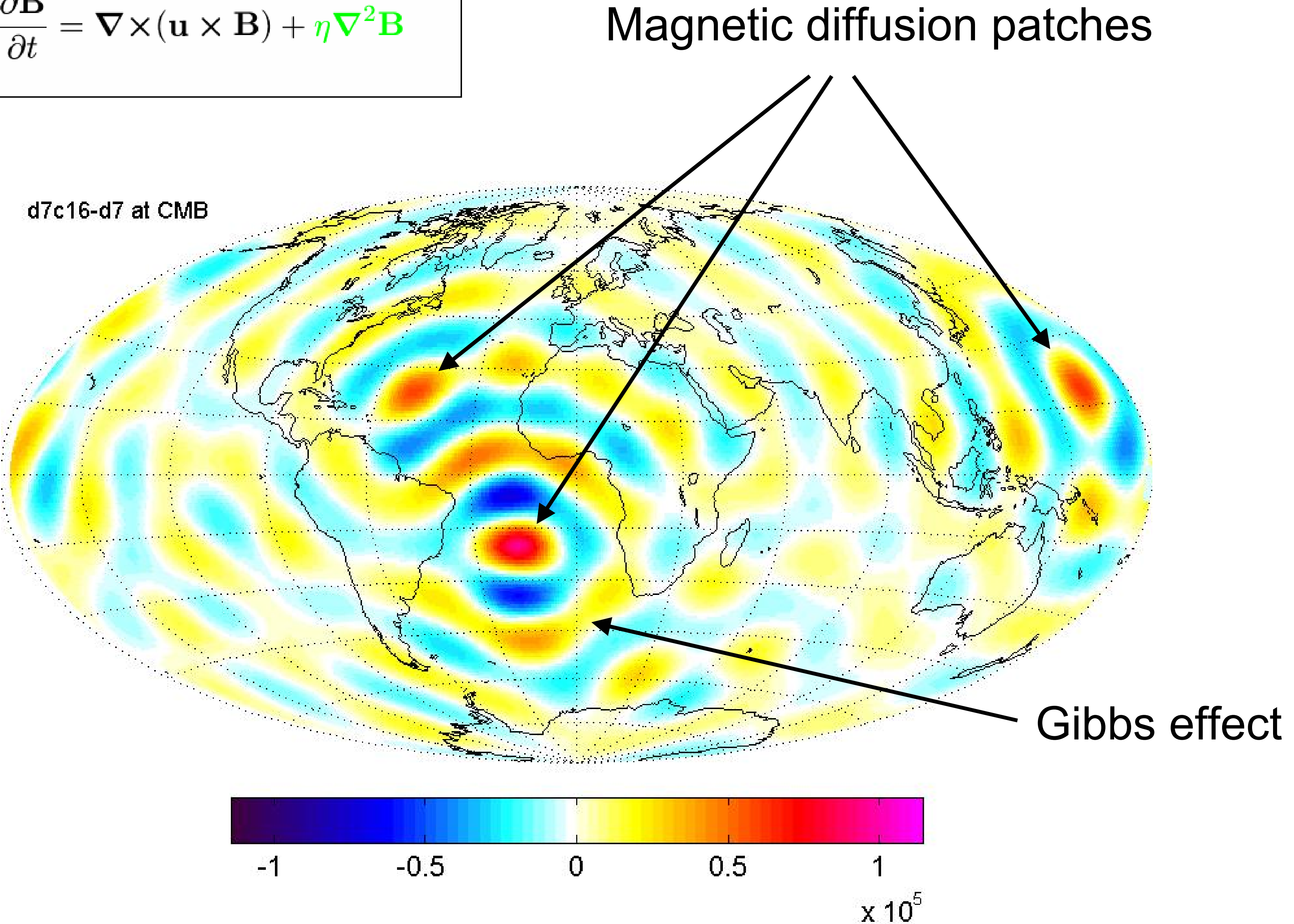


$\mu = 10^{-16}$

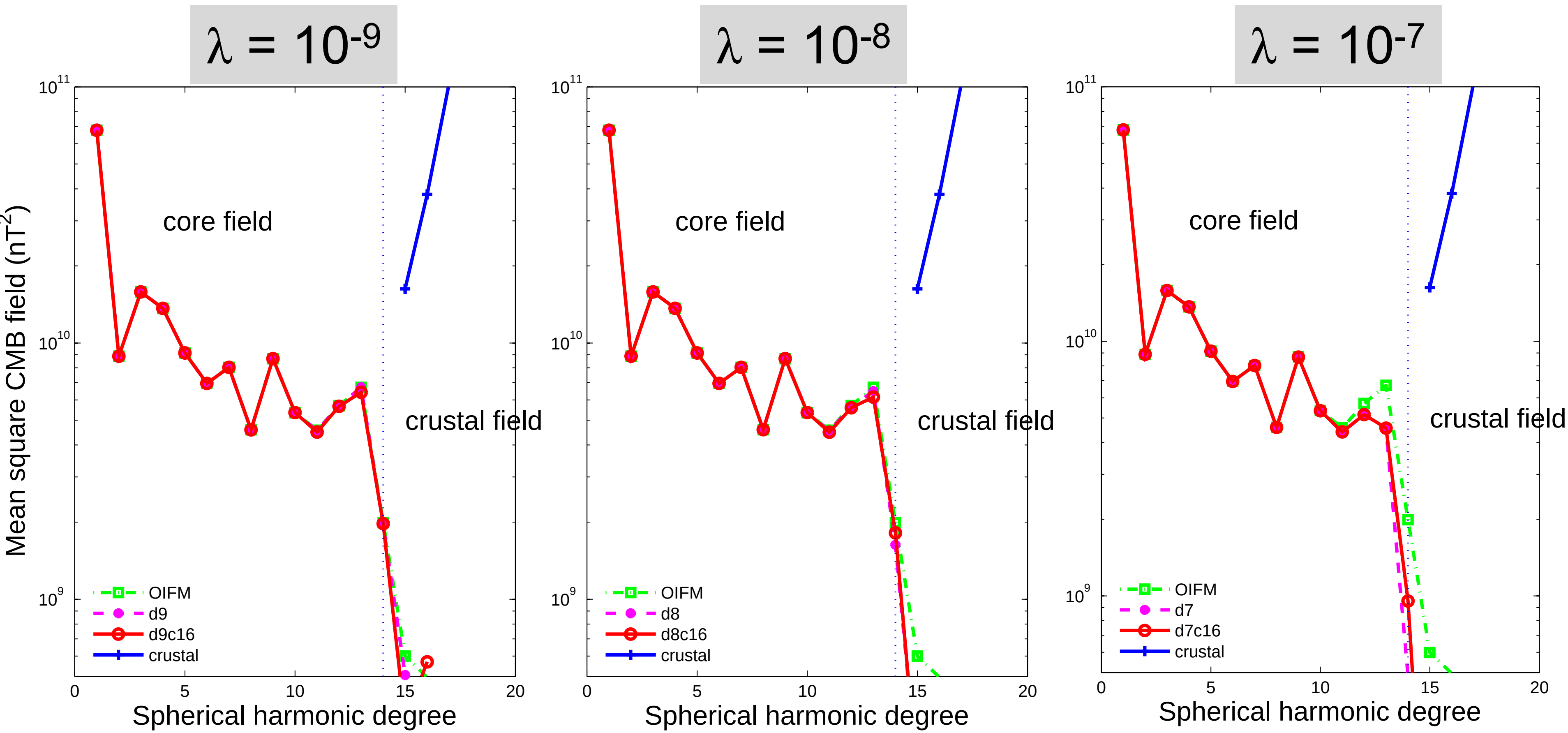




$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$



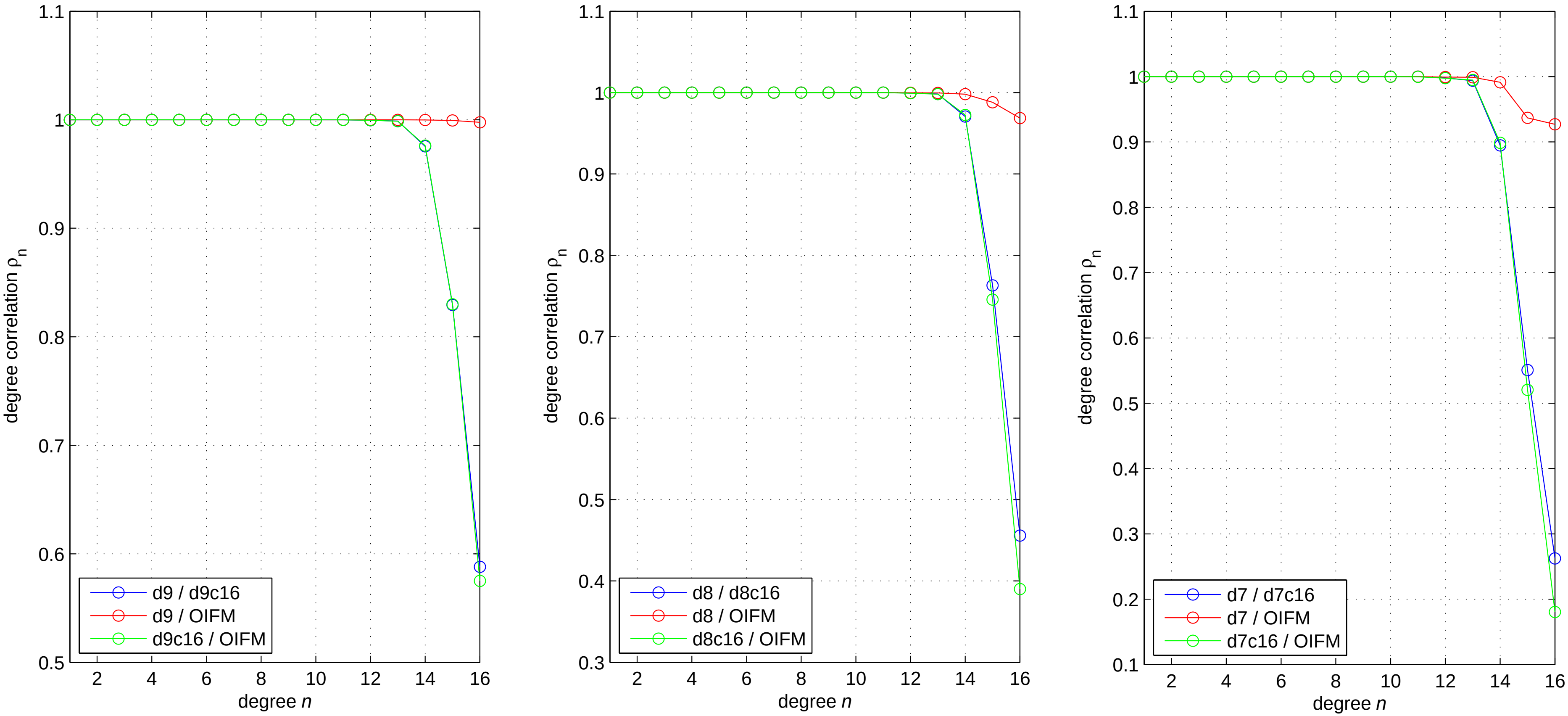
Power spectra at the CMB



=> The damping and FF constraint have an effect on the highest degrees only.

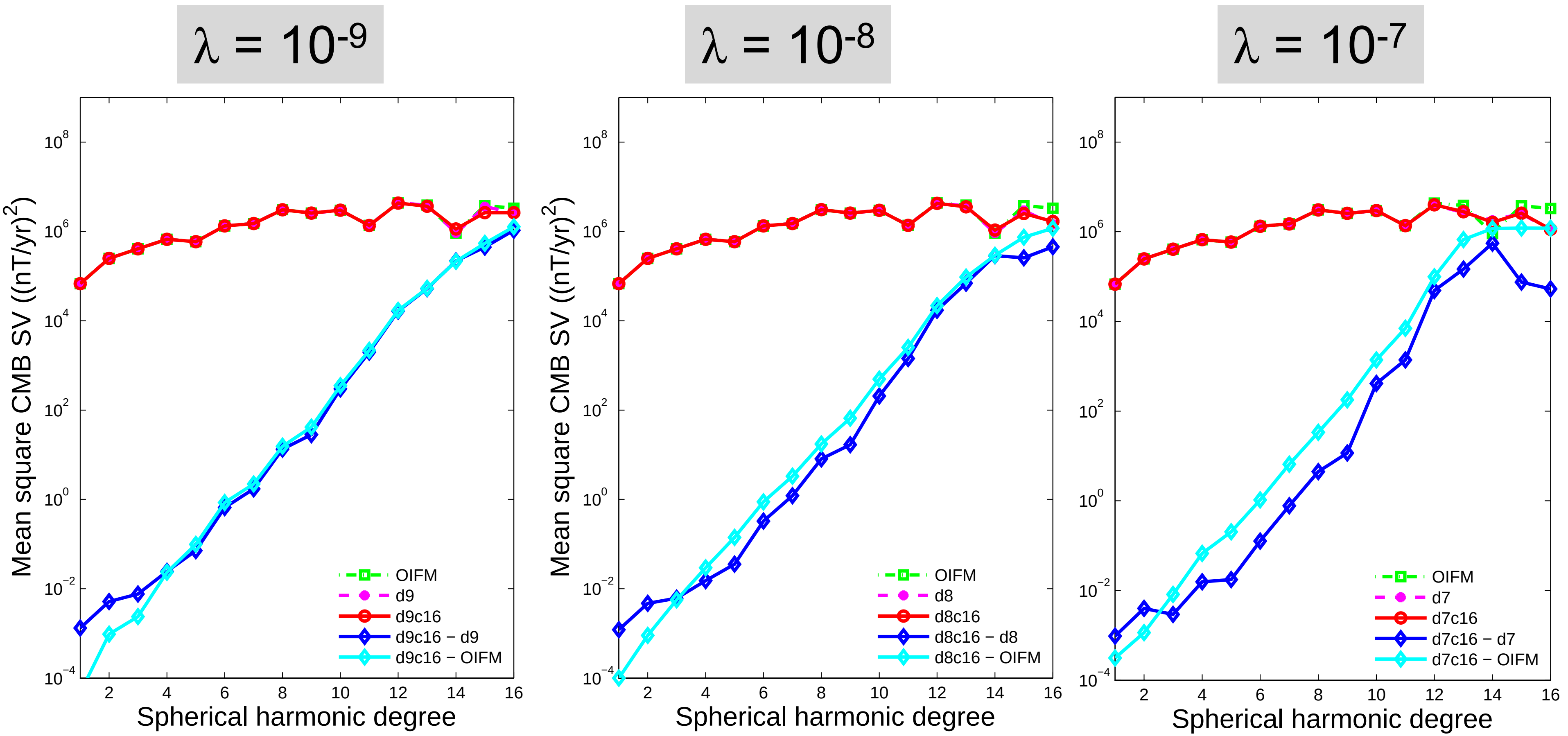
Degree correlations

$\lambda = 10^{-9}$ | $\lambda = 10^{-8}$ | $\lambda = 10^{-7}$



=> The FF constraint has an effect on the degrees larger than 12 ($\lambda = 10^{-7}$) or 13 ($\lambda = 10^{-8}$ and 10^{-9}).

Average SV power spectra at the CMB



=> For $\lambda = 10^{-7}$, the FF constraint has its largest effect on degree 14.

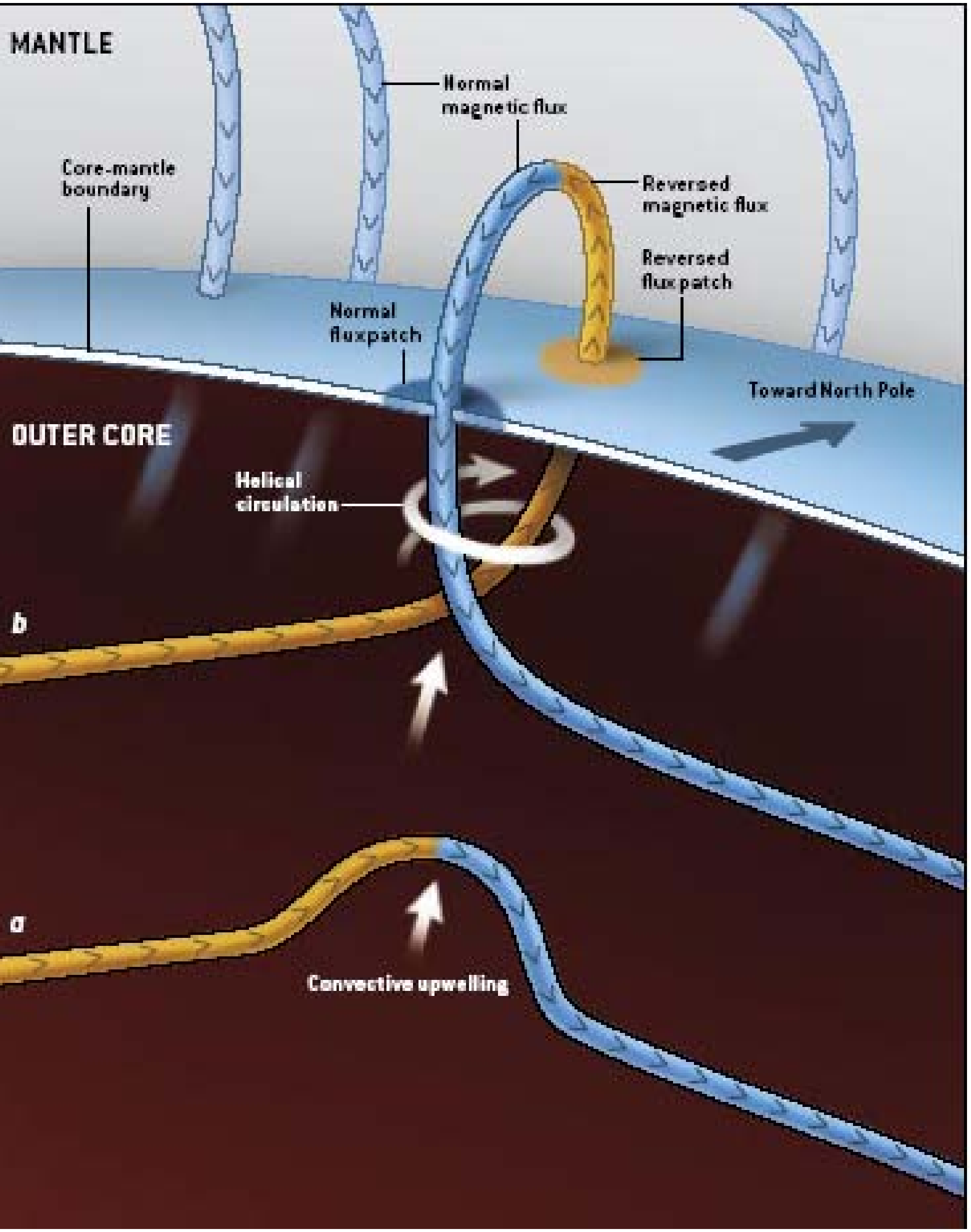
Conclusions

- The FF assumption is compatible with MAGSAT and Ørsted data over 20 yrs.

- But magnetic diffusion is also compatible with MAGSAT and Ørsted data, provided it occurs within three identified patches at the CMB (including one under South Atlantic).

- Within the identified patches, the SV due to magnetic diffusion could reach up to 50 % of the CMB field over 20 yrs. The corresponding length scale would be

$$\frac{\partial B_r}{\partial t} \approx \eta \frac{B_r}{l^2} \quad \Rightarrow \quad l \approx \sqrt{\eta \frac{B_r}{\left(\partial B_r / \partial t\right)}} \approx 35\ km$$



(Glatzmaier & Olson, 2005)

Diffusion could result from the formation of reversed flux patches at the CMB.